Computational Geometry Lectures

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Bibliography

Books

[30, 19, 18, 45, 66, 70]

1 Convex hull: definitions, classical algorithms.

- Definition, extremal point. [66]
- Jarvis algorithm. [54]
- Orientation predicate. [55]
- Buggy degenerate example. [36]
- Real RAM model and general position hypothesis. [66]
- Graham algorithm. [50]
- Lower bound. [66]
- Other results. [67, 68, 58, 56, 8, 65, 7]
- Higher dimensions. [82, 25]

2 Delaunay triangulation: definitions, motivations, properties, classical algorithms.

- Space of spheres [31, 32]
- Delaunay predicates. [42]
- Flipping trianagulation [52]
- Incremental algorithm [57]
- Sweep-line [48]
- Divide&conquer [74]
- Deletion [35, 20, 37]

3 Probabilistic analyses: randomized algorithms, evenly distributed points.

- Randomization [29, 73, 14]
  - Delaunay tree [16, 17] and variant [51]
  - Jump and walk [63, 62]
  - Delaunay hierarchy [34]
  - Biased random insertion order [6]
  - Accelerated algorithms [28, 72, 33, 26]
- Poisson point processes [71, 64]
  - Straight walk [44, 27]
  - Visibility walk [39]
  - Walks on vertices [27, 41, 59]
  - Smoothed analysis [75, 38]

4 Robustness issues: numerical issues, degenerate cases.

- IEEE754 [49]
- Orientation with double [55]
- Solution 1: no geometric theorems [76]
- Exact geometric computation paradigm [81]
- Perturbations
  - controlled [61]
  - symbolic: SoS [46], world [2], 4^{th} dim [43], "qualitative" [40]

5 Triangulations in the CGAL library.

- `https://doc.cgal.org/latest/Manual/packages.html#PartTriangulationsAndDelaunayTriangulations`
- CGAL [47]
- Triangulations [13, 77, 1]

6 Applications: reconstruction, meshing.

- Reconstruction [24]
  - Crust 2D [5]
  - Crust 3D [3, 4]
- Meshing [12]
  - Ruppert [69]
  - Off-centers [80]
  - Smoothing [78, 79]

7 Triangulations in non-Euclidean spaces.

- Delaunay triangulations of abstract compact manifolds [15]
- Delaunay triangulations of compact flat manifolds [60, 23]
- Delaunay triangulation on the sphere [22]
- Some background on hyperbolic geometry [21]
- Delaunay triangulations in hyperbolic space [9, 11]
- Delaunay triangulations of hyperbolic manifolds [10, 53]

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