

# Modèles d'environnements, planification de trajectoires

Olivier Devillers

Francis Colas

Computational geometry

Robotique

*Inria*

24 heures de cours (12x 2 heures)

<https://members.loria.fr/Olivier.Devillers/master/>

## Notation

Contrôle continu 40% [pas de rattrapage]

Exam 3h 30%, exposé sur article 30%

# Computational geometry



# Computational geometry

Design geometric algorithms

# Computational geometry

Design geometric algorithms

Study complexity

# Computational geometry

Design geometric algorithms

Study complexity

Model of computation

Worst-case or random analysis

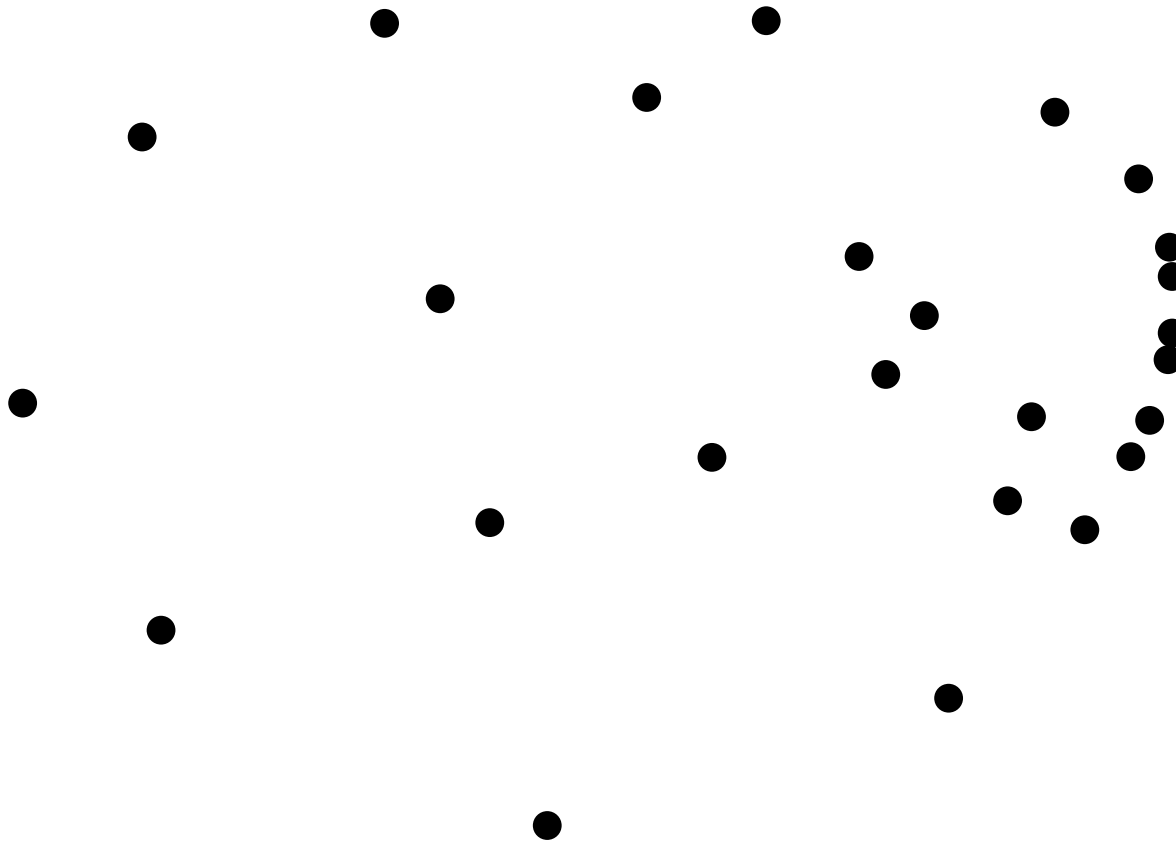
Lower bound

Asymptotic analysis

# Computational geometry problems

# Computational geometry problems

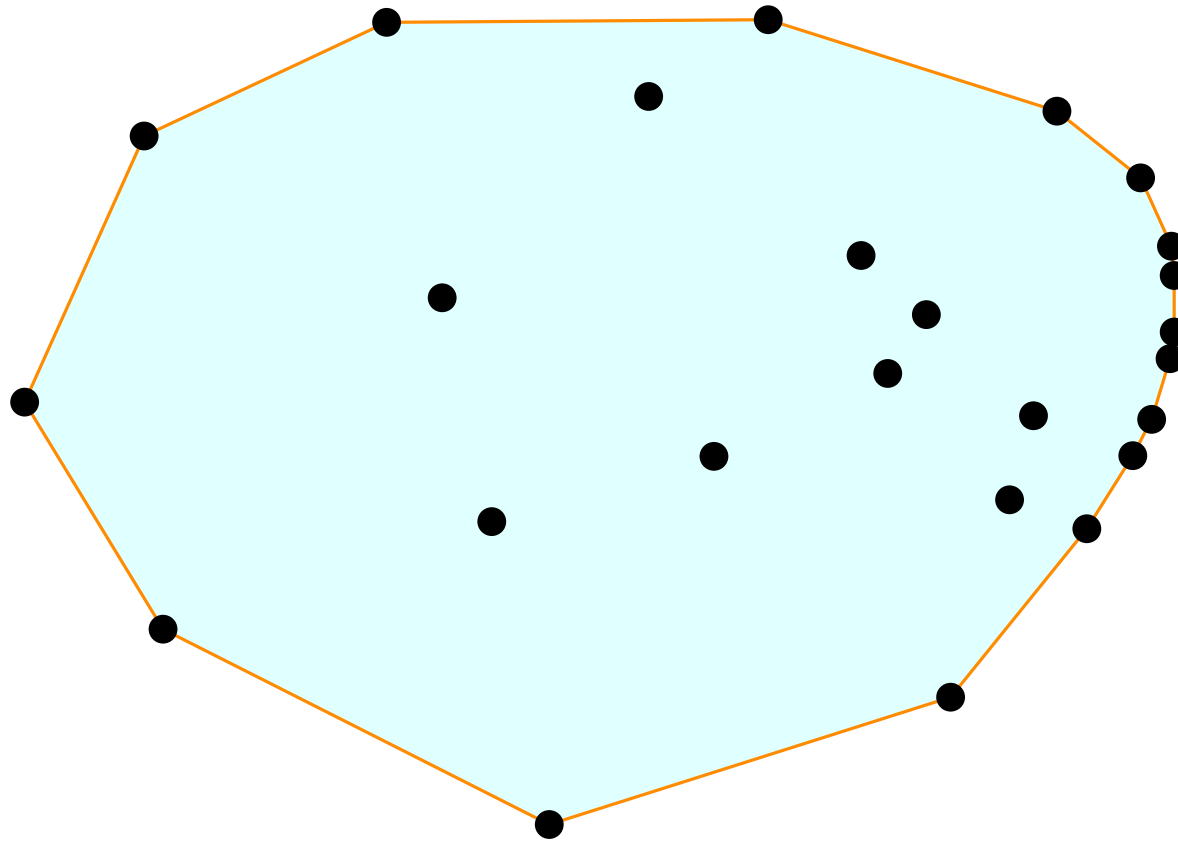
## Convex hull





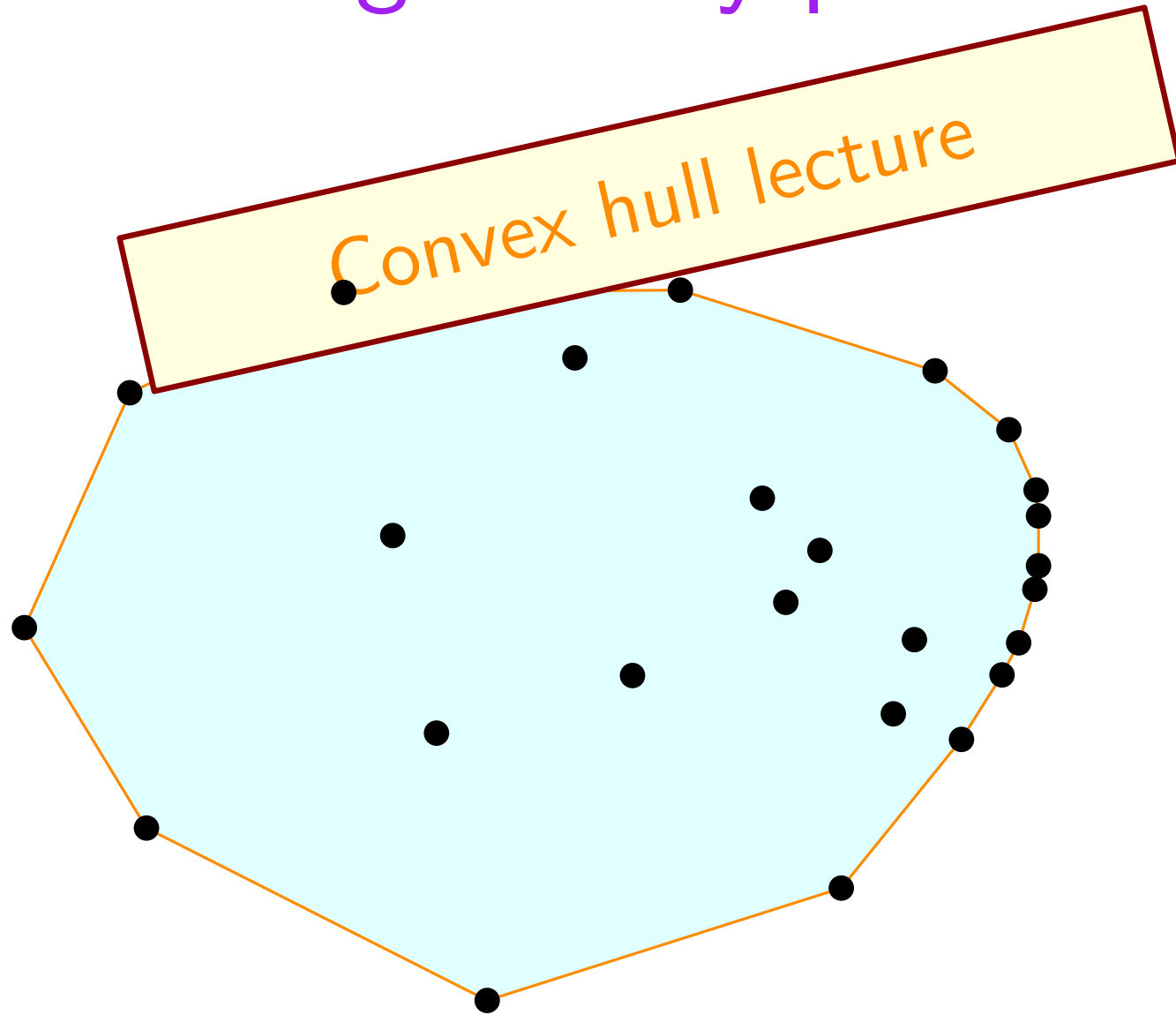
# Computational geometry problems

## Convex hull



# Computational geometry problems

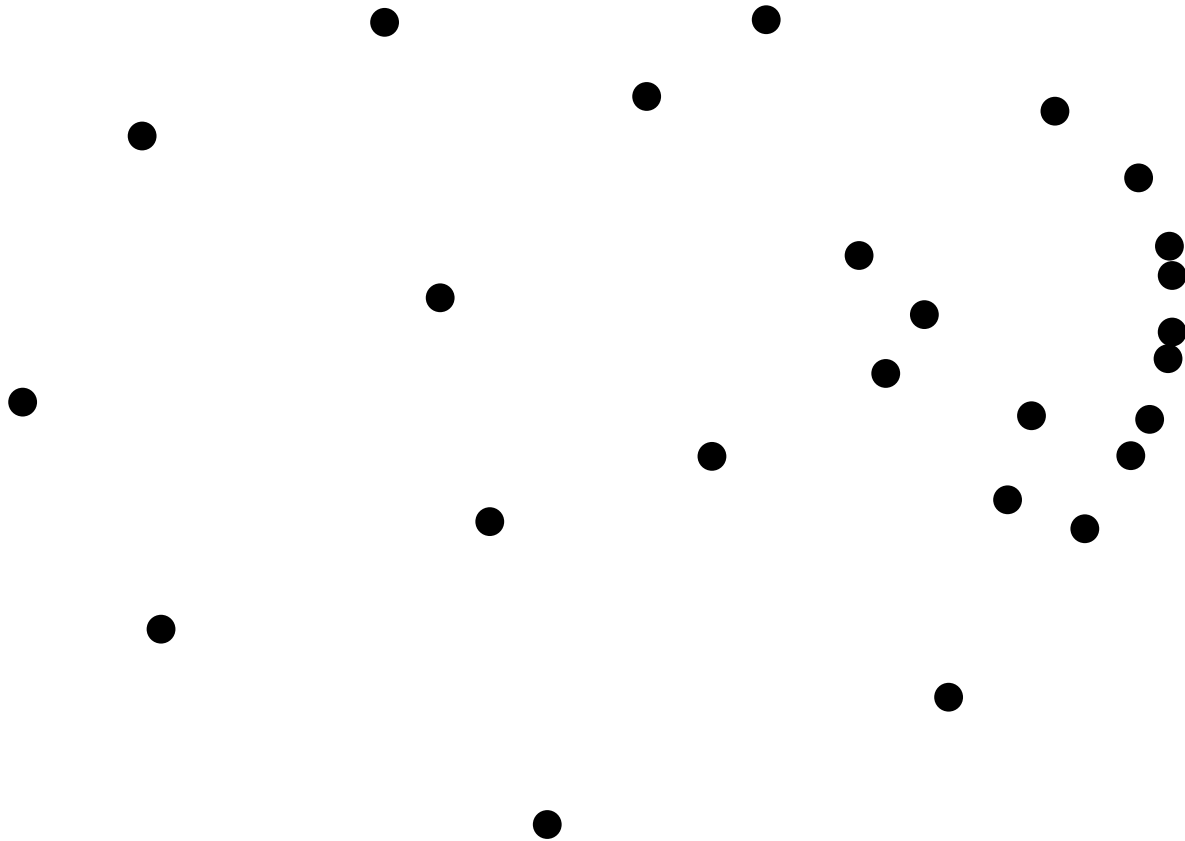
## Convex hull



# Computational geometry problems

Convex hull

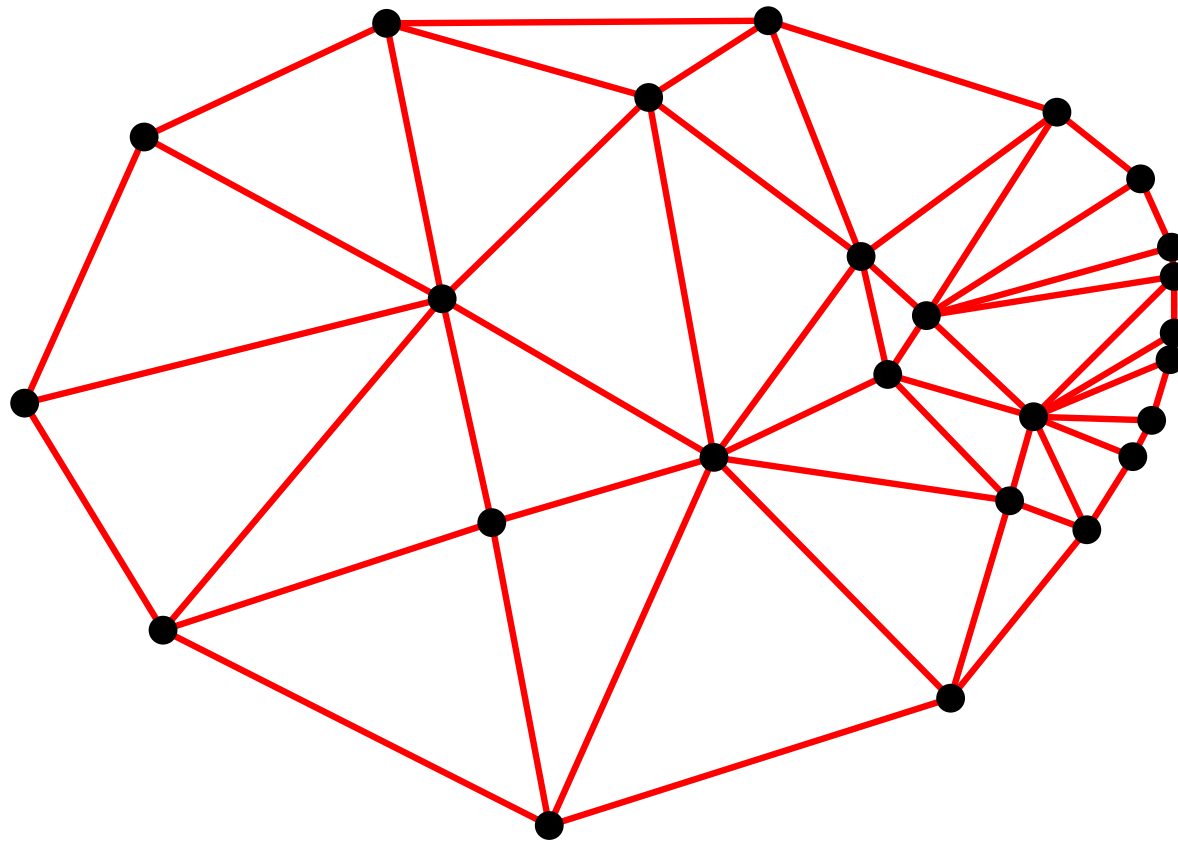
Delaunay triangulation / Voronoi diagrams



# Computational geometry problems

Convex hull

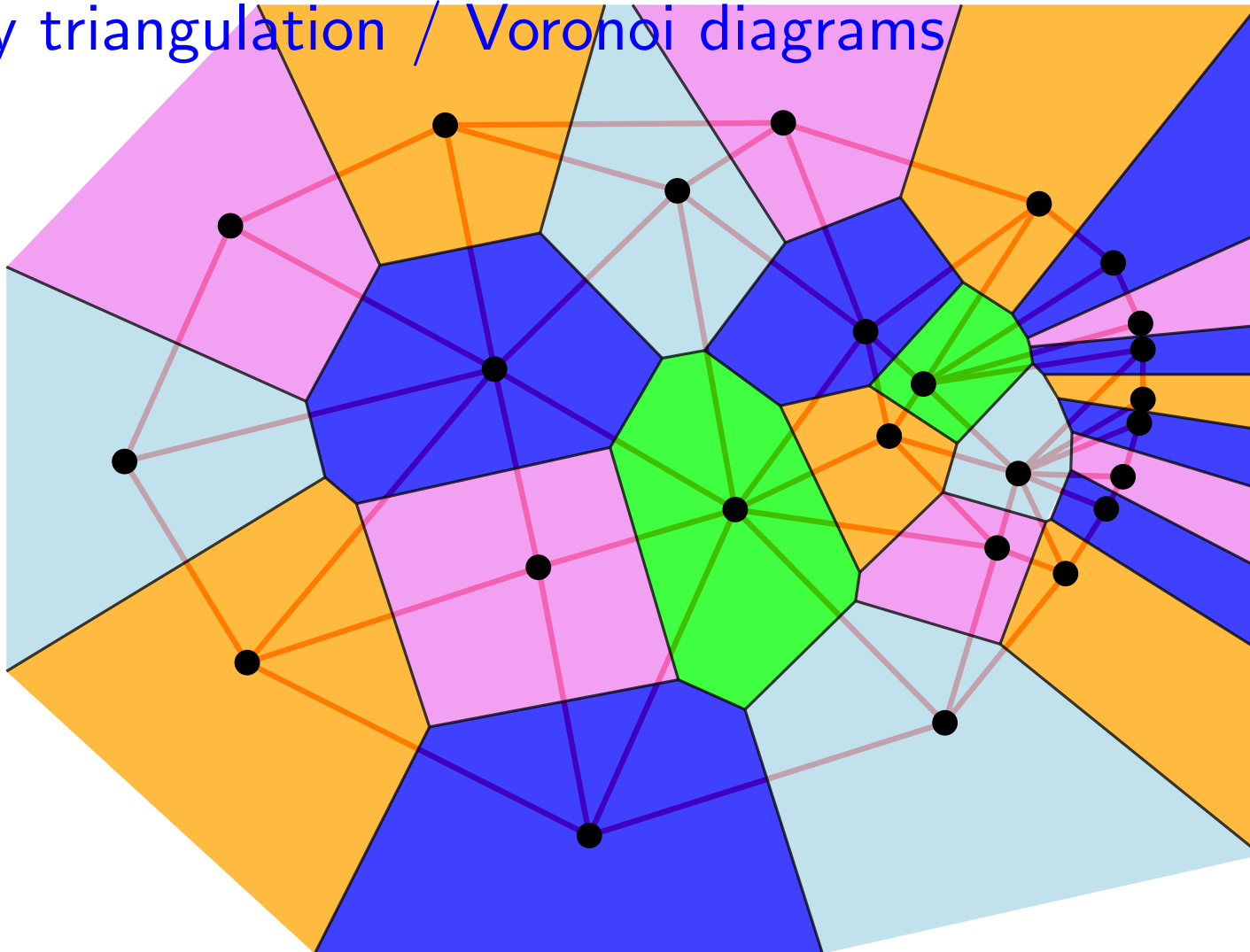
Delaunay triangulation / Voronoi diagrams



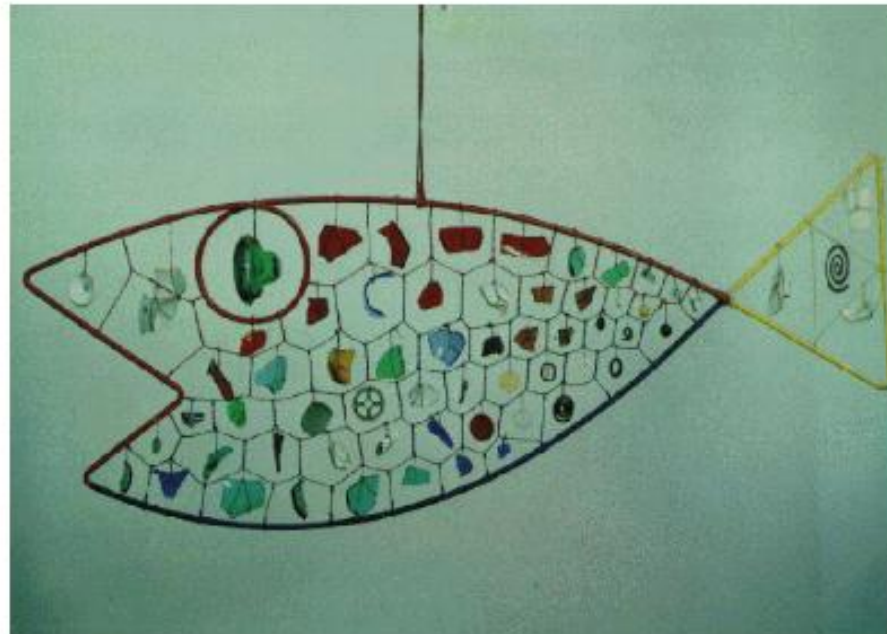
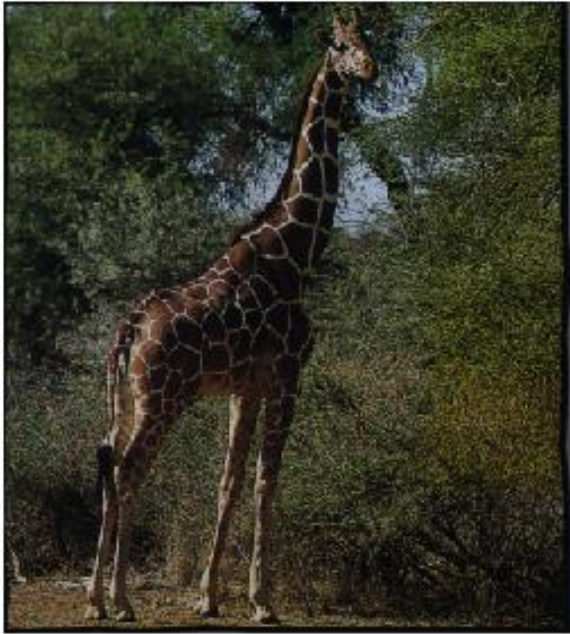
# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams



# Computational geometry problems



# Computational geometry problems



# Computational geometry problems





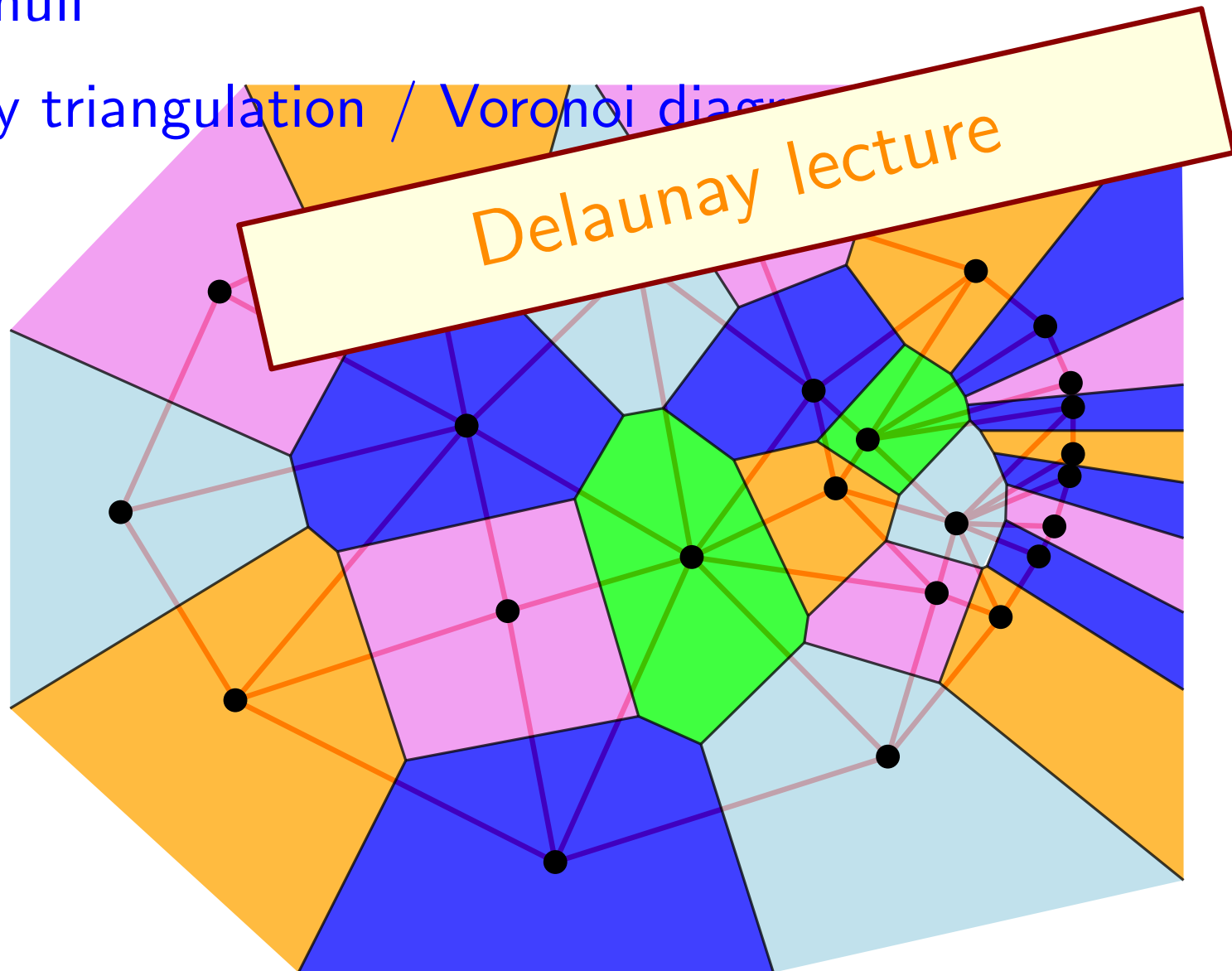
# Computational geometry problems



# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagram

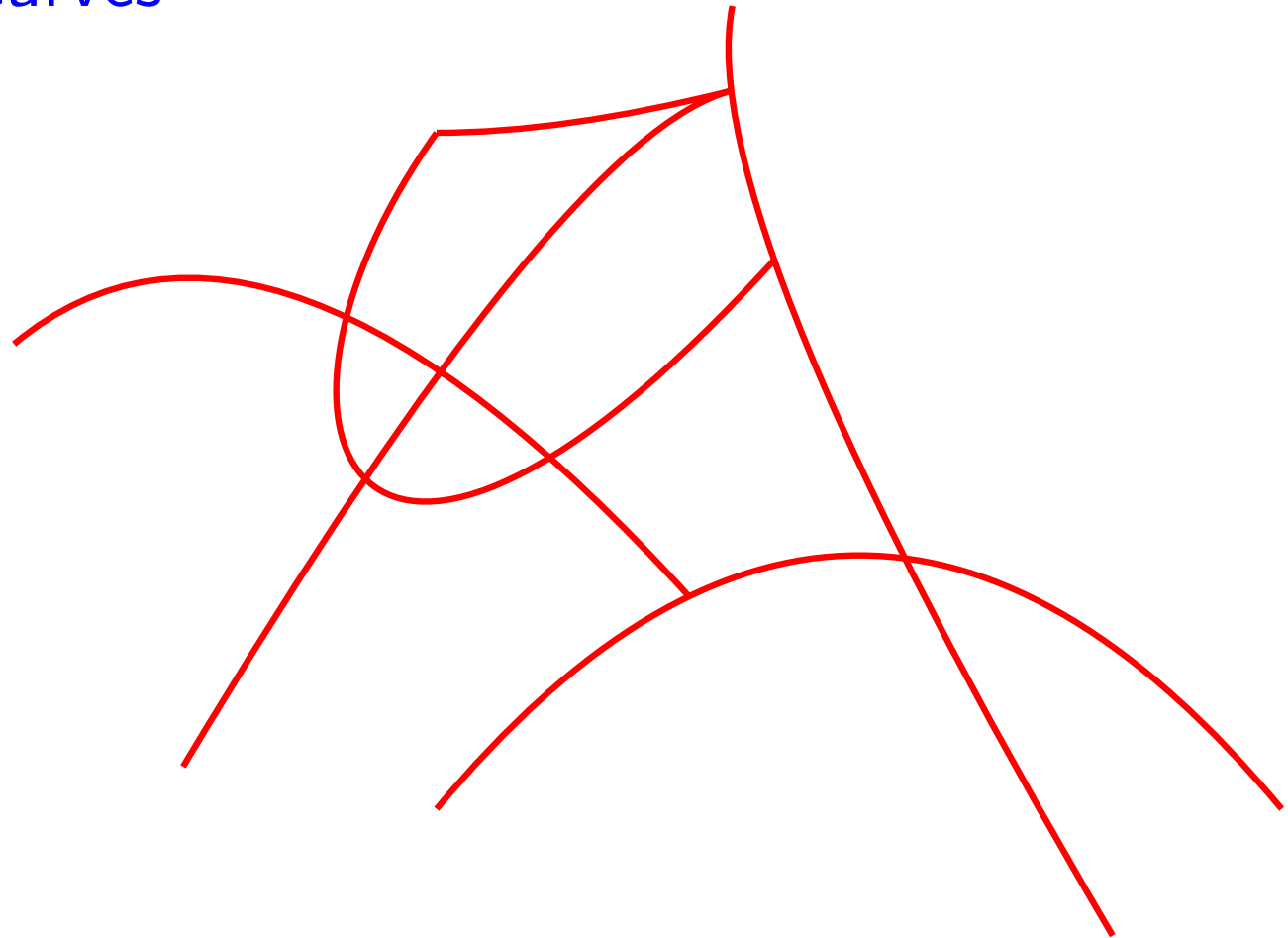


# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

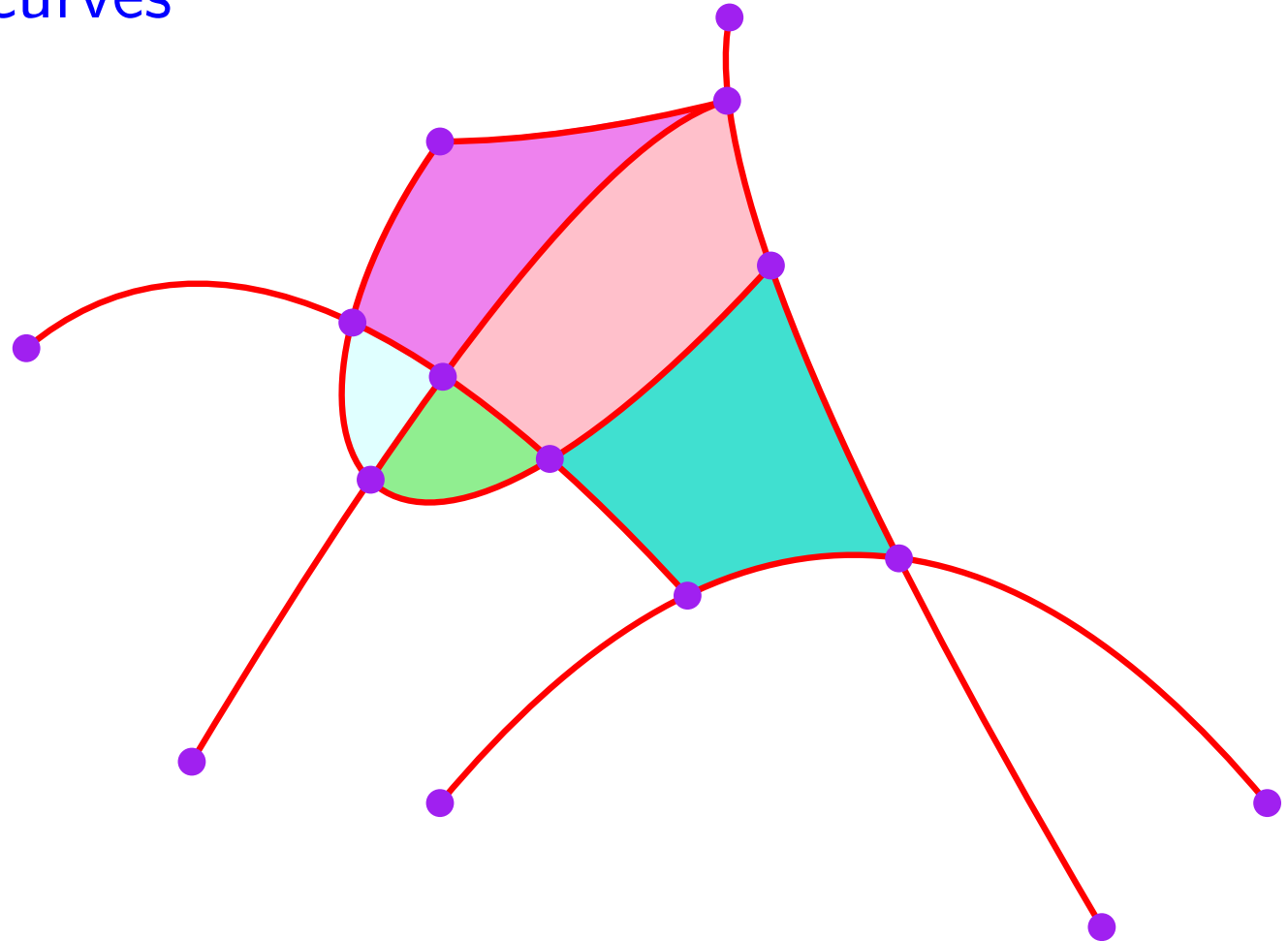


# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves



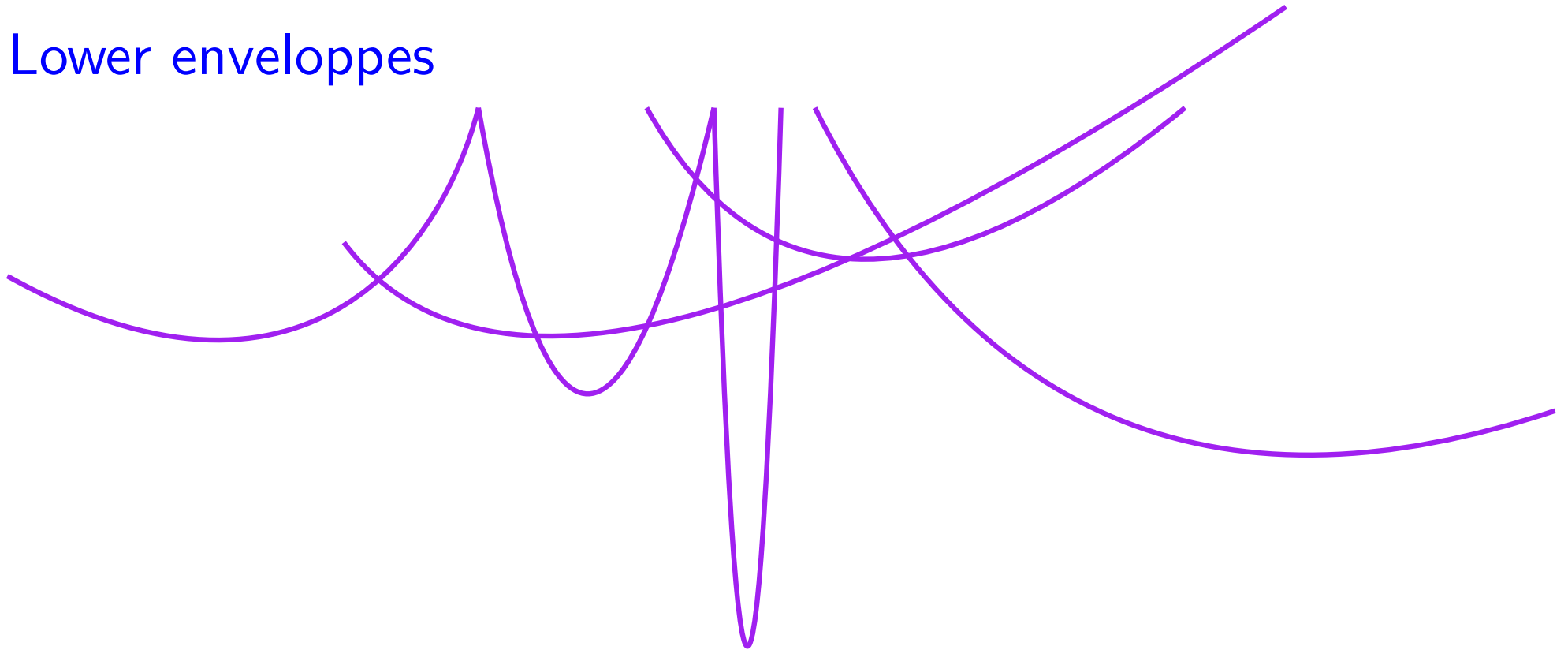
# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

Lower envelopes



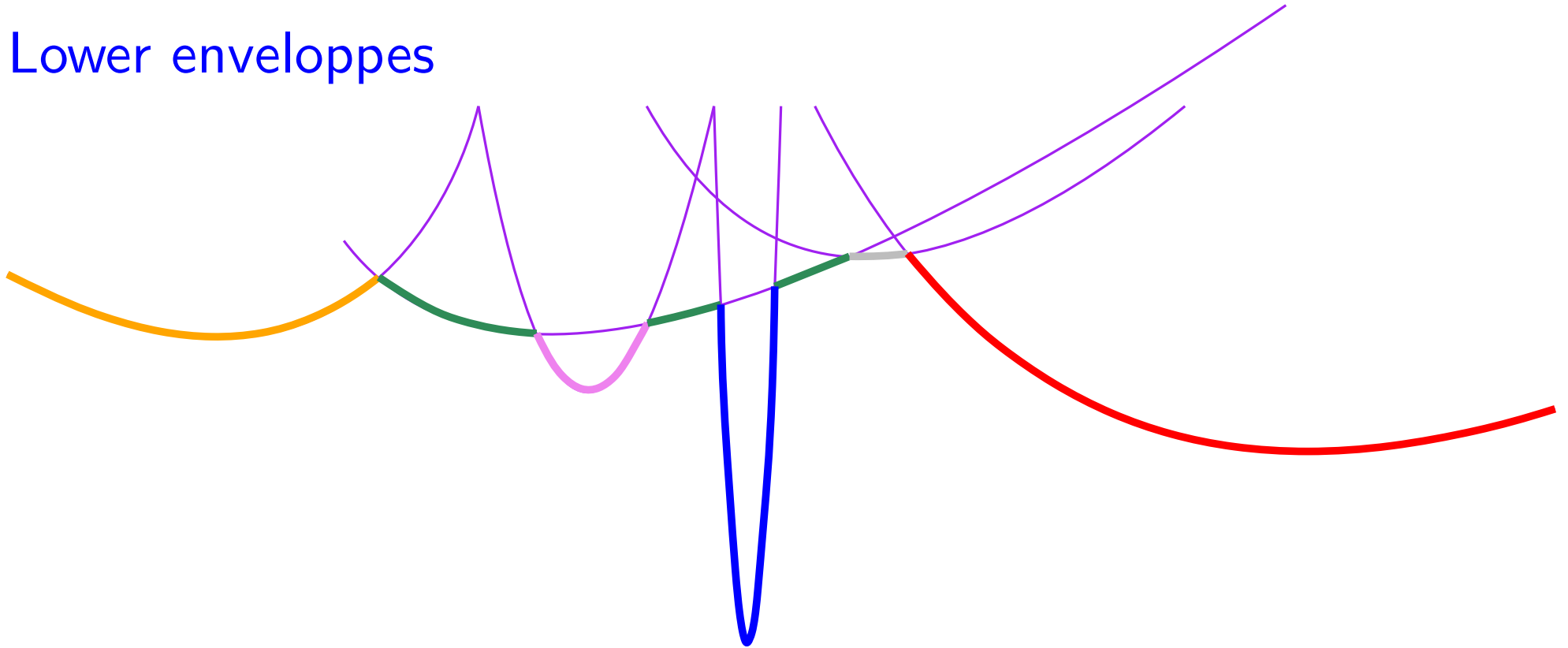
# Computational geometry problems

Convex hull

Delaunay triangulation / Voronoi diagrams

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# Computational geometry problems

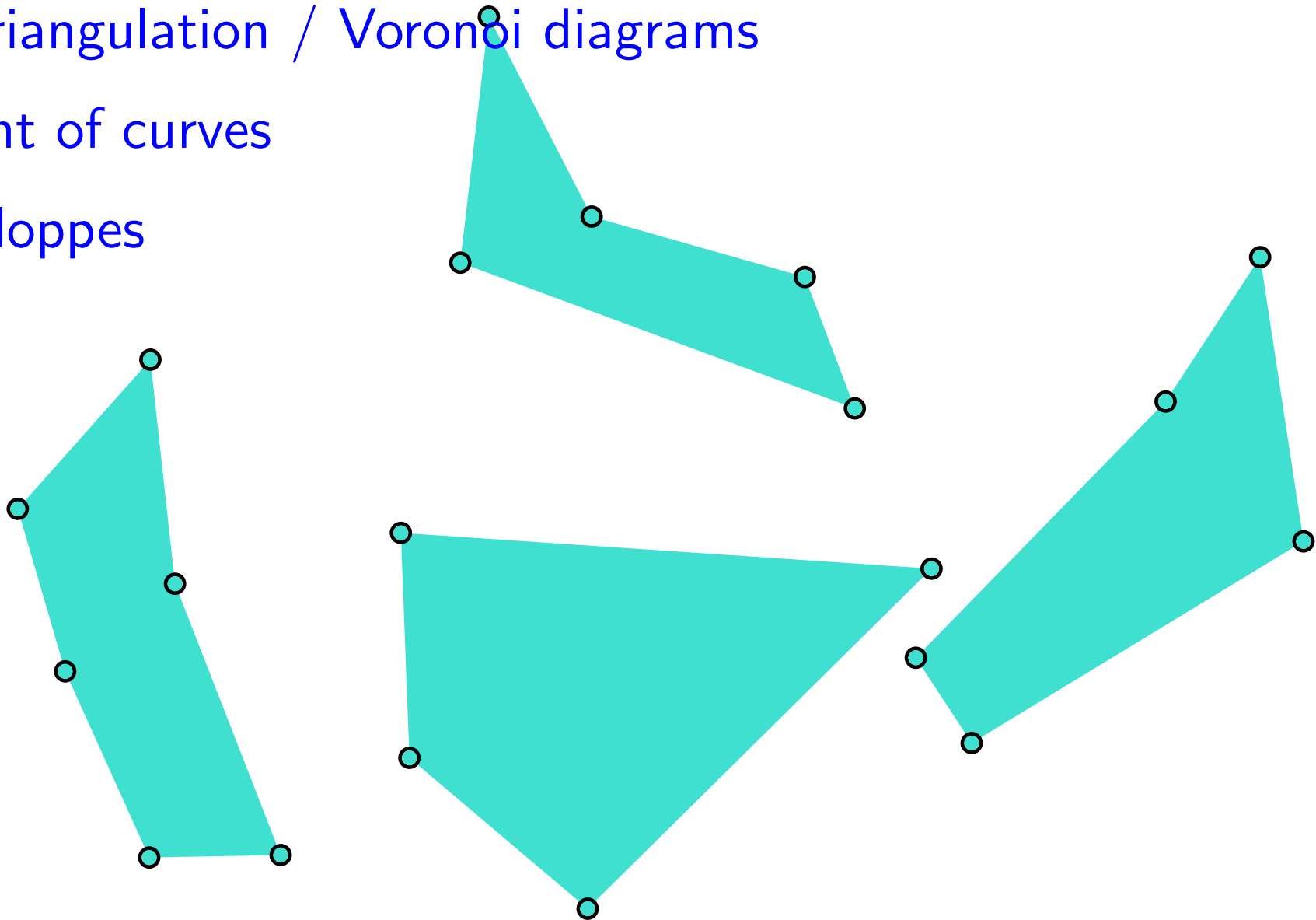
Convex hull

Delaunay triangulation / Voronoi diagrams

Arrangement of curves

Lower envelopes

Visibility



# Computational geometry problems

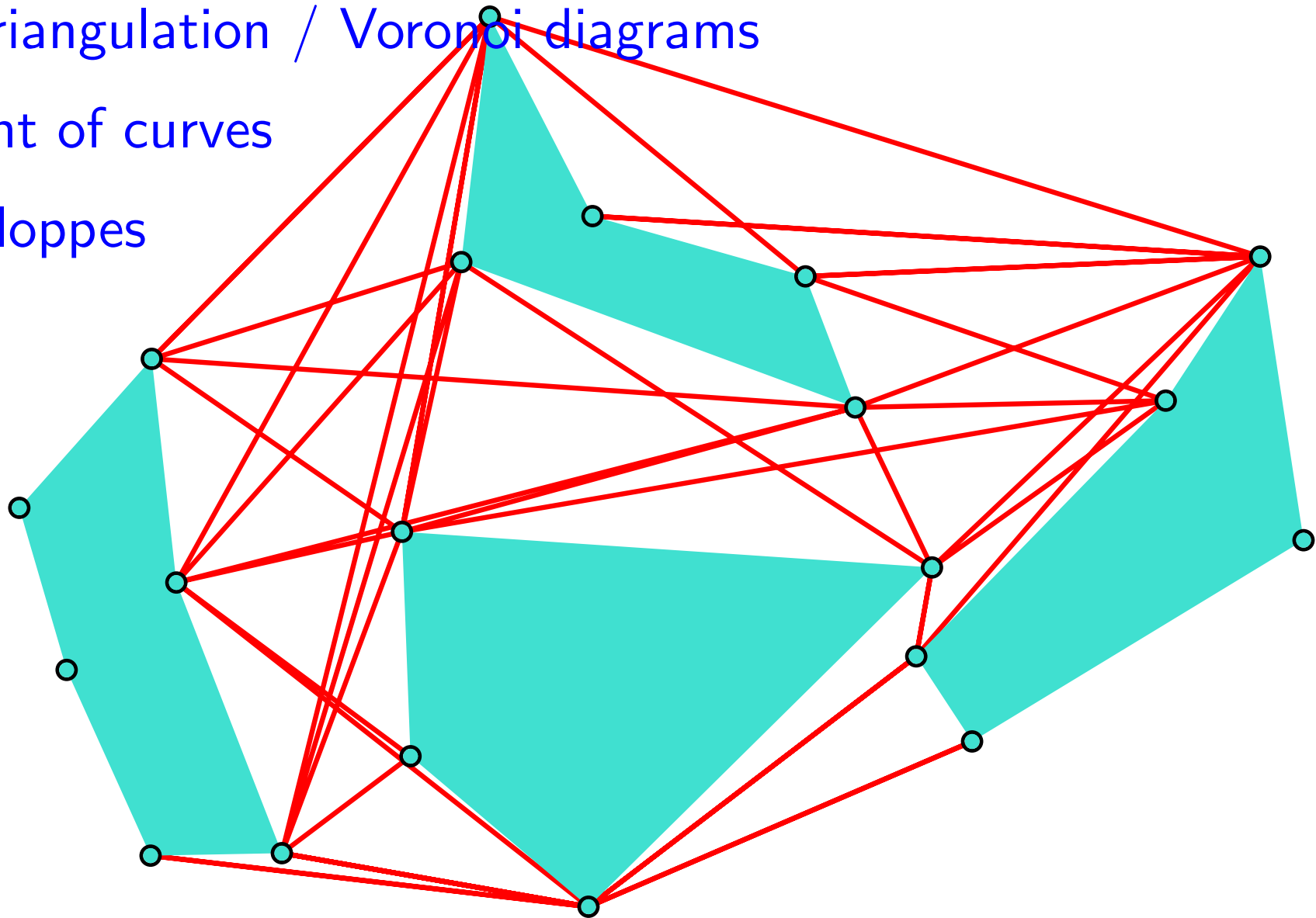
Convex hull

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Arrangement of curves

Lower envelopes

Visibility

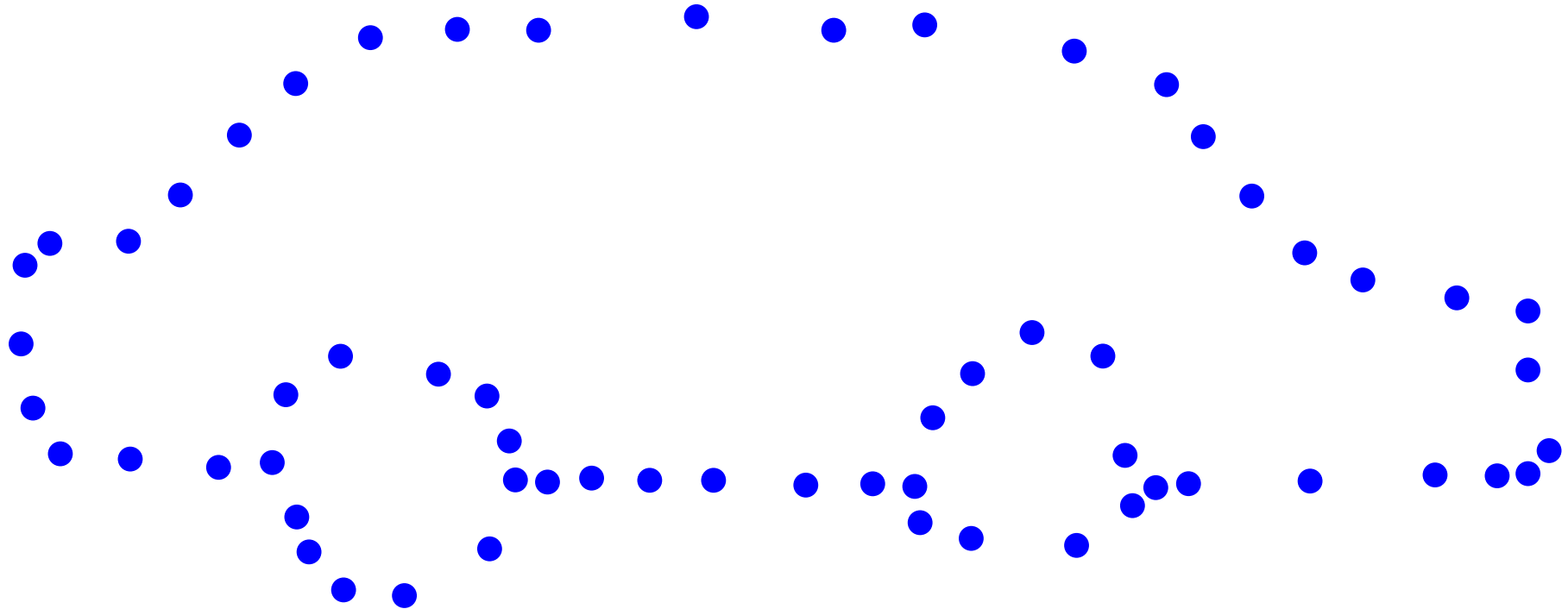




# Computational geometry usage

# Computational geometry usage

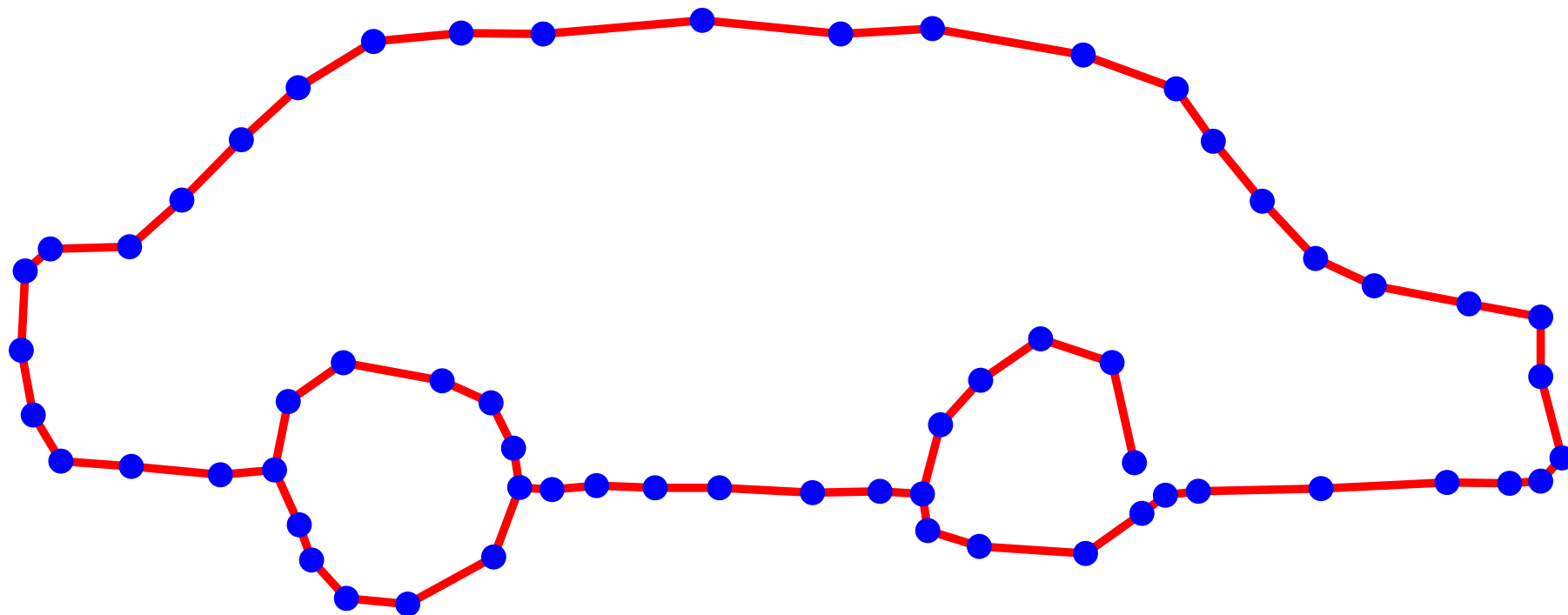
Points to shape



# Computational geometry usage

Points to shape

Reconstruction lecture

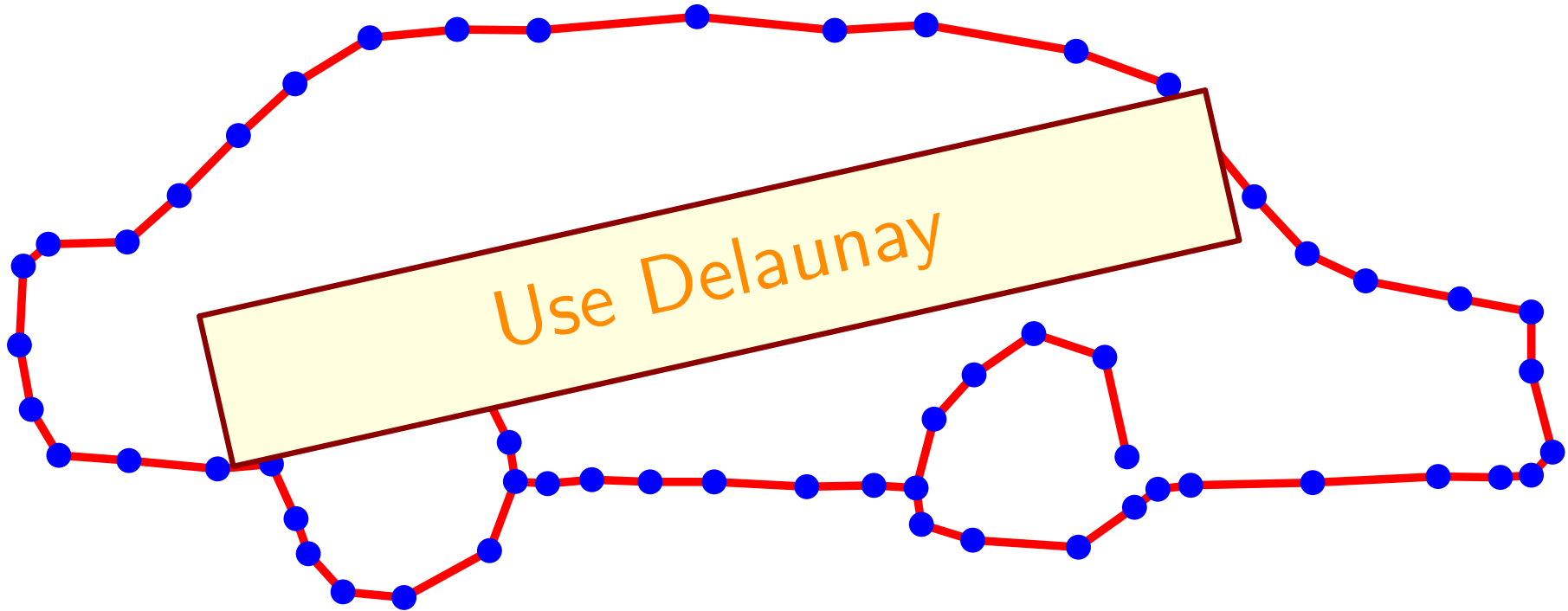


# Computational geometry usage

Points to shape

Reconstruction lecture

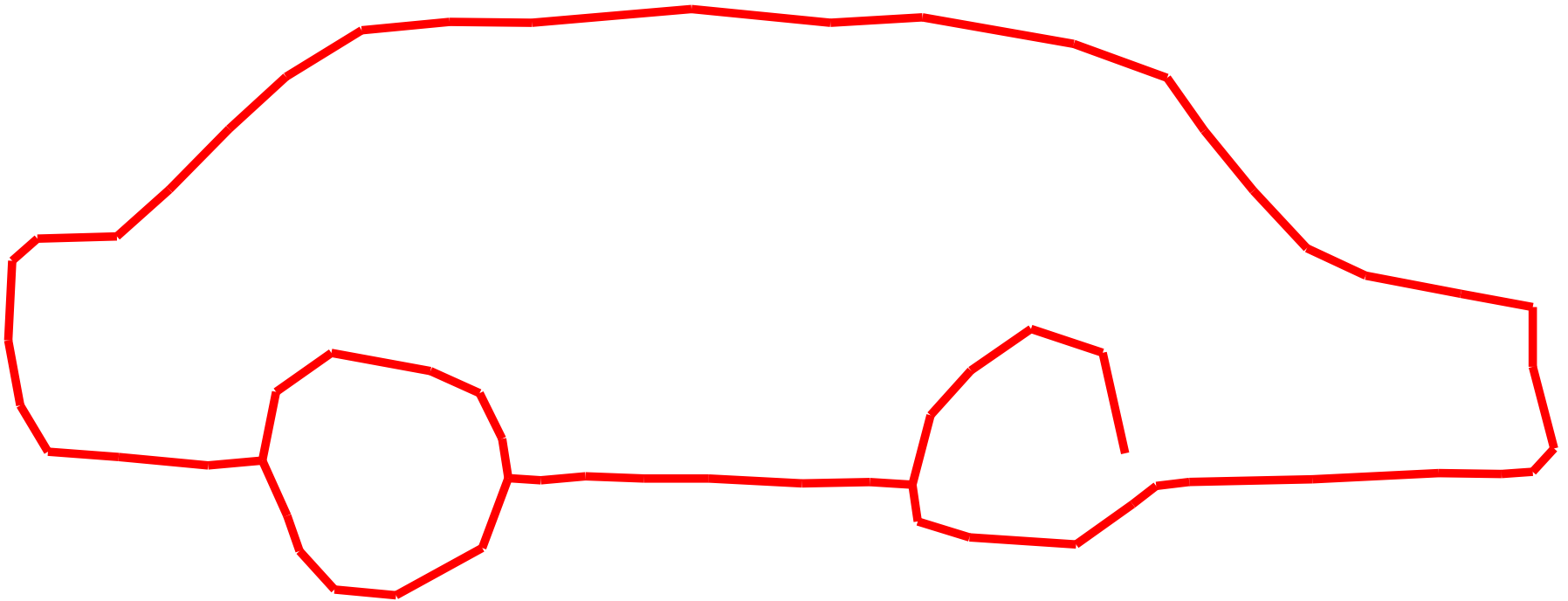
Use Delaunay



# Computational geometry usage

# Computational geometry usage

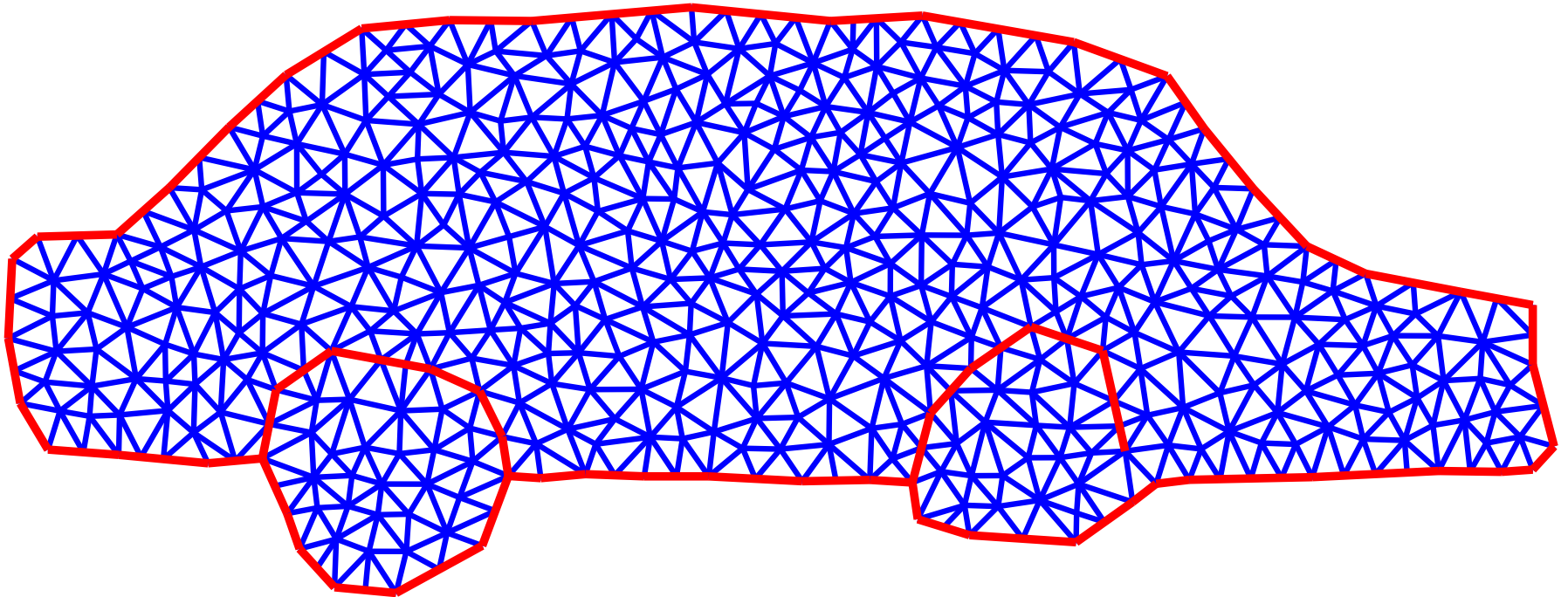
Shape to mesh



# Computational geometry usage

Shape to mesh

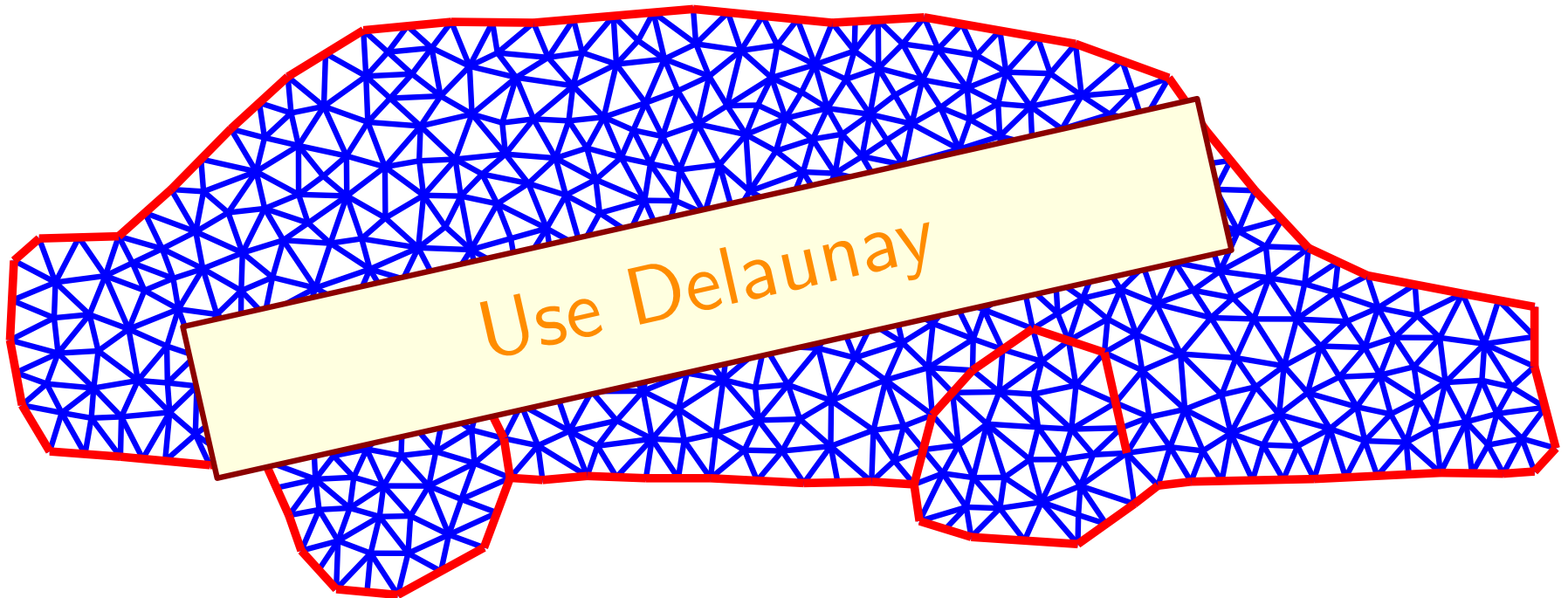
Meshing lecture



# Computational geometry usage

Shape to mesh

Meshing lecture





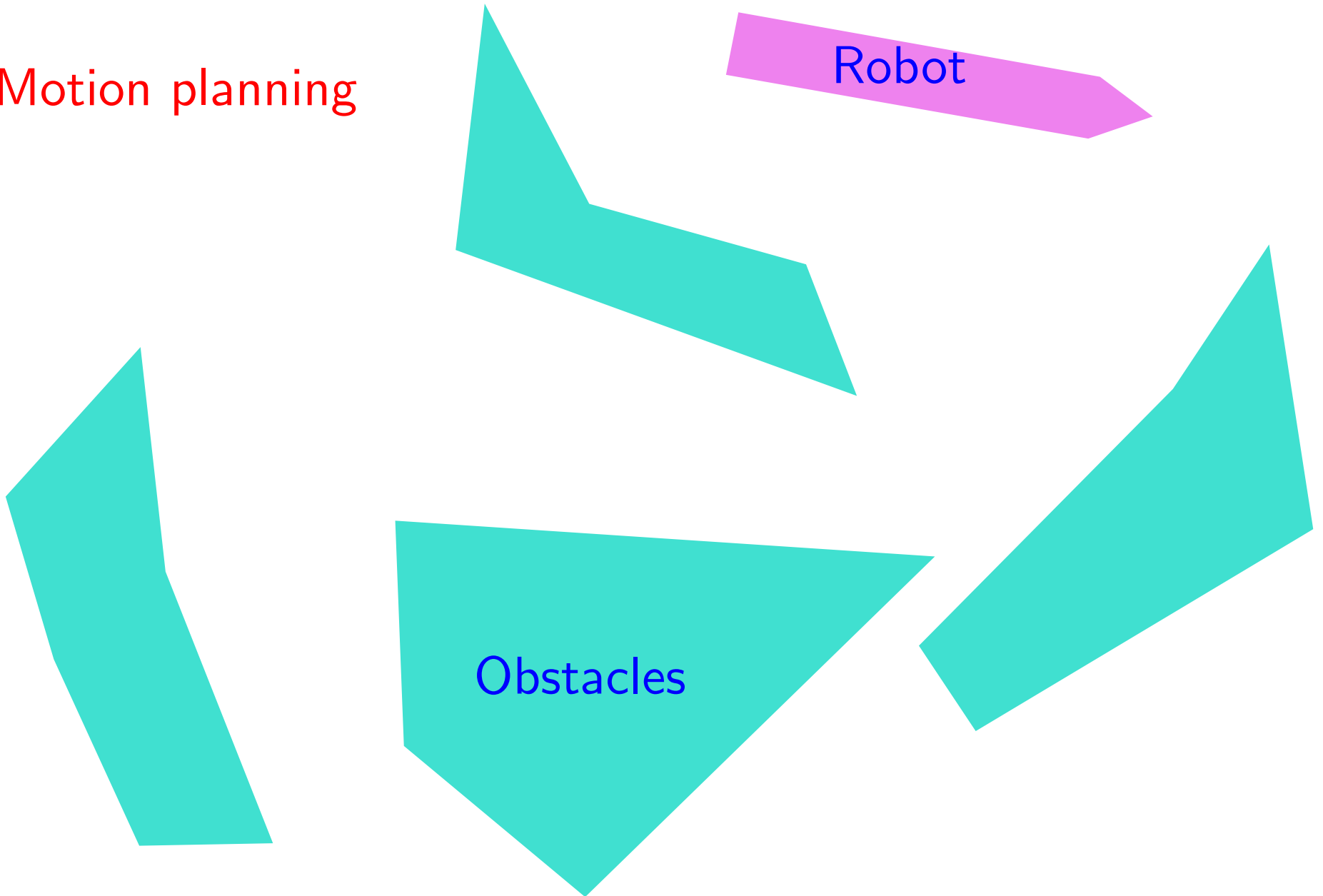
# Computational geometry usage

# Computational geometry usage

Motion planning

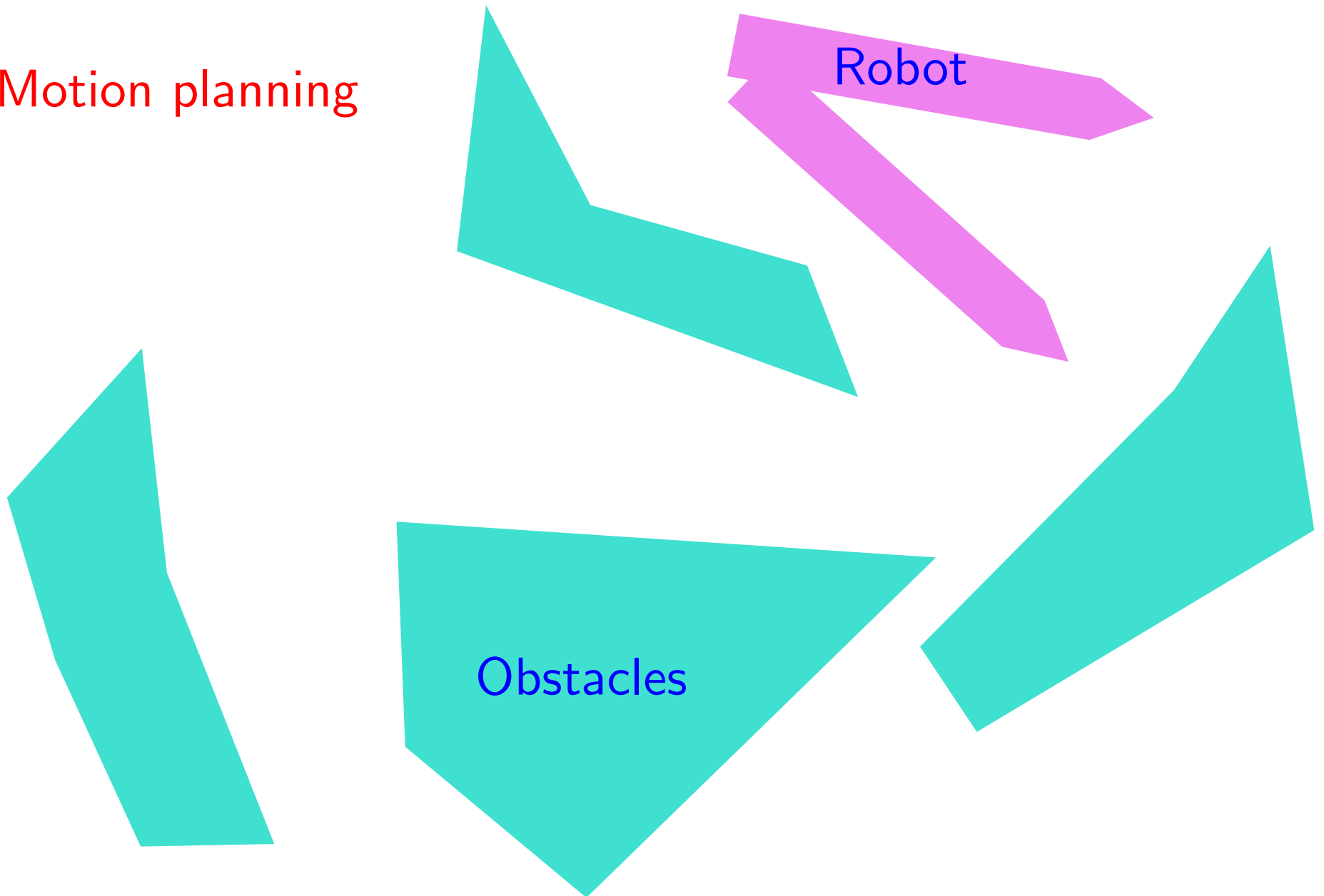
Robot

Obstacles



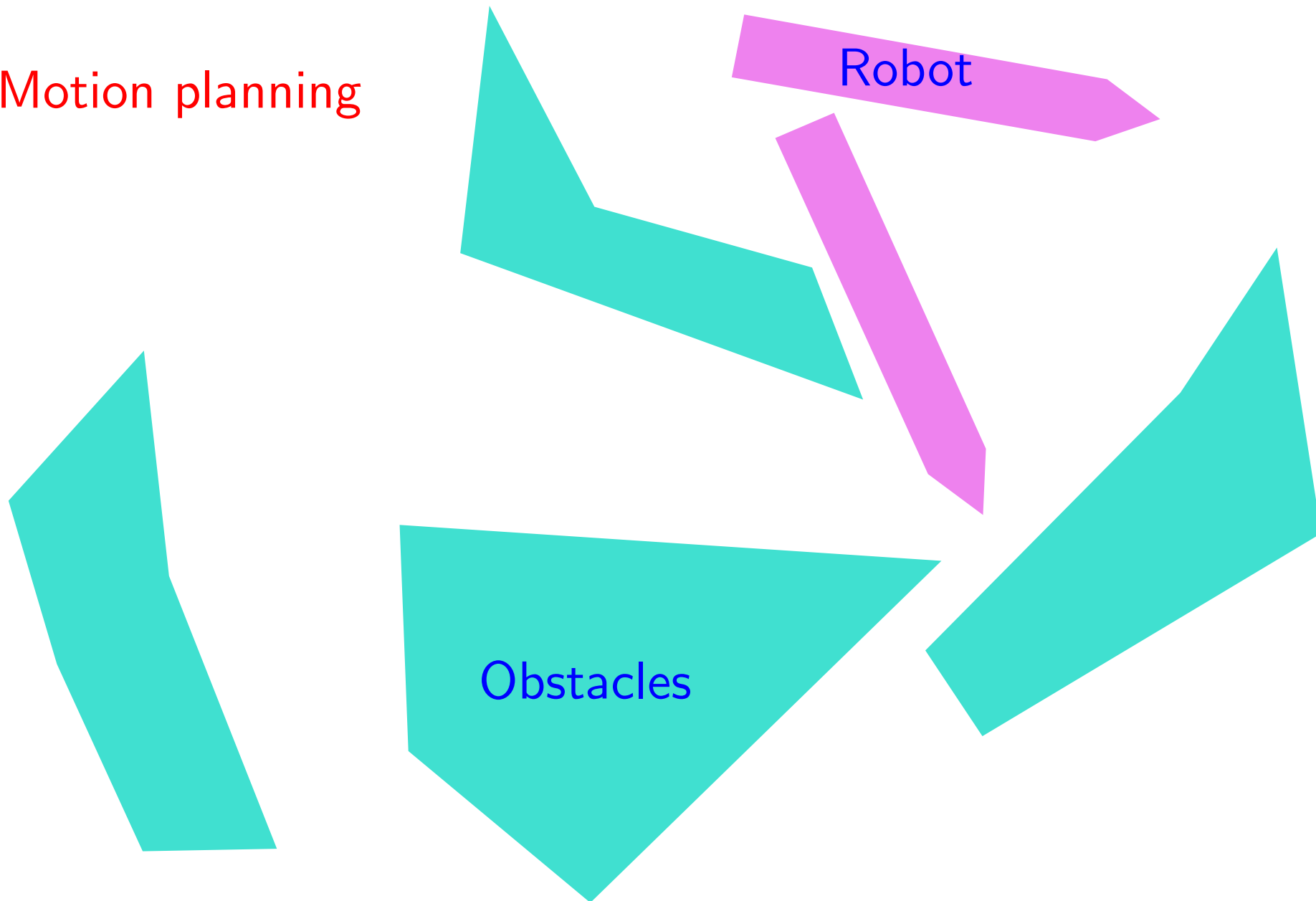
# Computational geometry usage

Motion planning



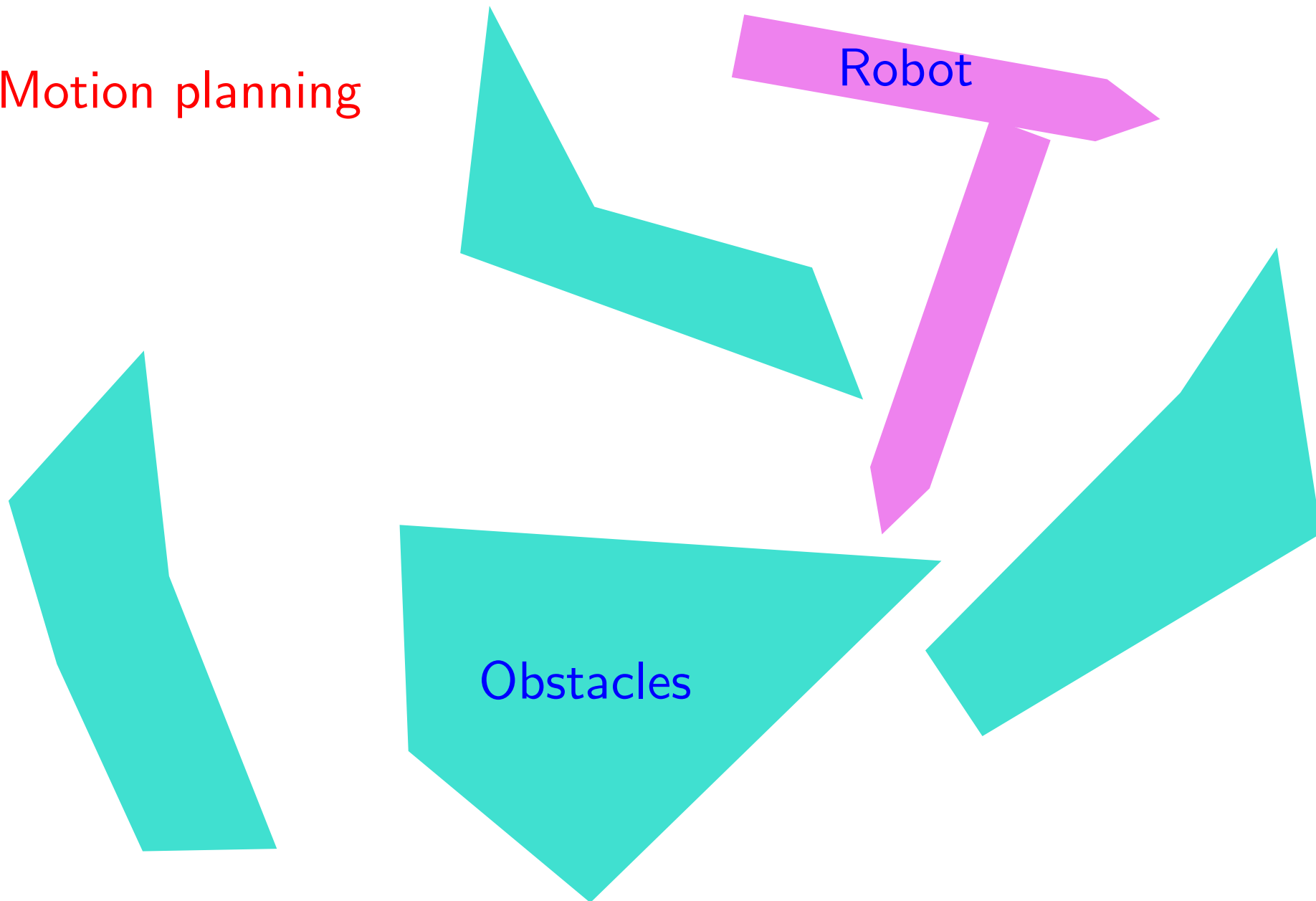
# Computational geometry usage

Motion planning



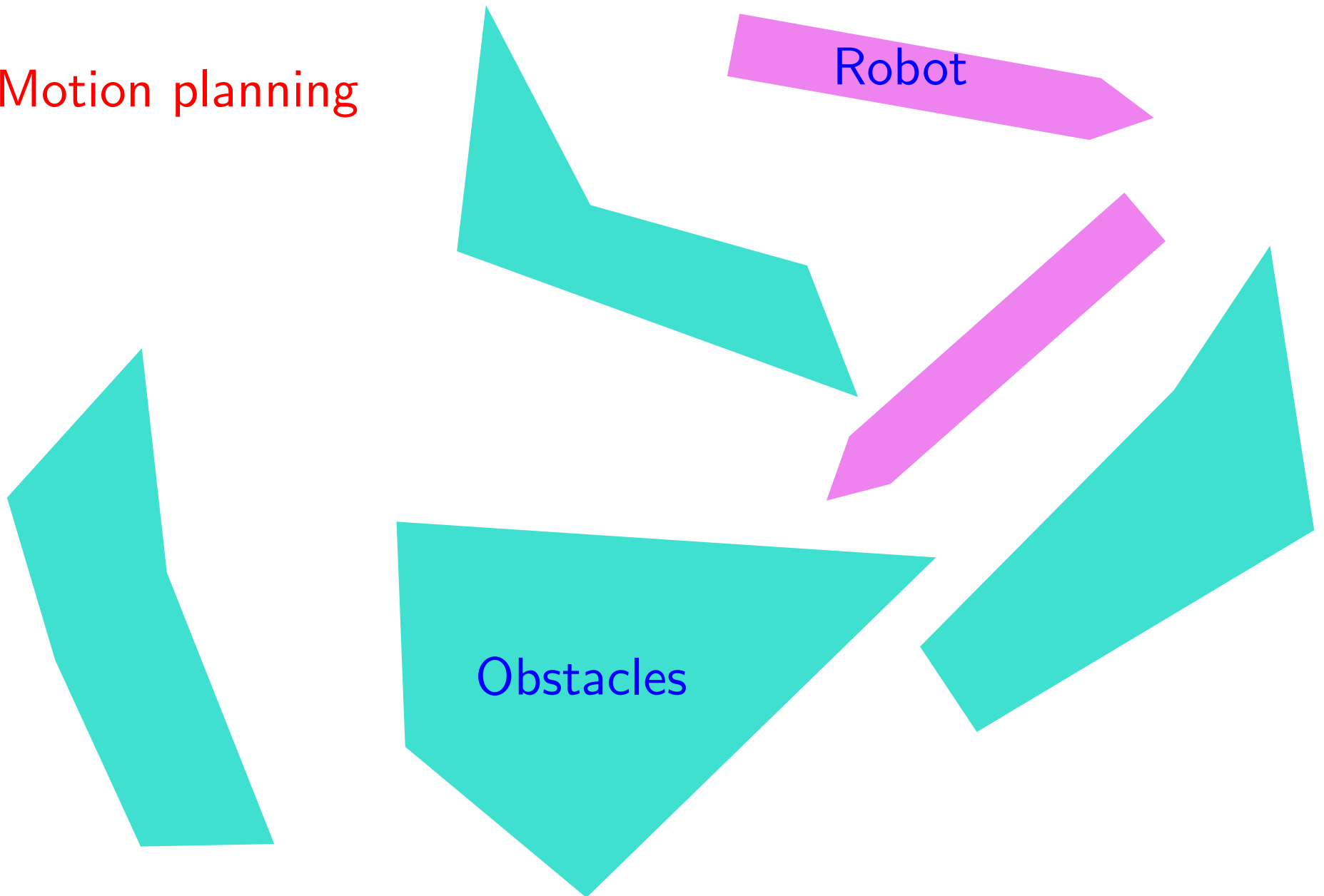
# Computational geometry usage

Motion planning



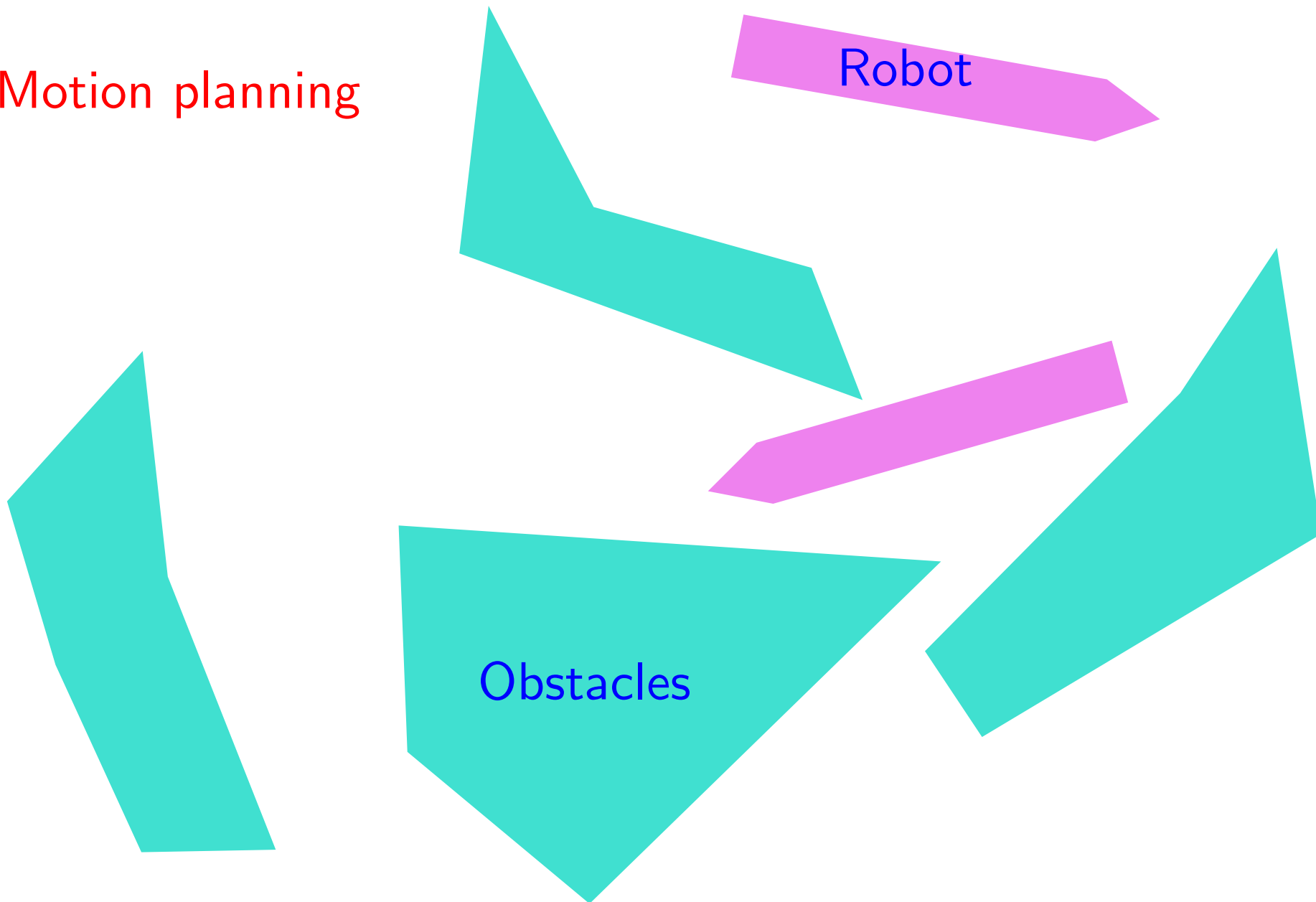
# Computational geometry usage

Motion planning



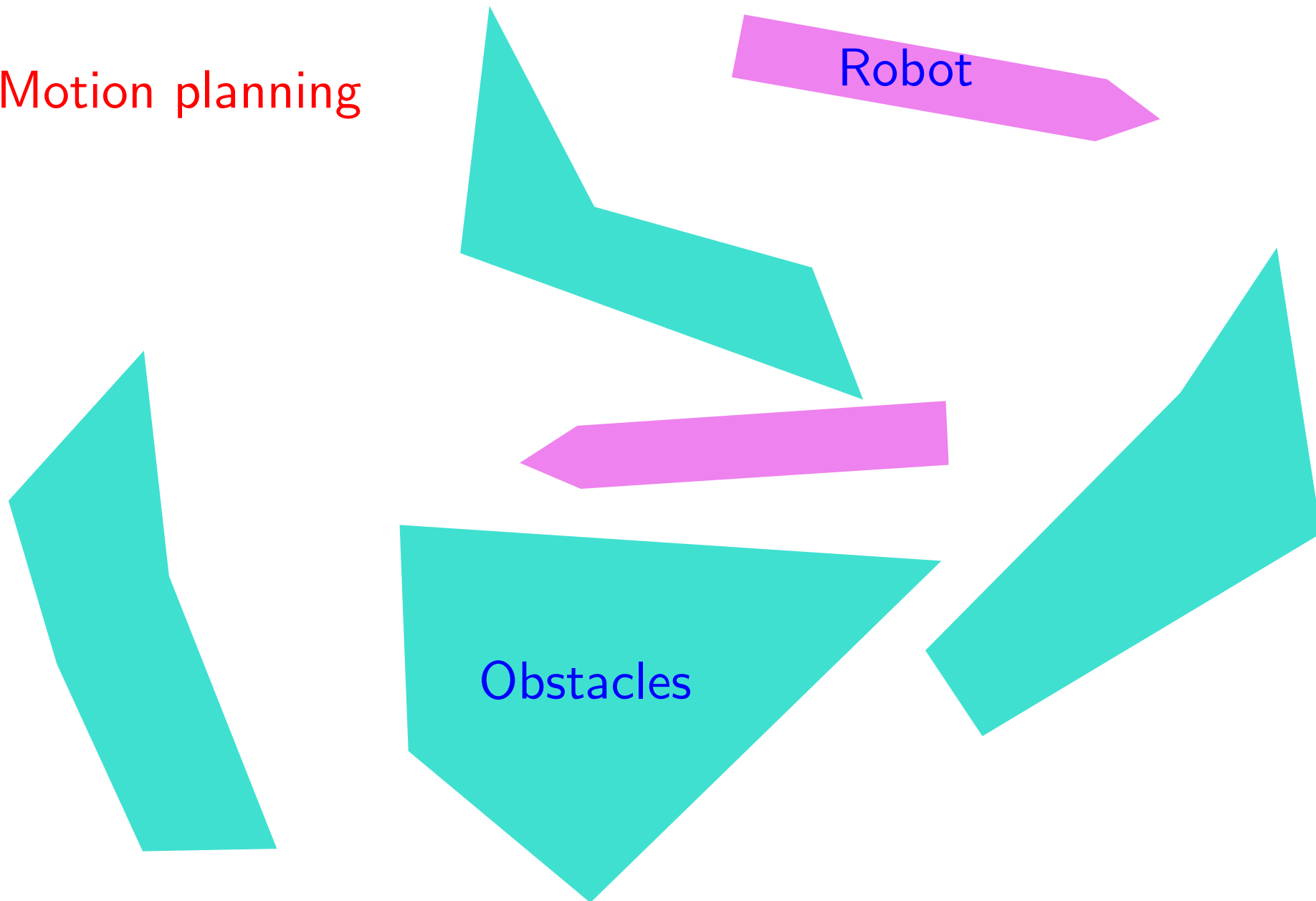
# Computational geometry usage

Motion planning



# Computational geometry usage

Motion planning



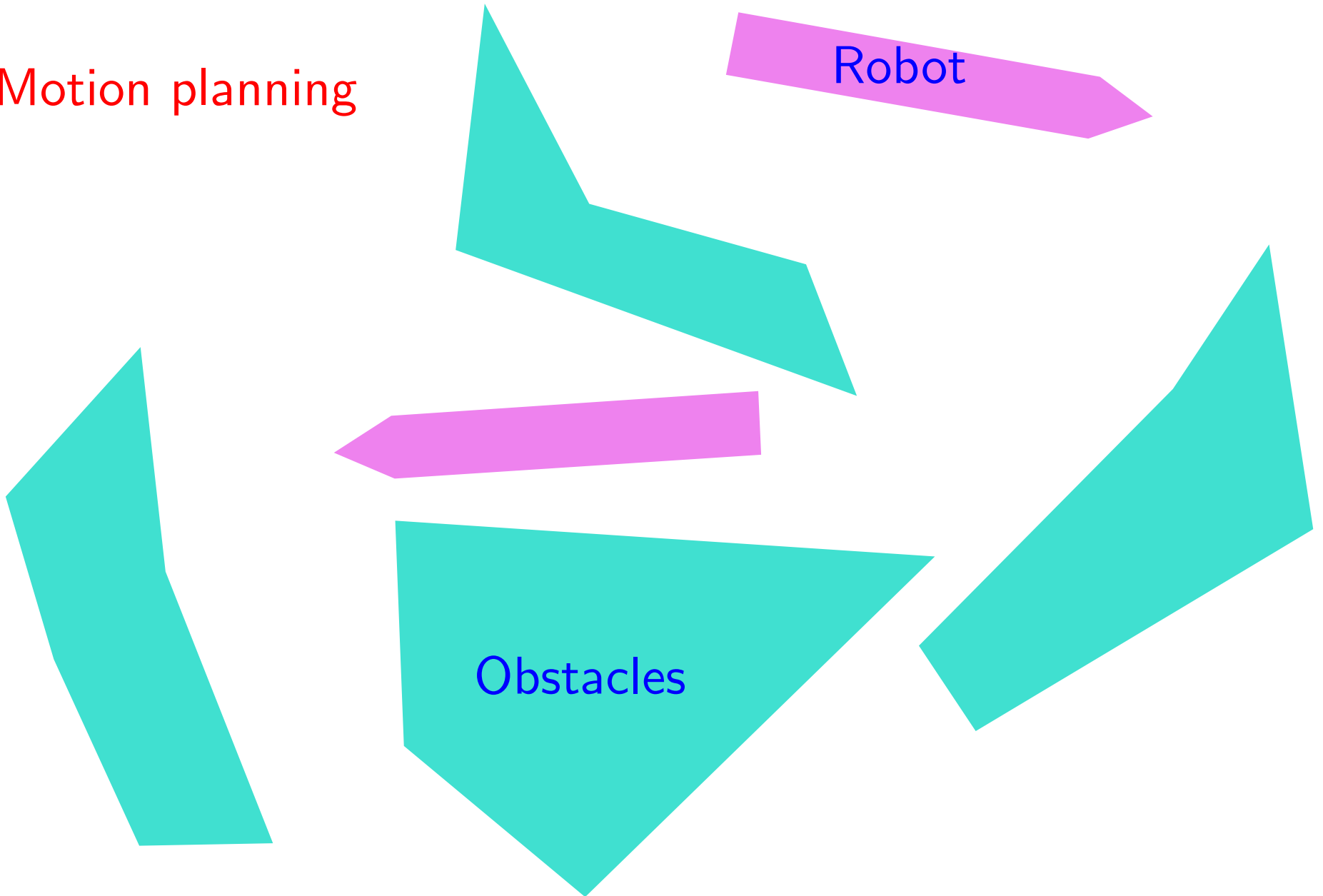


# Computational geometry usage

Motion planning

Robot

Obstacles

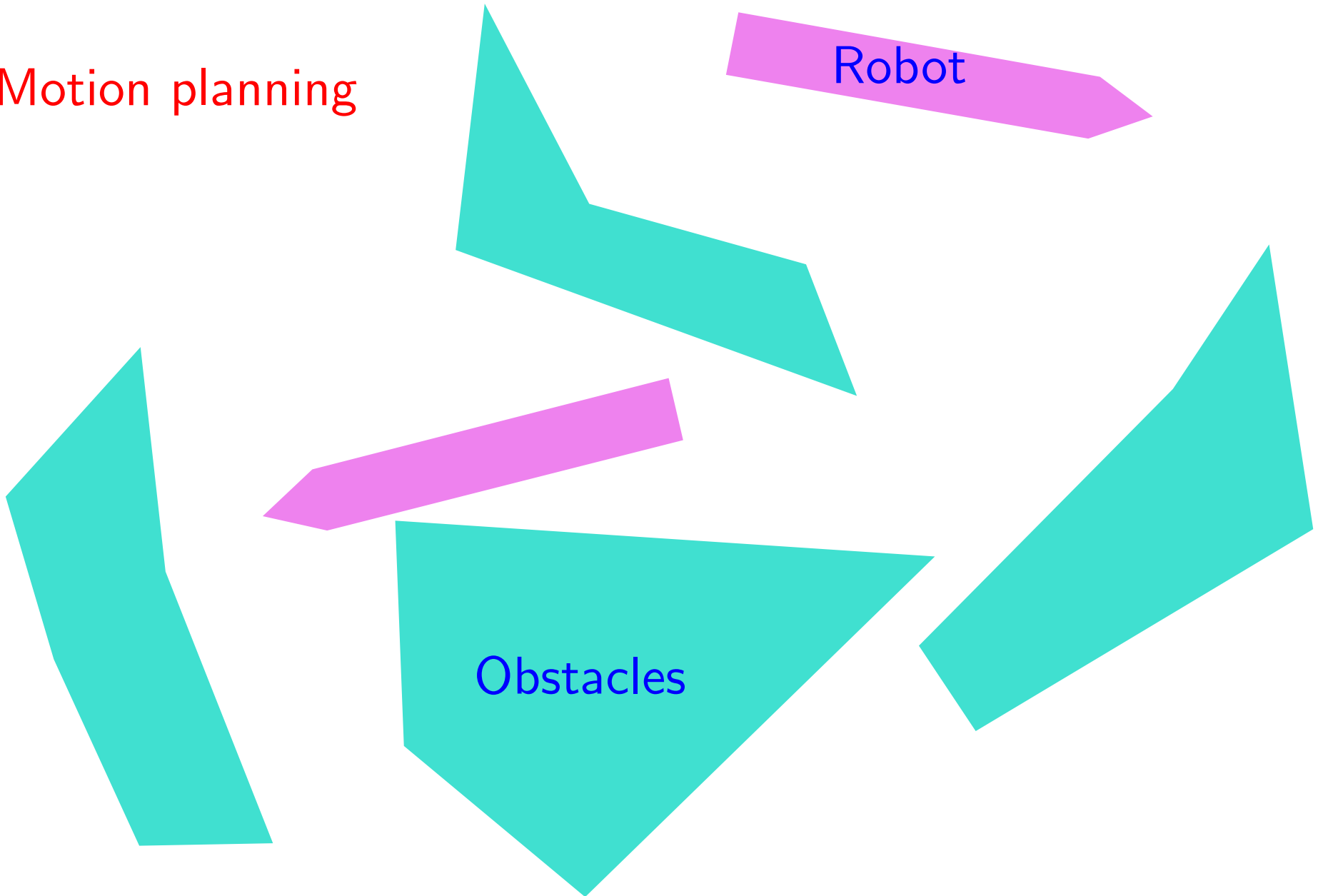


# Computational geometry usage

Motion planning

Robot

Obstacles

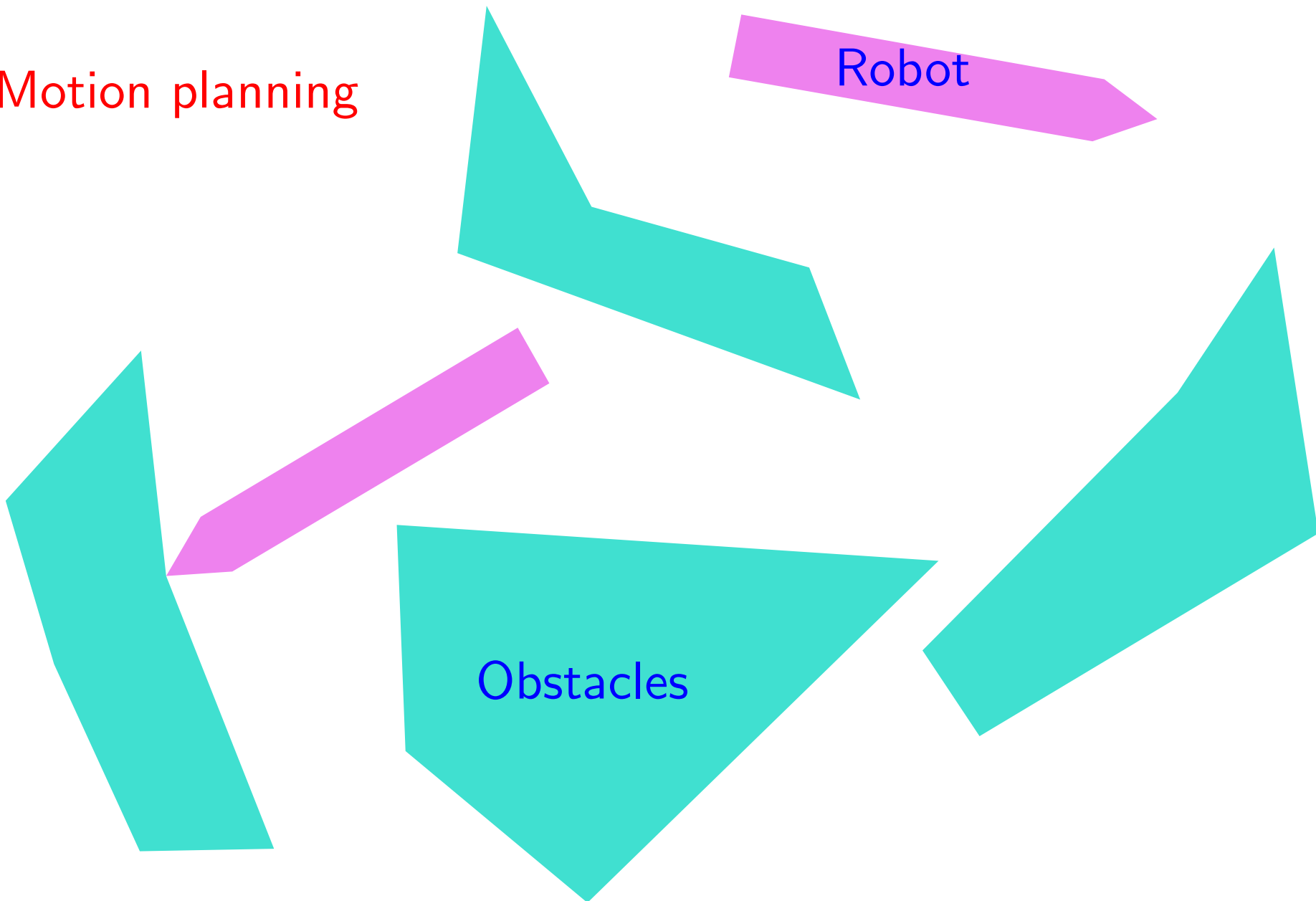


# Computational geometry usage

Motion planning

Robot

Obstacles

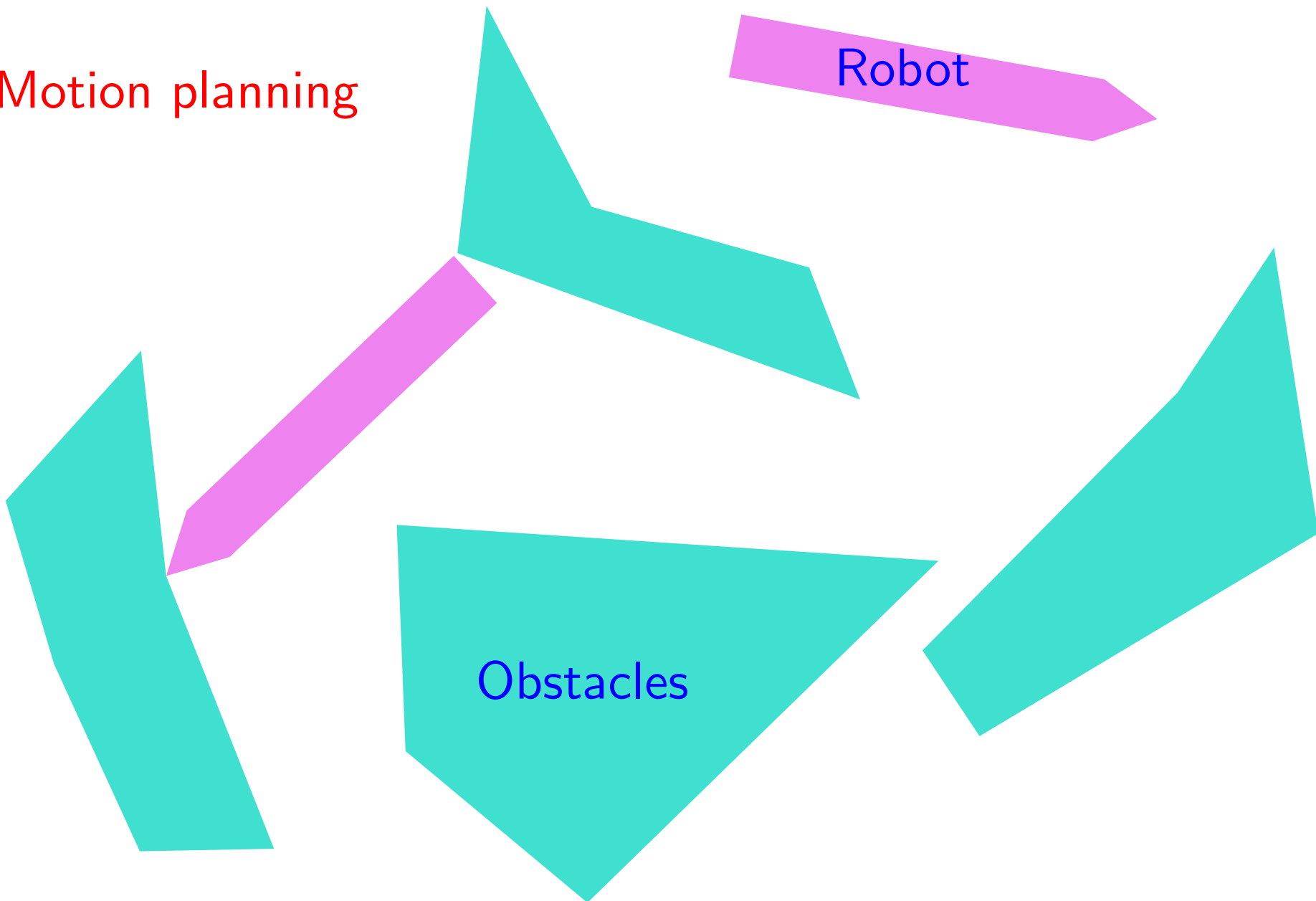


# Computational geometry usage

Motion planning

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Obstacles

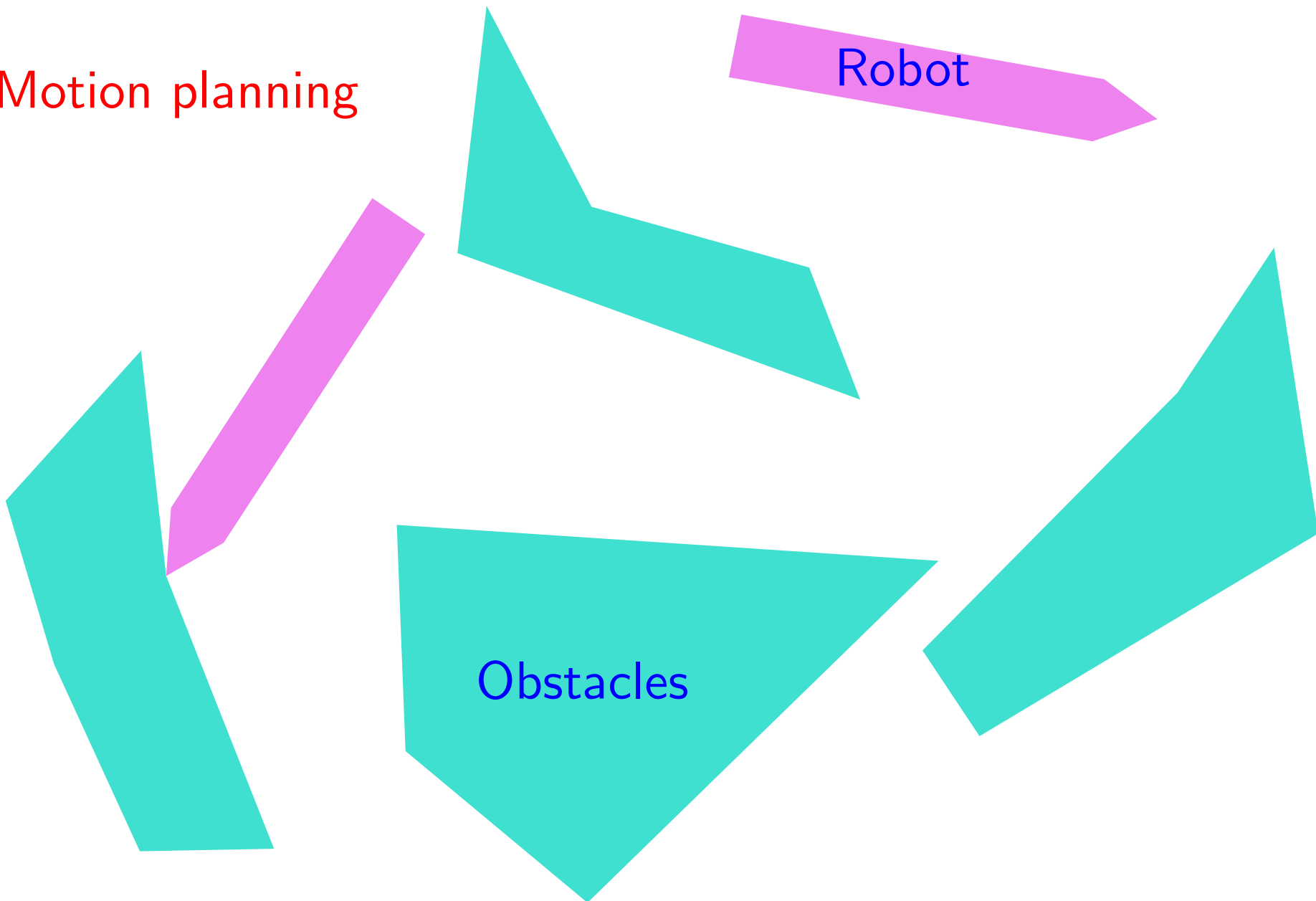


# Computational geometry usage

Motion planning

Robot

Obstacles

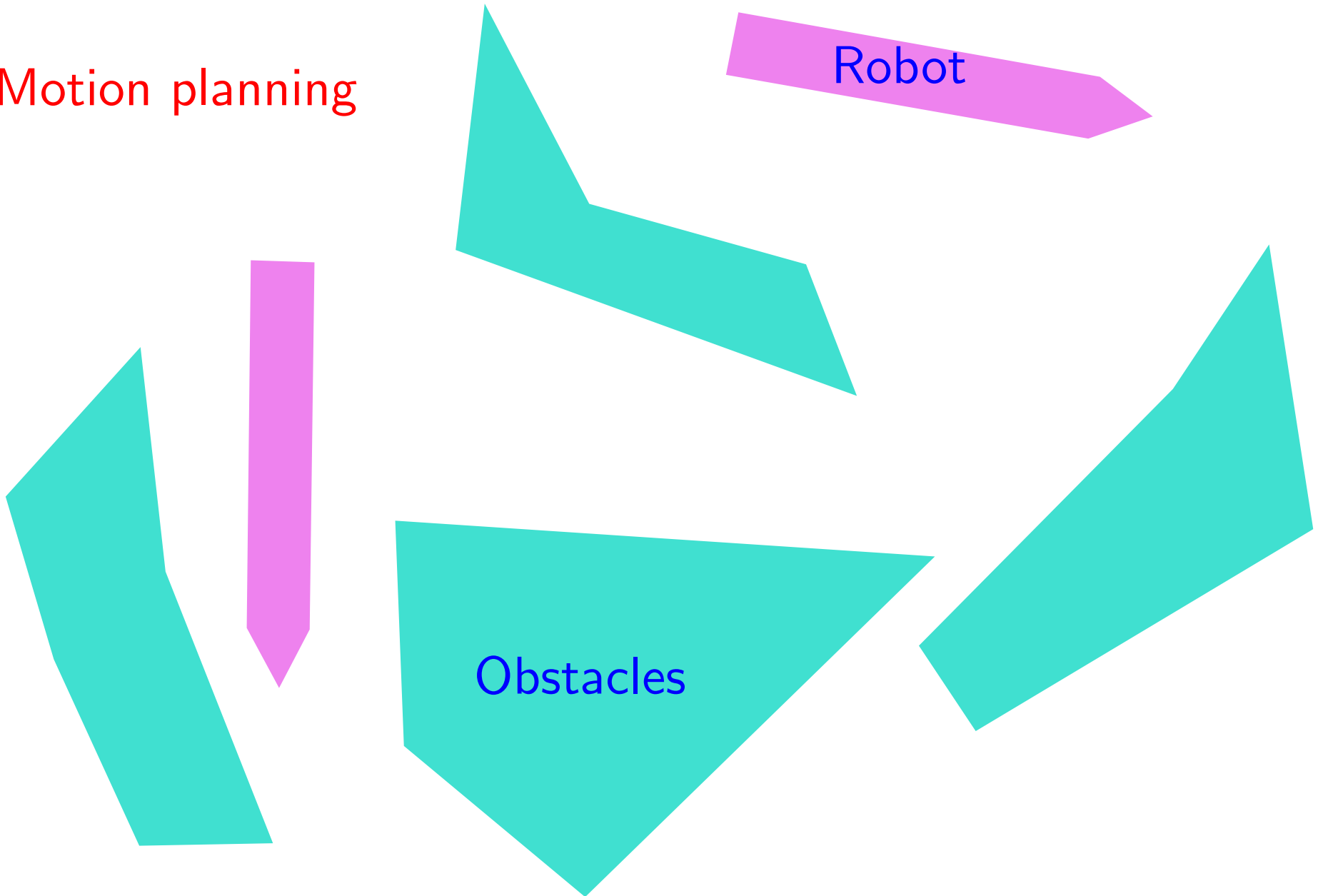


# Computational geometry usage

Motion planning

Robot

Obstacles

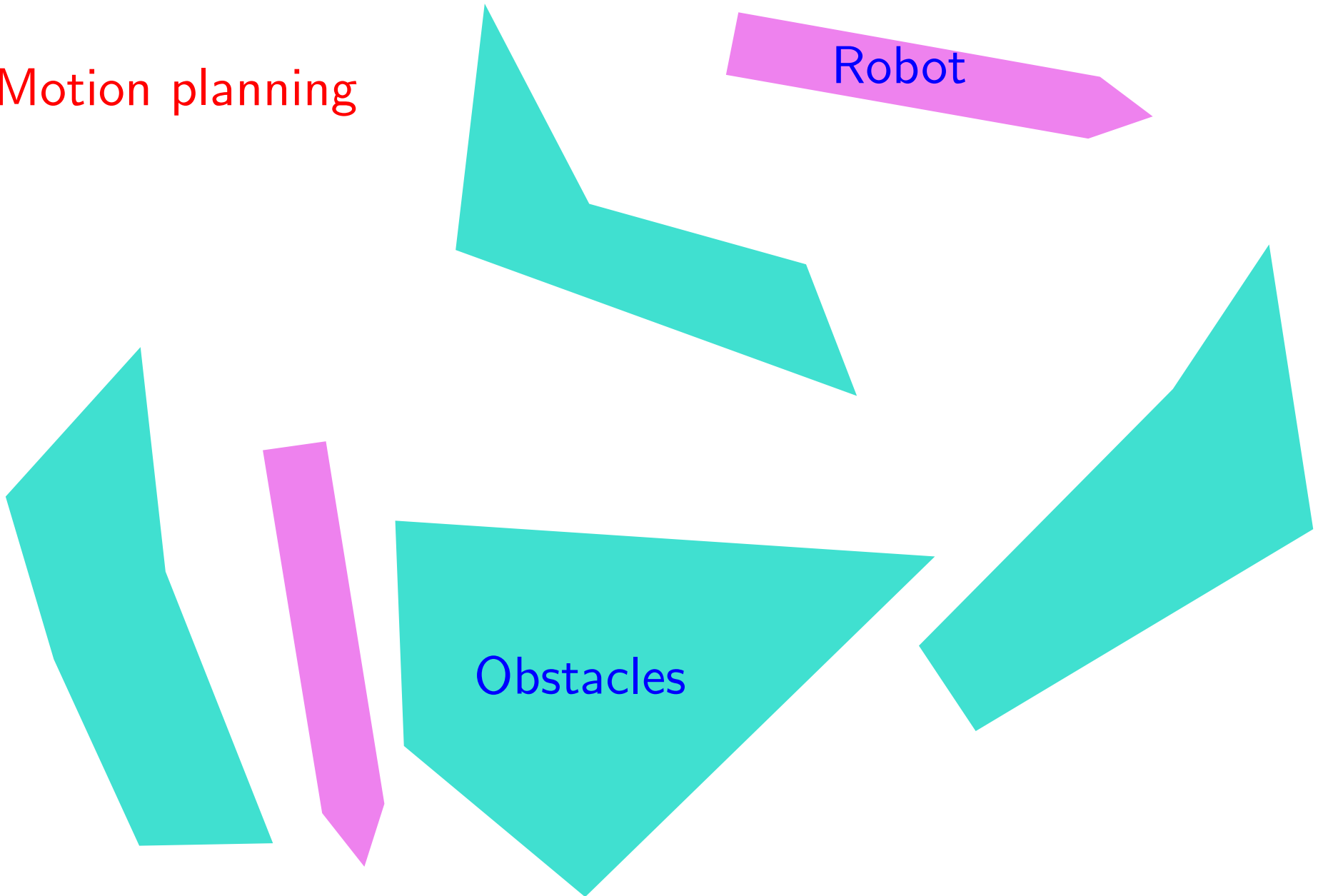


# Computational geometry usage

Motion planning

Robot

Obstacles



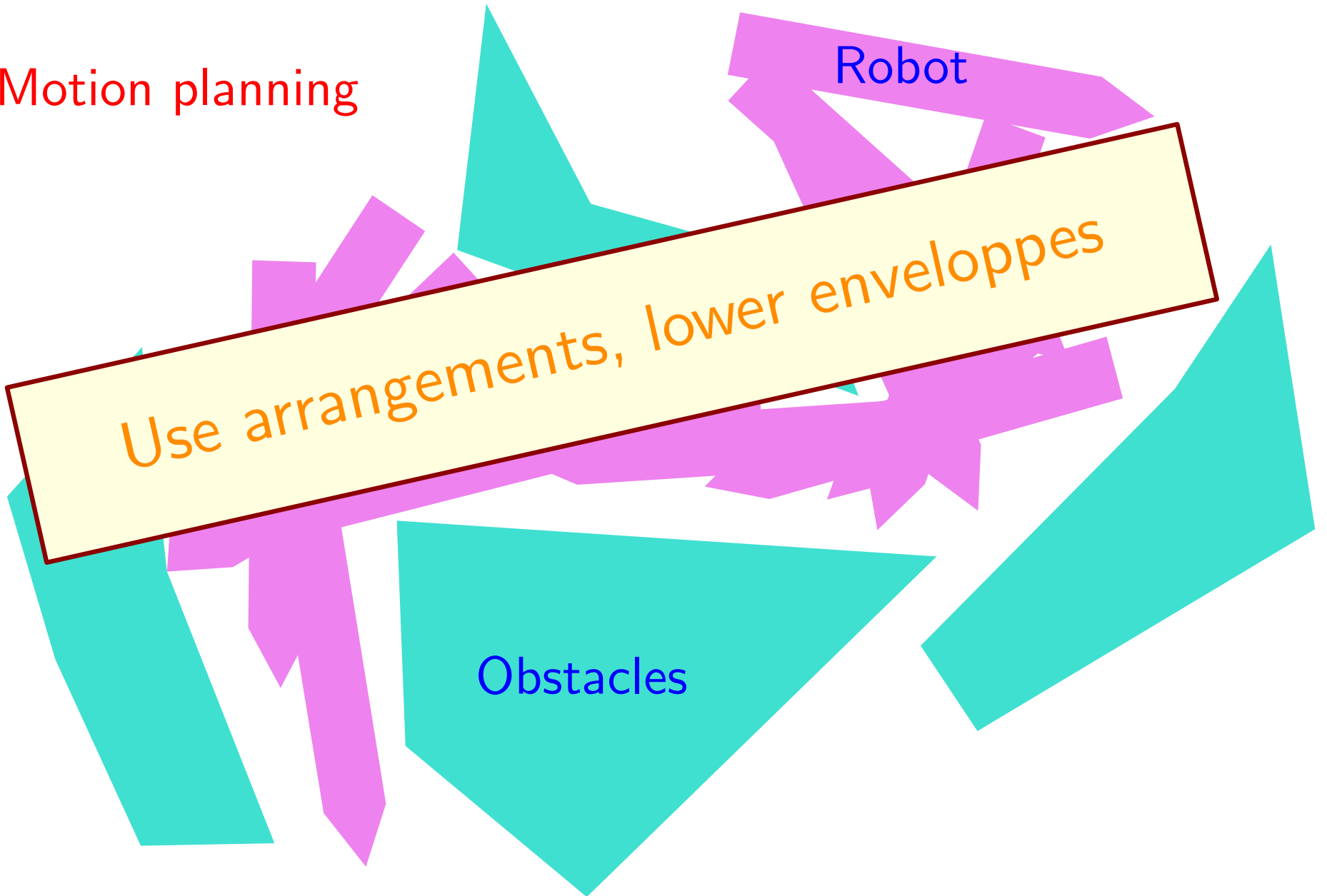
# Computational geometry usage

Motion planning

Robot

Use arrangements, lower envelopes

Obstacles





# Computational geometry, 1975-1985

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 1975-1985

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

Fit real life data

For  $n$  big enough

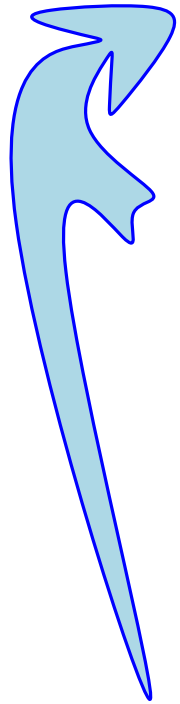
Does it exist

Real life data

Don't degeneracies exist?



# Computational geometry, 1975-1985



Complicated algorithms

Worst case complexities

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Fit real life data ?

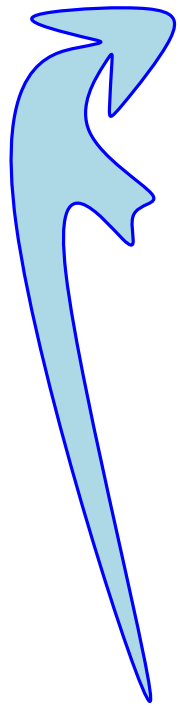
For  $n$  big enough ?

Does it exist ?

Real life data ?

Don't degeneracies exist ?

# Computational geometry, 1975-1985



Complicated algorithms

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Not used in practice

Fit real life data ?

For  $n$  big enough ?

Does it exist ?

Real life data ?

Don't degeneracies exist ?

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

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General position hypothesis

# Computational geometry, 1985-2000

Simpler

~~Complicated algorithms~~

Worst case complexities

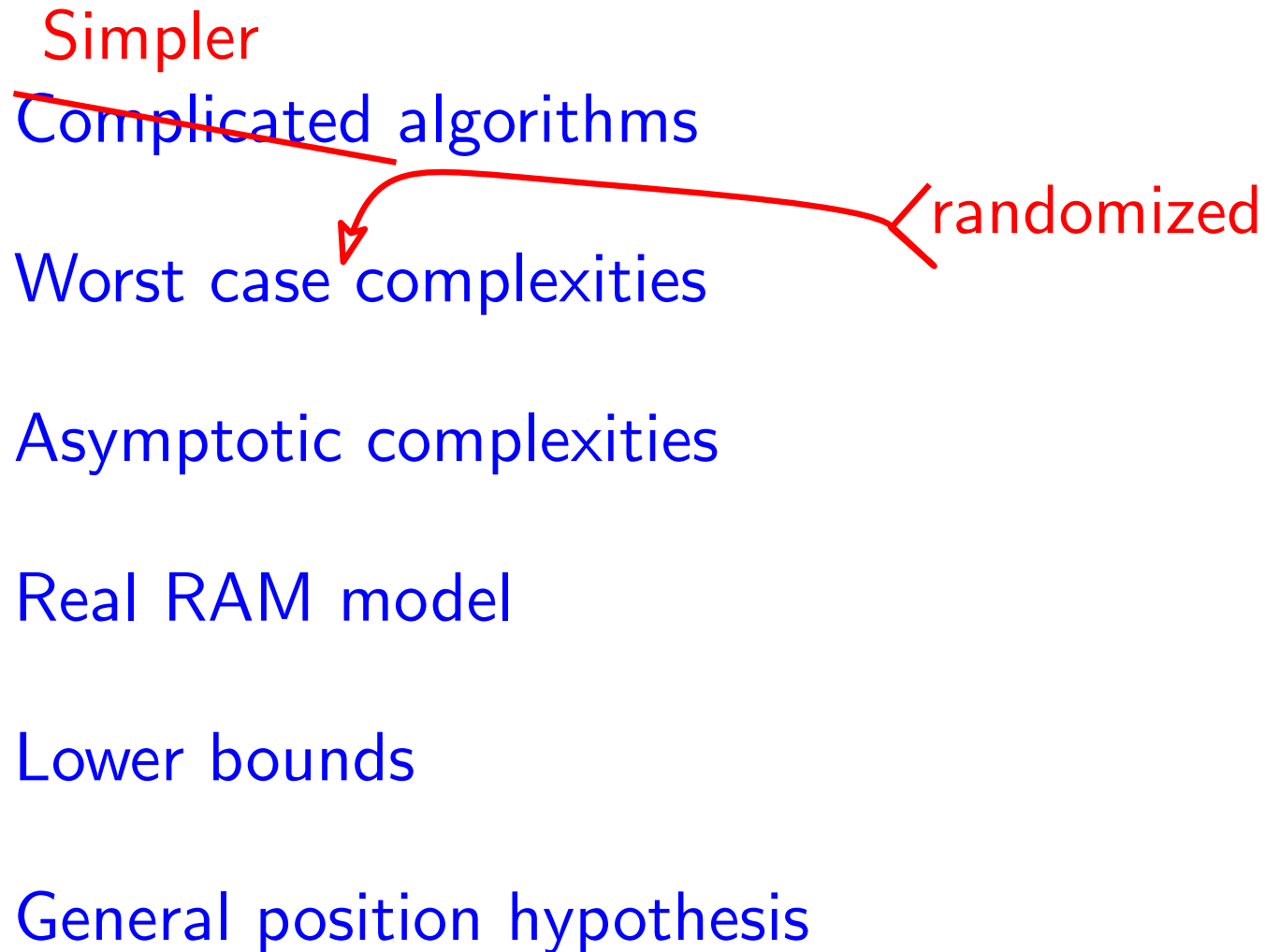
Asymptotic complexities

Real RAM model

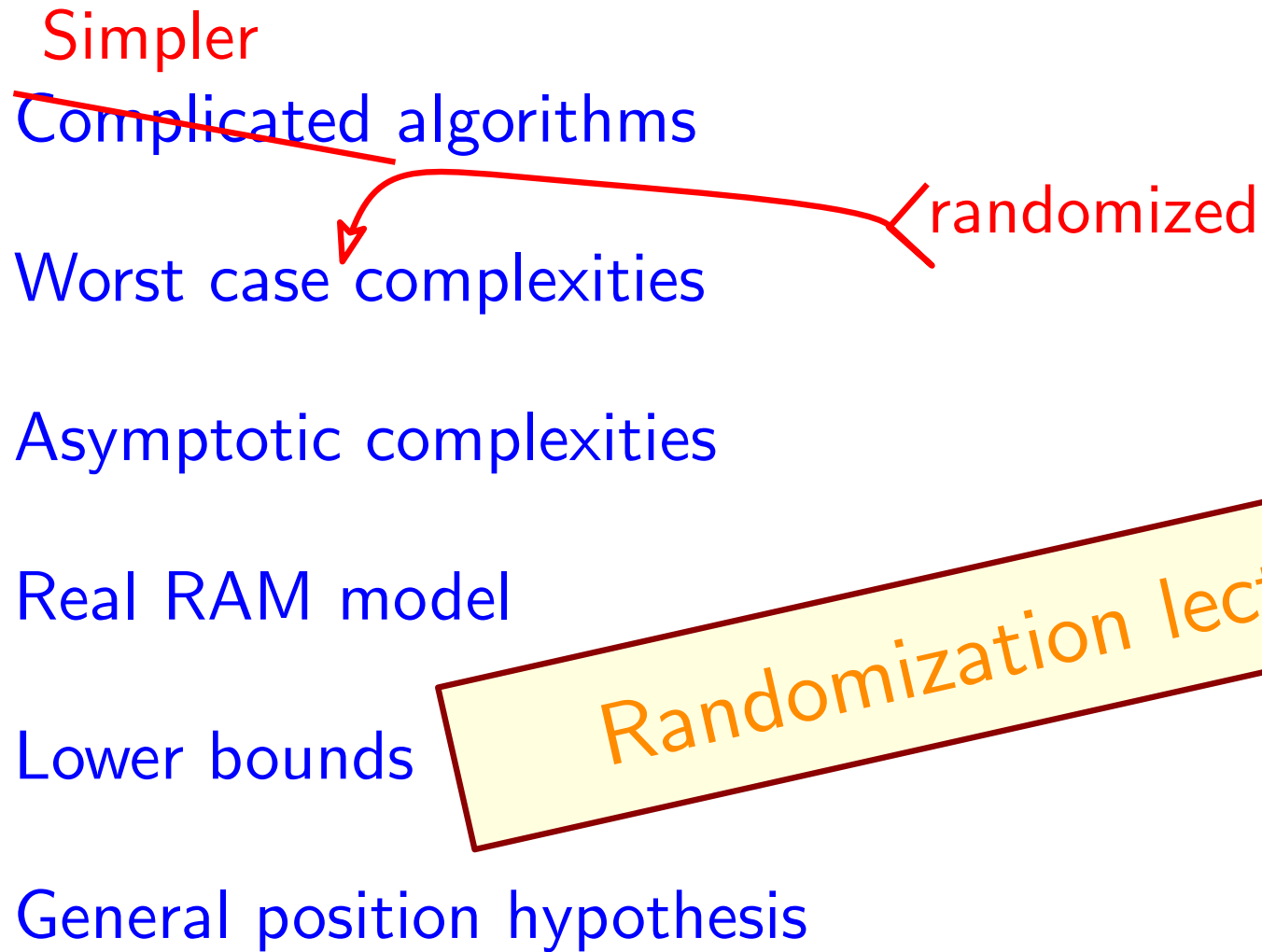
Lower bounds

General position hypothesis

# Computational geometry, 1985-2000



# Computational geometry, 1985-2000





# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model      address robustness issues

Lower bounds

General position hypothesis      solve degeneracies

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic

Robustness lecture

Real RAM model

address robustness issues

Lower bounds

General position hypothesis

solve degeneracies

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Just really code it

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 1985-2000

Complicated algorithms

Worst case complexities

Asymptotic complexities

Just really code it

Real RAM model

Lower bounds

General position hypothesis

A yellow rectangular box with a red border, tilted upwards from left to right. Inside the box, the letters 'CGAL' are written in a bold, yellow, sans-serif font. Each letter is superimposed on a faint, light gray geometric diagram: 'C' is on a circle with radial lines, 'G' is on a square with internal lines, 'A' is on a triangle with internal lines, and 'L' is on a square with internal lines.

CGAL

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Probabilistic hypotheses

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# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Probabilistic hypotheses

Old (and recent) math literature

Asymptotic complexities

Real RAM model

Lower bounds

General position hypothesis

# Computational geometry, 2000-

Complicated algorithms

Worst case complexities

Probabilistic hypotheses

Old (and recent) math literature

Asymptotic complexities

Real RAM model

Lower bounds

Poisson Delaunay lecture

General position hypothesis



# Computational geometry, 2000-

Complicated algorithms

Beyond the Euclidean realm

Worst case complexities

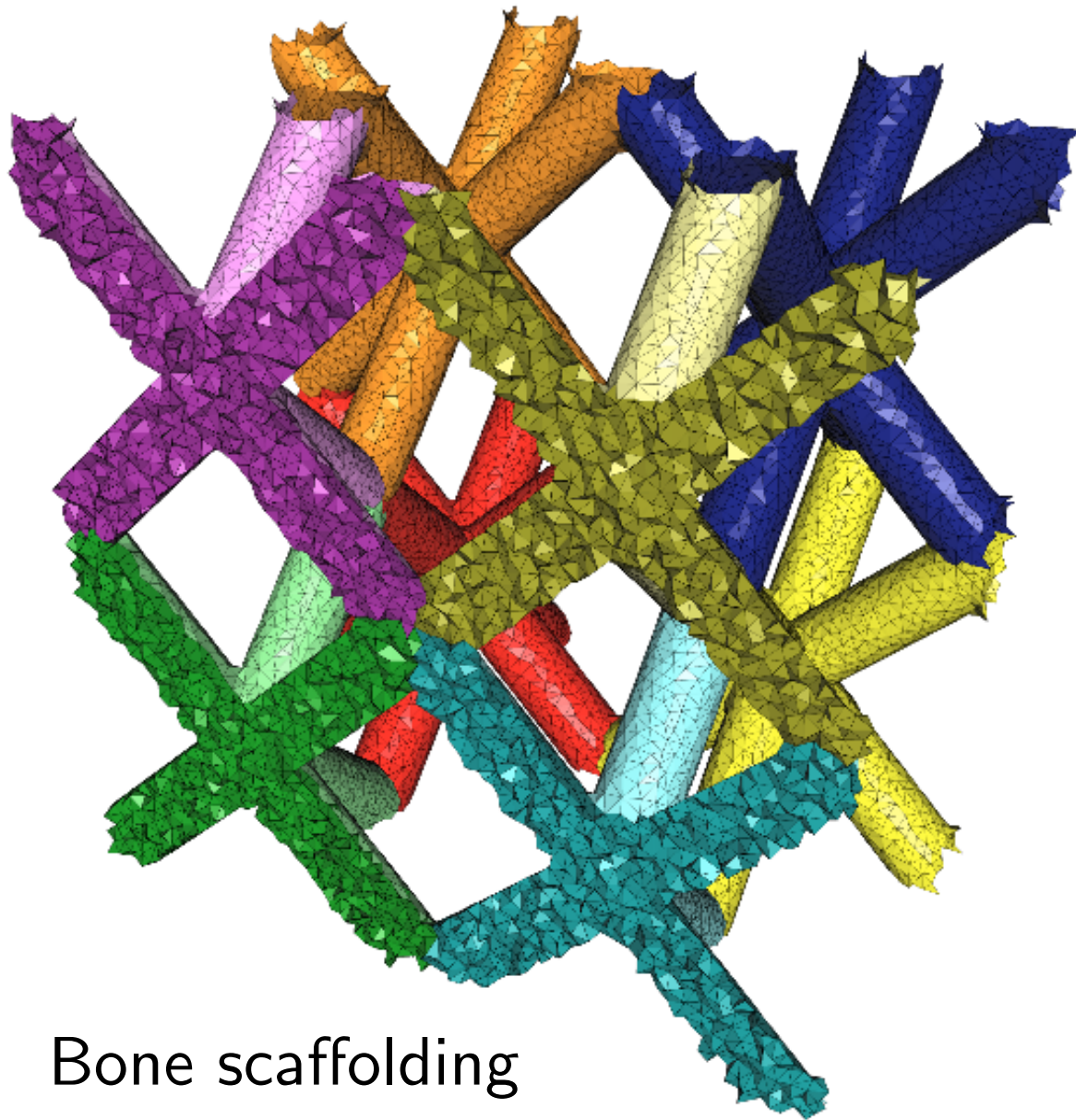
Asymptotic complexities

Real RAM model

Lower bounds

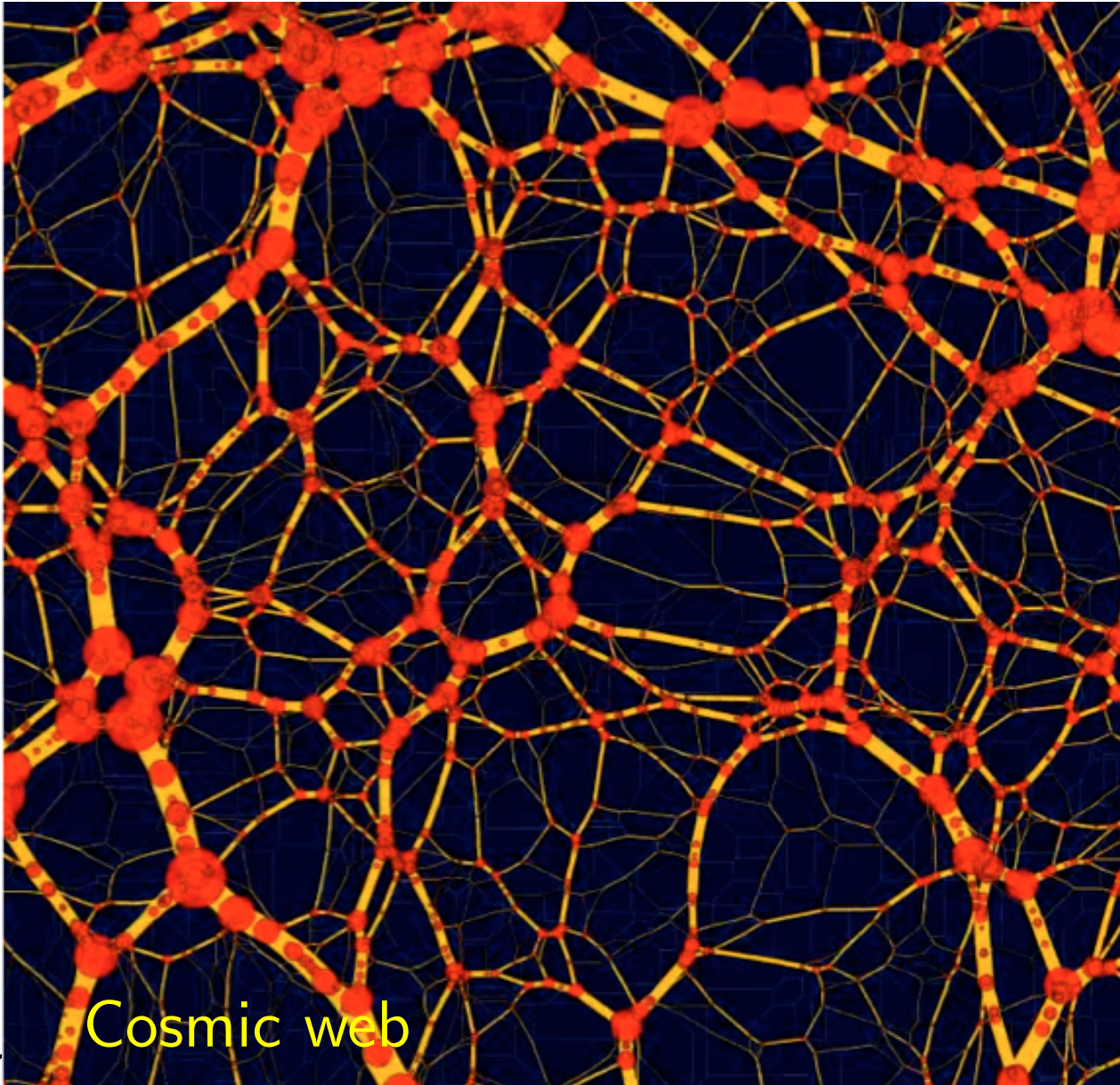
General position hypothesis

# Computational geometry, 2000-



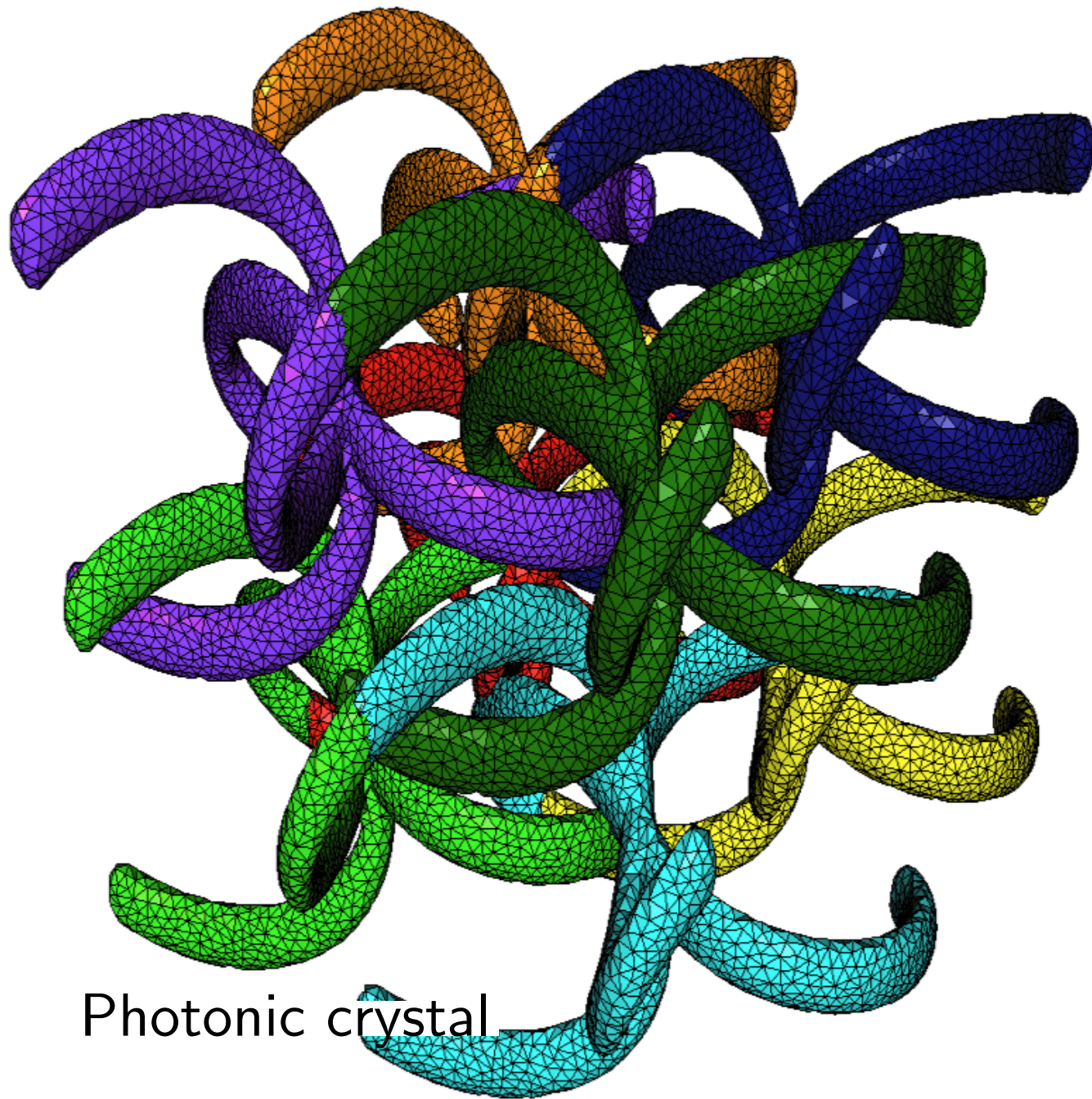
Bone scaffolding

# Computational geometry, 2000-



Cosmic web

# Computational geometry, 2000-



Photonic crystal

# Computational geometry, 2000-

Complicated algorithms

Beyond the Euclidean realm

Worst case complexities

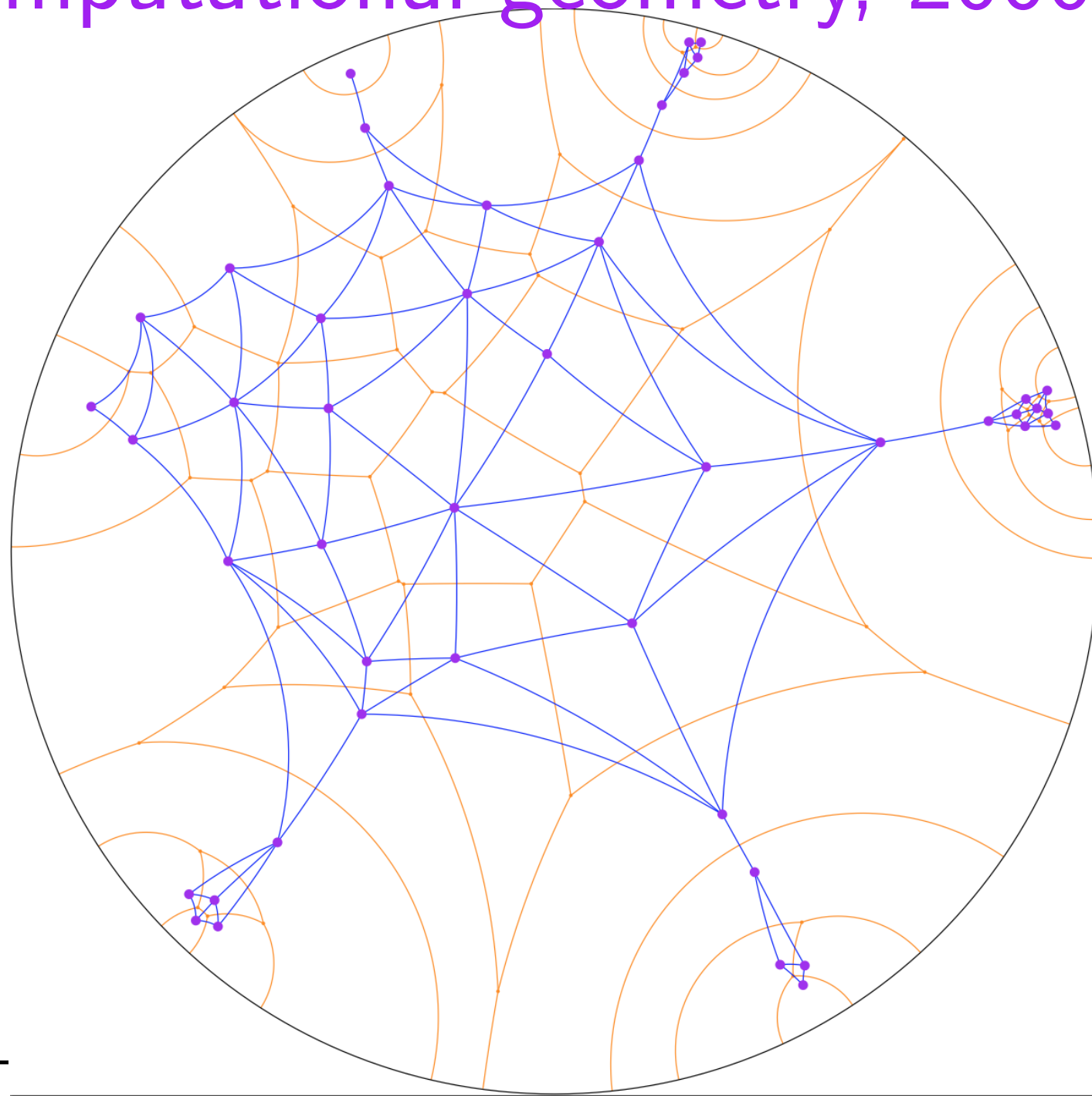
Asymptotic complexities

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# Computational geometry, 2000-



# Computational geometry, 2000-

