Appendix

Let us complete here the proof of Proposition 1.

The *InCircle* predicate.

Here we consider that p_4 is always inside $\mathcal{D}$, so we need to examine cases for the other three points.

- All points are inside $\mathcal{D}$. In this case, all the arguments of the predicate are rational and from (4) we get a rational polynomial expression of total degree 4.
- Three points are inside $\mathcal{D}$—let $p_2, p_3, p_4 \in \mathcal{D}$, and suppose p_1 is outside $\mathcal{D}$. As for the *Orientation* predicate, we have a total of 14 cases.
- Two points are inside $\mathcal{D}$—we consider $p_3, p_4 \in \mathcal{D}$. Both p_1 and p_2 can be images of input points under translations around V_0 and V_1. We get 56 cases.
- Only p_4 is inside $\mathcal{D}$—here p_1, p_2 and p_3 can be images of input points under all translations around V_0 and V_1. Again avoiding redundancies, the total number of cases is 168 (84 combinations around V_0 and another 84 around V_1).

Similarly to the *Orientation* predicate, in all listed cases the expressions resulting from (4) have strictly positive denominators and their numerators can be brought into a form (5). Here the maximum total degree of the expressions A, B, C, D is 18. By squaring twice to eliminate square roots, we get degree 72.

The *SideOfOctagon* predicate. This predicate is much simpler than the previous two ones, as it only takes one point p as argument. Taking the symmetries into account, we reduce the number of sides of the octagon to test p against by rotating it to a point p' in the first octant: $x' = |x|, y' = |y|$, and if $y' > x'$ we swap them. Then we test whether p' is outside both K_0 and K_1. Since some sides of $\mathcal{D}$ are open and some are closed, if p' lies on K_0 or on K_1, we must take into account the octant in which p lies to conclude: if $y < -\tan(\pi/8)x$, then p is inside $\mathcal{D}$; otherwise, it is outside.

Evaluating whether p' lies in K_0 (resp. K_1) is a call to the *InCircle* predicate with V_0, M_0 and V_1 (resp. V_1, M_1, V_2) as other three arguments (the coordinates of these points were given in Table 2). For these specific points, the maximum algebraic degree of A, B, C, D in the expressions of the form (5) is 2, and by squaring twice we get algebraic expressions of total degree 8 in the input coordinates.