

4 Robustness issues: numerical issues, degenerate cases.

4.1 double arithmetic

(Assume rounding mode is to the nearest representable `double`).

4.1.1 Multiplication

For real numbers we have

$$\forall a, b, c \in \mathbb{R}, a, b, c > 0 \quad a < b \Rightarrow a \cdot c < b \cdot c$$

Now if `a`, `b`, and `c` are three non negative `double` such that `(a<b)` evaluates to `true`.

— Is `a*c<b*c` always `true` ? (Prove or give a counter-example [write numbers in binary])

— Is `a*c<=b*c` always `true` ? (Prove or give a counter-example [write numbers in binary])

4.1.2 Integers in double

Let $x_1, x_2, x_3, y_1, y_2, y_3$ integers between -2^b and 2^b .

Find the largest value of b so that you can prove that the expressions

$$(x_2 - x_1) * (y_3 - y_1) - (x_3 - x_1) * (y_2 - y_1)$$

and

$$x_2 * y_3 + x_3 * y_1 + x_1 * y_2 - x_3 * y_2 - x_1 * y_3 - x_2 * y_1$$

certainly evaluates the same.

4.1.3 A function

What does the following function return when called on a `double` in the open interval $] -2^{51}, 2^{51}[$?

```
double WhoAmI{double x}
{
    double a = 6755399441055744.0;    // 2^51 + 2^52
    double s = x+0.5+a;
    double r = s-a;
    return r;
}
```

4.2 Segment intersection

Let S_1 and S_2 be two line segments with endpoints (x_1, y_1) , (x'_1, y'_1) , (x_2, y_2) , and (x'_2, y'_2) .

4.2.1 Orientation

Recall the expression of the orientation predicate: `is_ccw`($x_p, y_p, x_q, y_q, x_r, y_r$).

4.2.2 Predicate for segment intersections

Write the predicate testing if S_1 and S_2 intersect using calls to `is_ccw`.