Exercices 23 septembre 2021

2.1 Dessiner...

2.1 Correction:
2.2 Diameter

Let $S$ be a set of points in the plane. The diameter of $S$ is the pair of point in $S^2$ that realizes the largest distance (assume no degenracies).

The disjointness problem is: “given two sets of $n$ real numbers in some interval, determine if if the intersection of the two sets is non empty”

**Theorem:** The disjointness problem has an $\Omega(n \log n)$ lower bound in the real-RAM model.

2.2.1 Diameter lower bound

Prove that the diameter problem has an $\Omega(n \log n)$ lower bound in the real-RAM model.

Hint: design a stupid algorithm for the disjointness problem for two set of numbers in $[0, \frac{\pi}{2}]$.

2.2 Correction:

Stupid algorithm for the collision problem:

Let $\alpha_i$ and $\beta_i$ be two sets of $n$ numbers in $[0, \frac{\pi}{2}]$.

Create $2n$ points $p_i = (\cos \alpha_i, \sin \alpha_i)$ and $q_i = (-\cos \beta_i, -\sin \beta_i)$.

Solve the diameter problem on this two set of points.

If the diameter $p_iq_j$ has length 2 answer that $\alpha_i = \beta_j$ as a witness of non empty intersection.

If the length is strictly less than 2, then answer “empty intersection”.

The complexity of this algorithm is linear plus the complexity of solving the diameter problem, thus the diameter problem cannot be solved faster than $n \log n$ without contradicting the lower bound on the disjointness problem.