2 Delaunay triangulation: definitions, motivations, properties, classical algorithms.

2.1 Drawing

Draw the Delaunay triangulation of the attached point set.

2.2 Nearest neighbors

S a set of n points.

Denote $NN(q)$ the nearest neighbor of q in $S \setminus \{q\}$ (q may belong to S or not).

For a point q, cut the plane in four quadrants using the two lines through q of slopes 1 and -1, and call these quadrants North, West, South, and East. Denote $NN_N(q)$, $NN_W(q)$, $NN_S(q)$, and $NN_E(q)$ the nearest neighbor of q in $S \setminus \{q\}$ in each quadrant.

2.2.1 Nearest neighbor

For $q \in S$, is $NN(q)$ a neighbor of q in the Delaunay triangulation of S ? Prove it or draw a counter example.

If the answer is no, then prove that there exist w a Delaunay neigbor of q such that the distance $\|qw\|$ is not larger than $\alpha \cdot \|qNN(q)\|$ for some constant α. Give the best value for α.

2.2.2 Oriented nearest neighbor

For $q \in S$, is $NN_N(q)$ a neighbor of q in the Delaunay triangulation of S ? Prove it or draw a counter example.

If the answer is no, then prove that there exist w a Delaunay neigbor of q in the half-plane above q such that the distance $\|qw\|$ is not larger than $\alpha \cdot \|qNN_N(q)\|$ for some constant α. Give the best value for α.

2.2.3 Nearest neighbor path

Let T be a triangulation of S, p be a point (not in S) and $q \in S$. Define the sequence $(q_i)_{i\in\mathbb{N}}$ by $q_0 = q$ and q_{i+1} the closest point of p amongst the neighbors of q_i in the triangulation T, and q_i itself (assume no ties).

What are the possible behavior of this sequence ? Does it have a limit ? If yes, what is this limit ?

If T is not any triangulation but the Delaunay triangulation, what are the possible behavior of this sequence ? Does it have a limit ? If yes, what is this limit ?

2.3 Diameter

Let S be a set of points in the plane. The diameter of S is the pair of point in S^2 that realizes the largest distance (assume no degenracies).

The collision problem is: "given a two set of n real numbers in some interval, determine if if the intersection of the two sets is non empty"

Theorem: The collision problem has an $\Omega(n \log n)$ lower bound in the real-RAM model.

2.3.1 Diameter lower bound

Prove that the diameter problem has an $\Omega(n \log n)$ lower bound in the real-RAM model.

Hint: design a stupid algorithm for the collision problem for a set numbers in $[0, \frac{\pi}{2}]$.

Draw the Delaunay triangulation