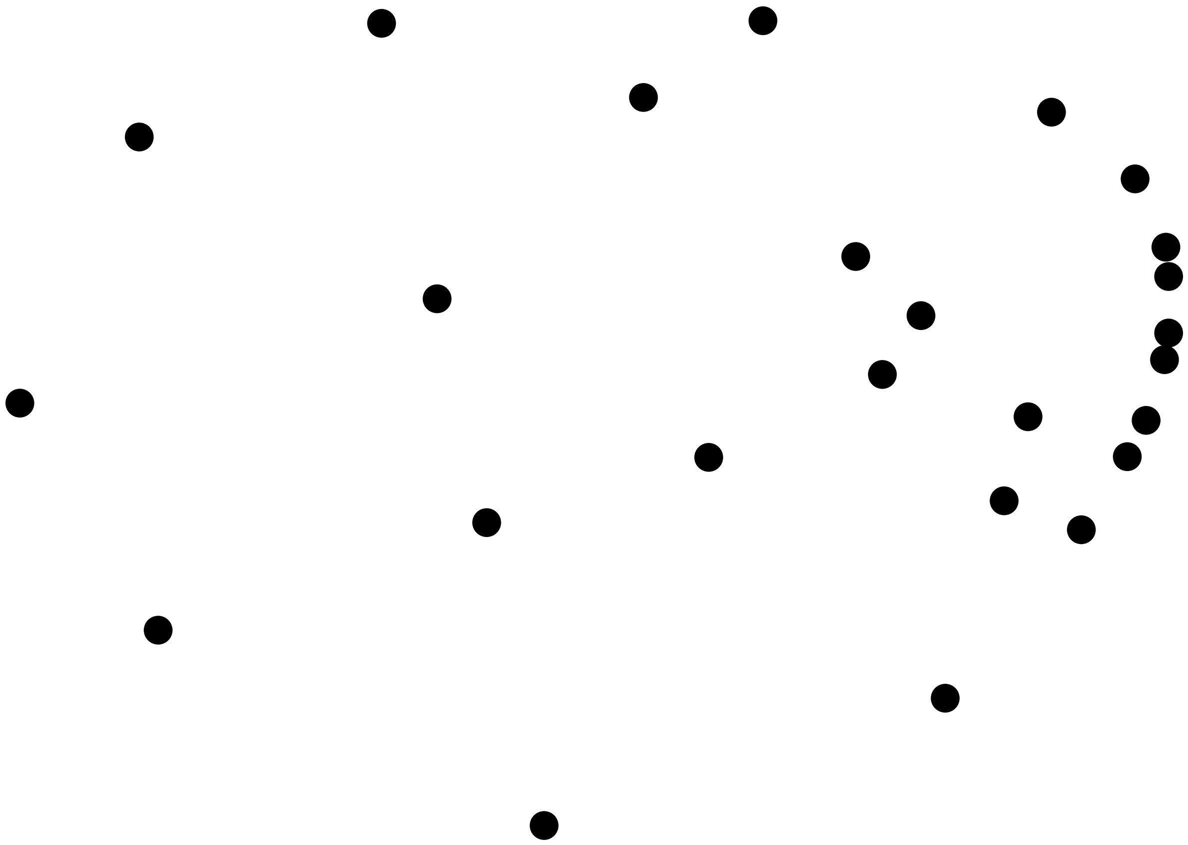
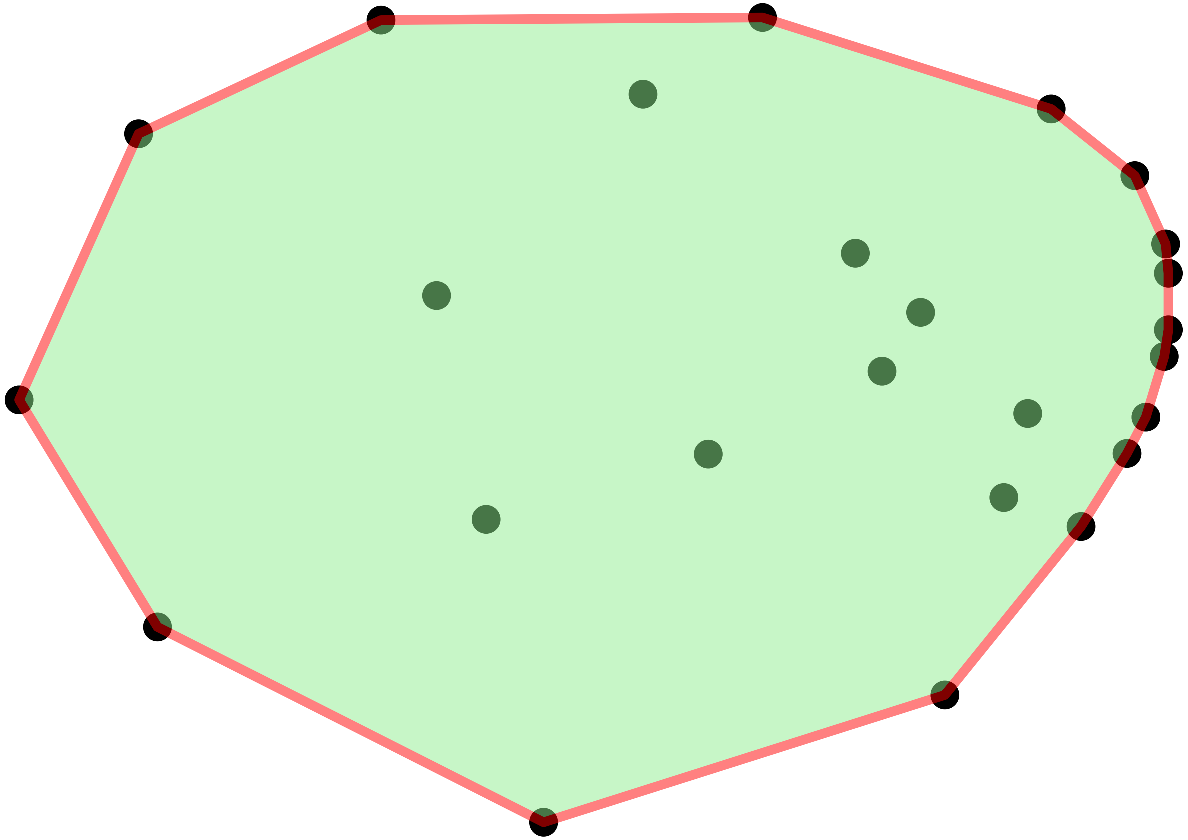
Convex hull

Convex hull



Convex hull



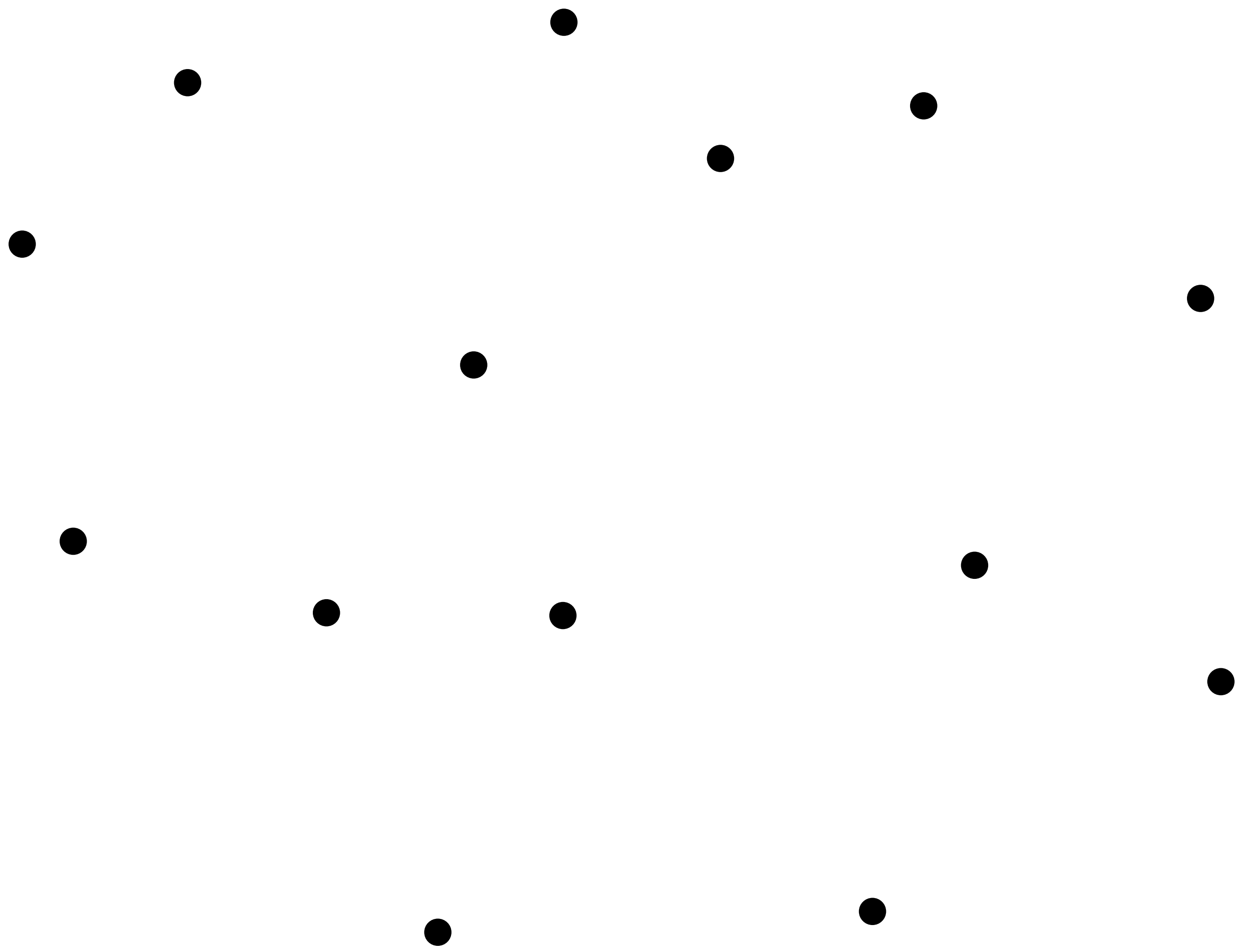
Convex hull

- Definition, extremal point
- Jarvis algorithm
- Orientation predicate
- Buggy degenerate example
- Real RAM model and general position hypothesis
- Graham algorithm
- Lower bound
- Three dimensions

Convex hull

Convex hull

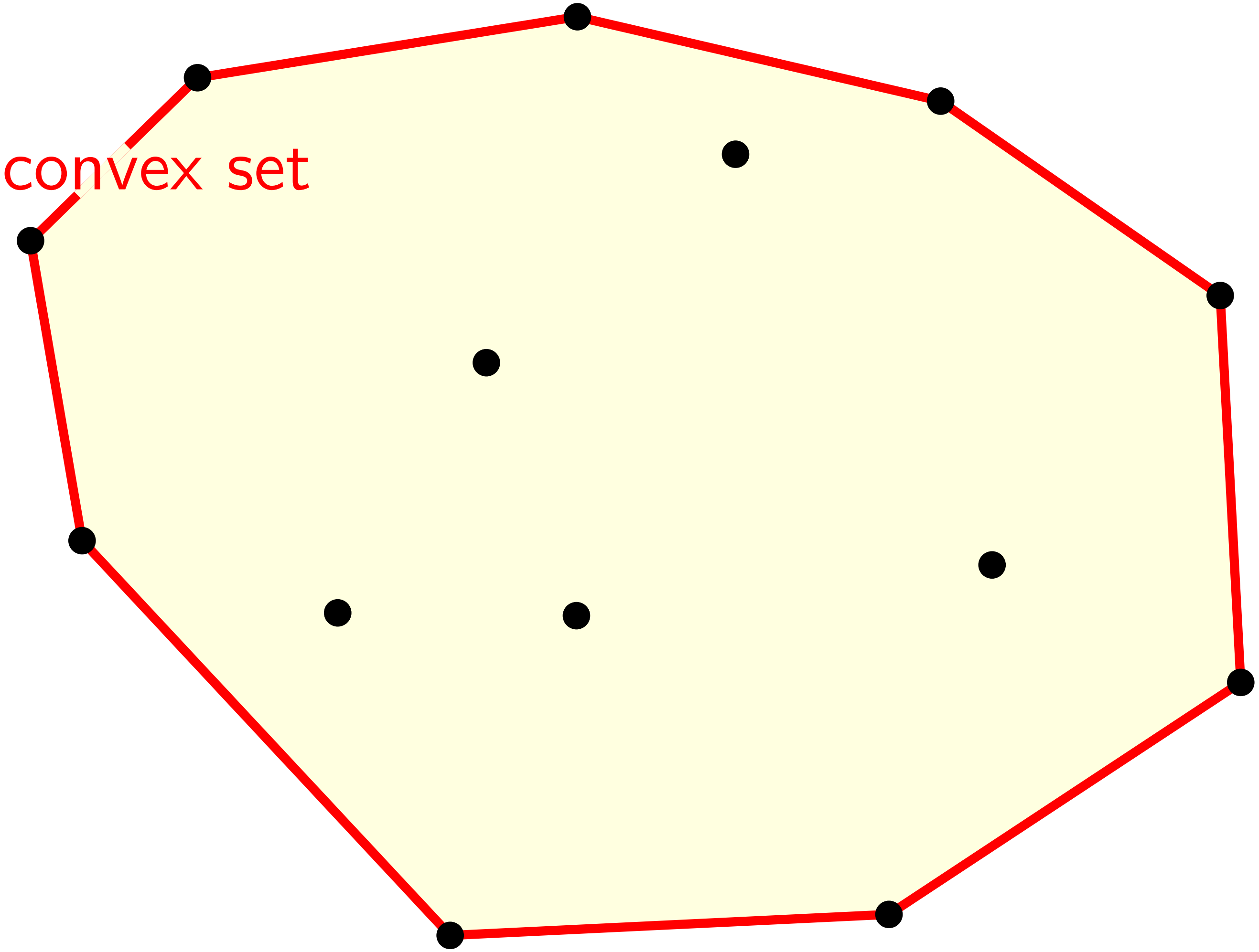
Set of points



Convex hull

Set of points

Smallest enclosing convex set

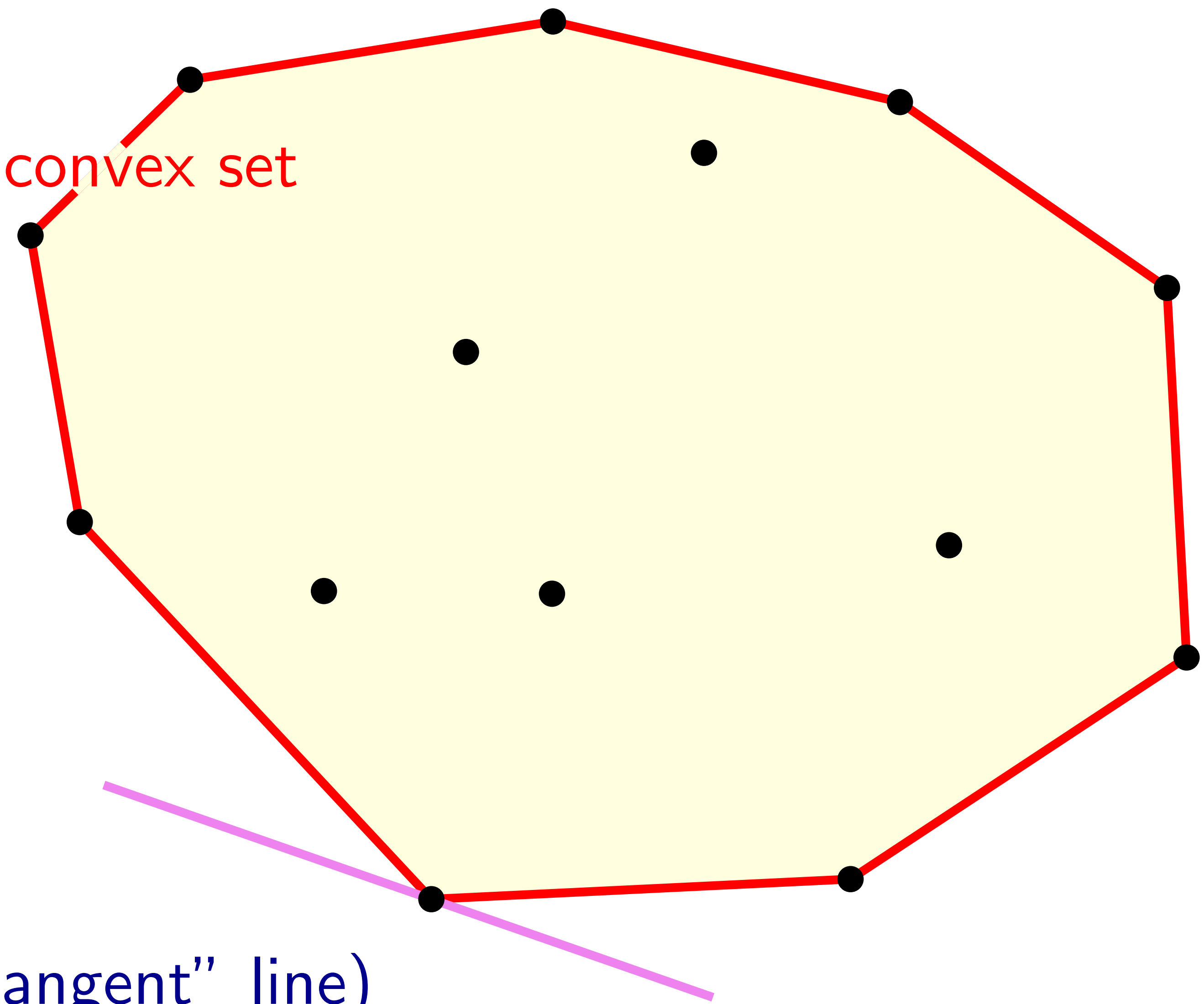


Convex hull

Definition, extremal point

Set of points

Smallest enclosing convex set

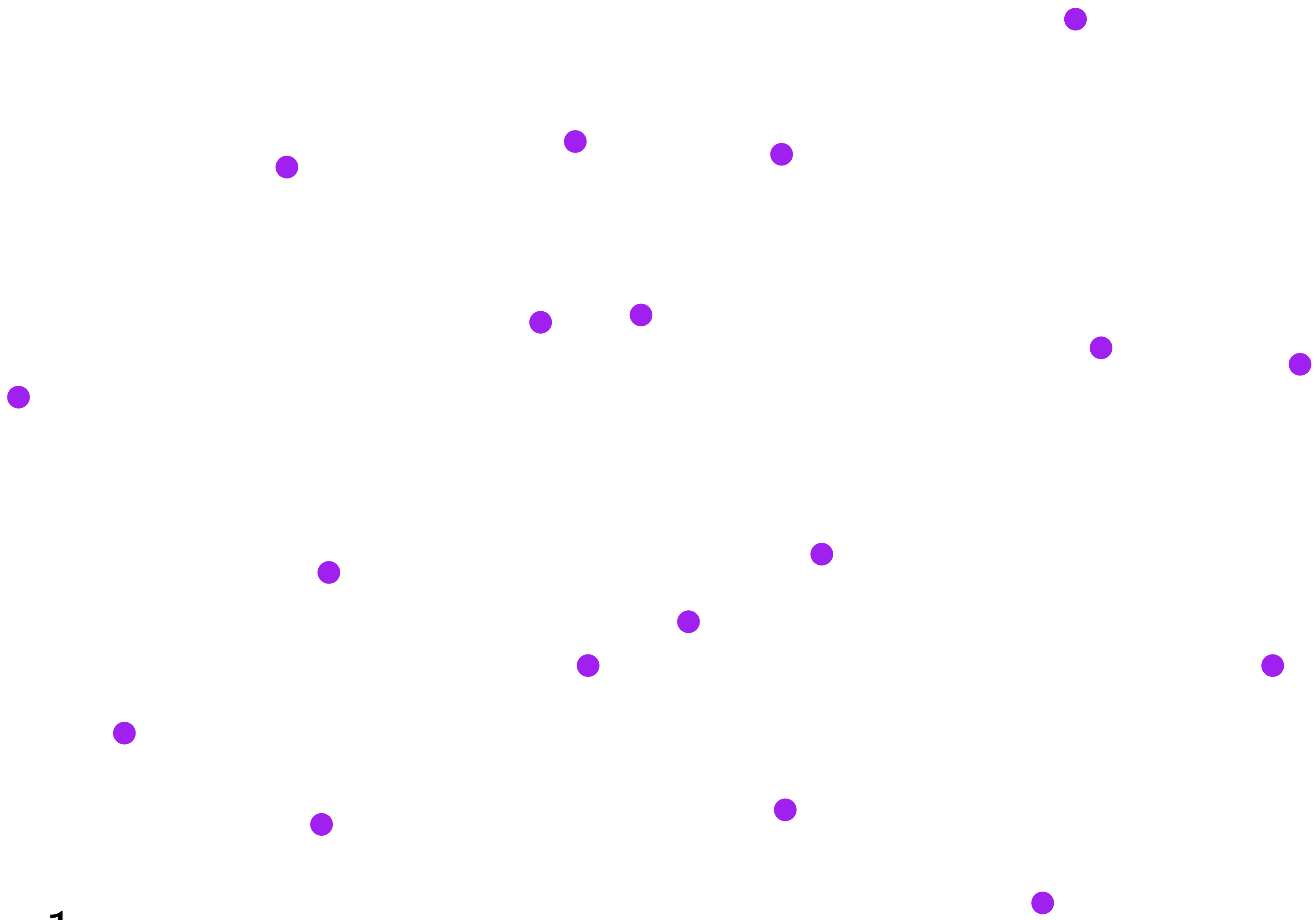


Extremal point

Supporting line ("tangent" line)

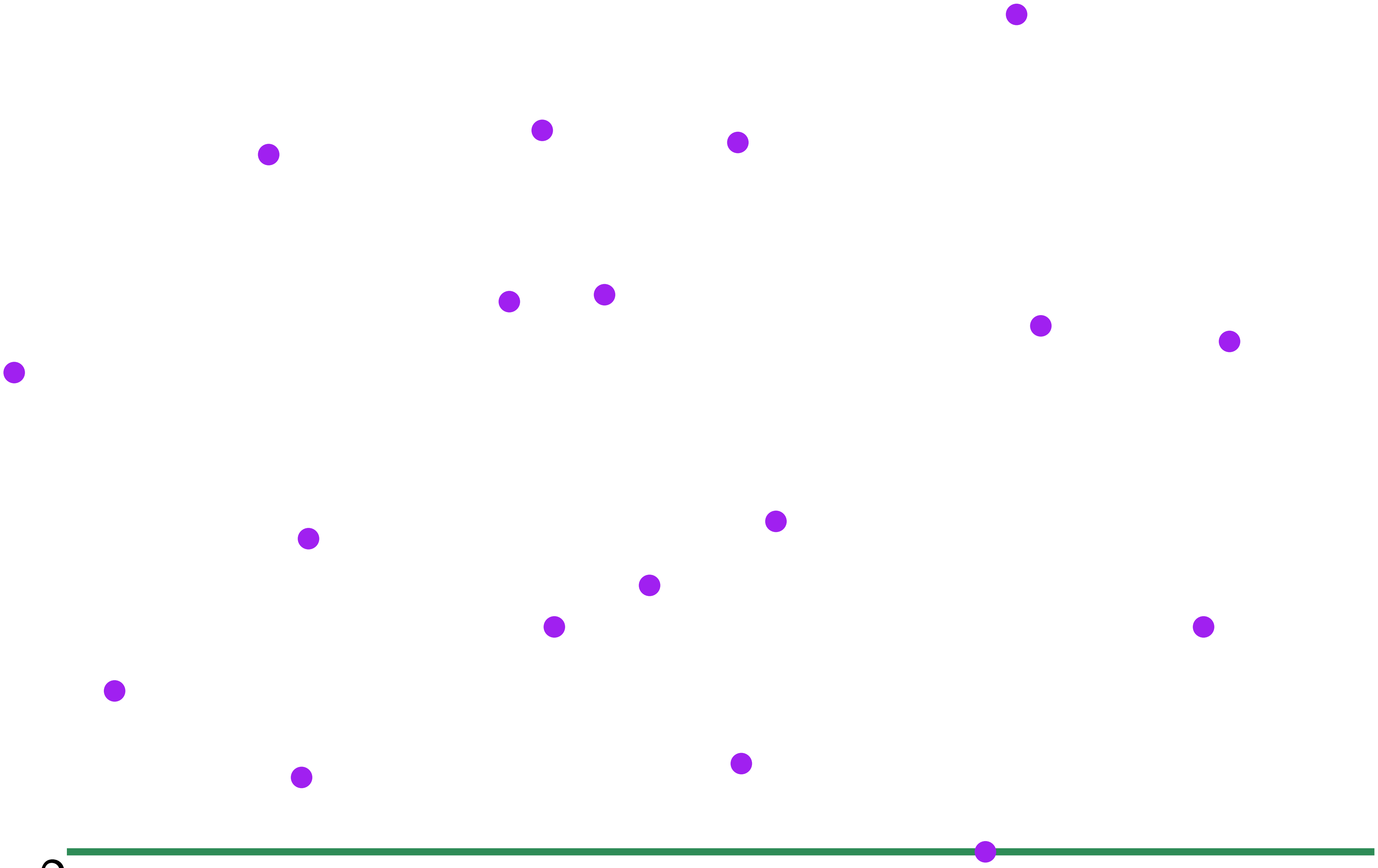
Convex hull

Jarvis algorithm



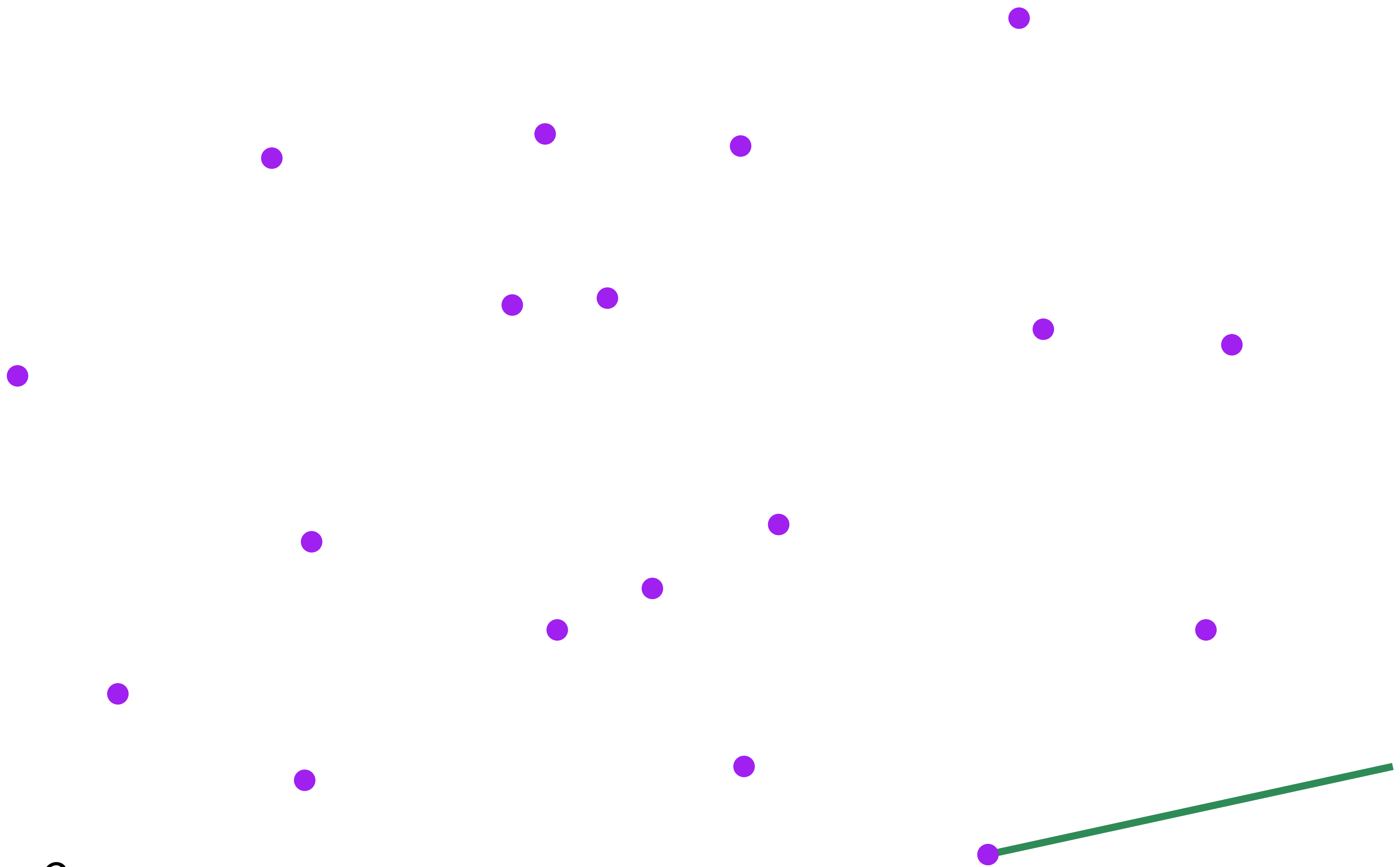
Convex hull

lowest point is extremal



Convex hull

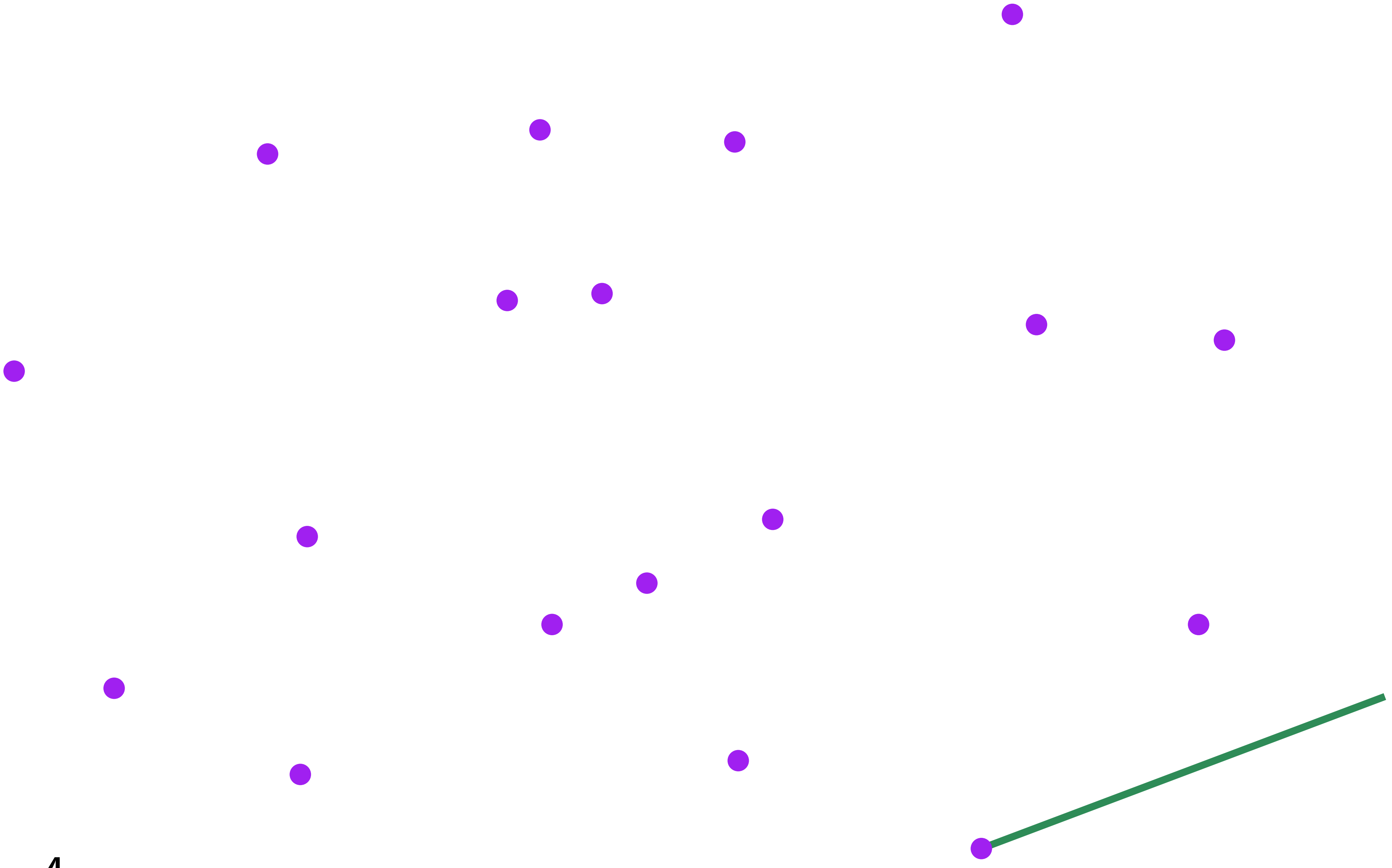
rotate



Convex hull

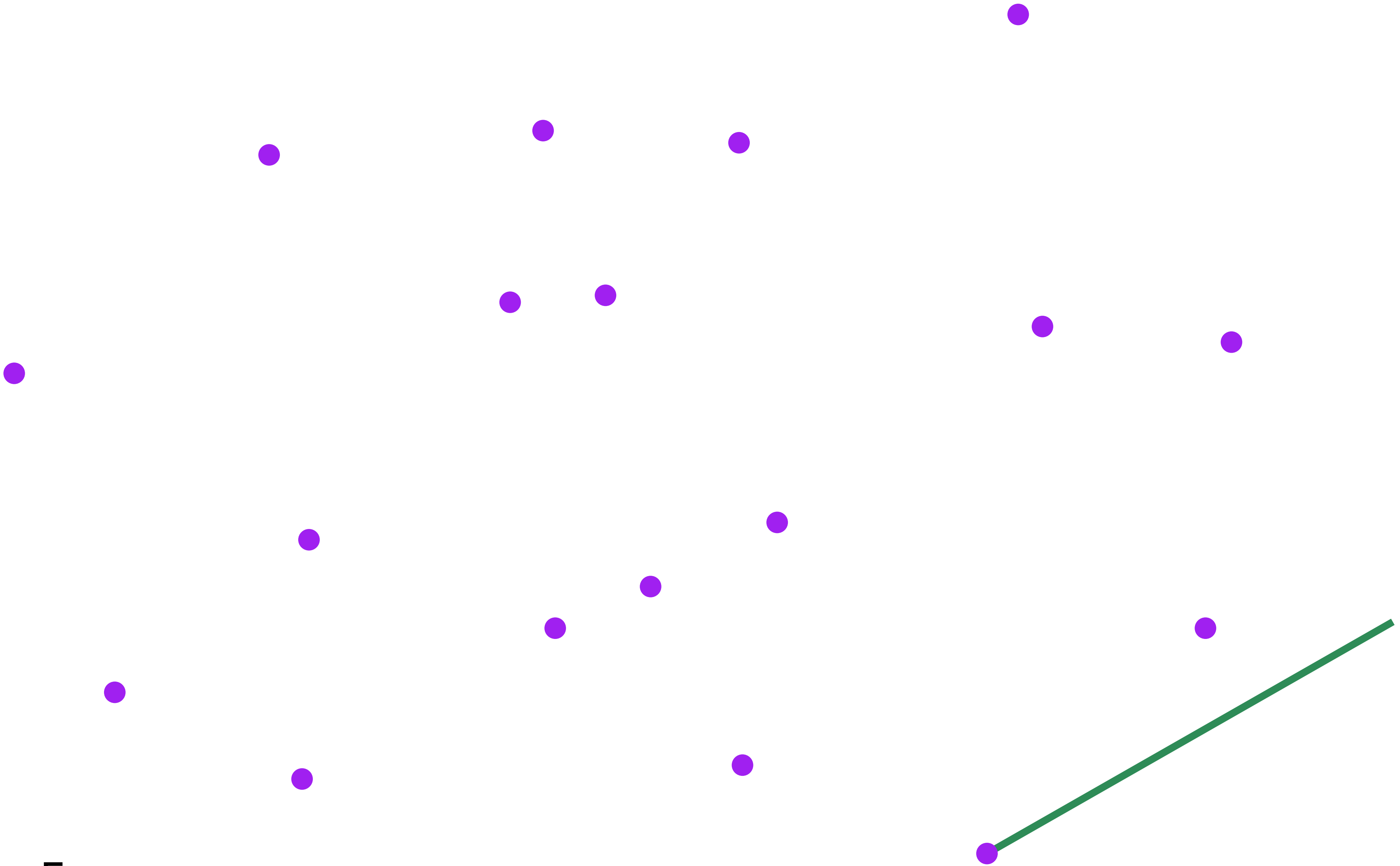
Jarvis algorithm

rotate



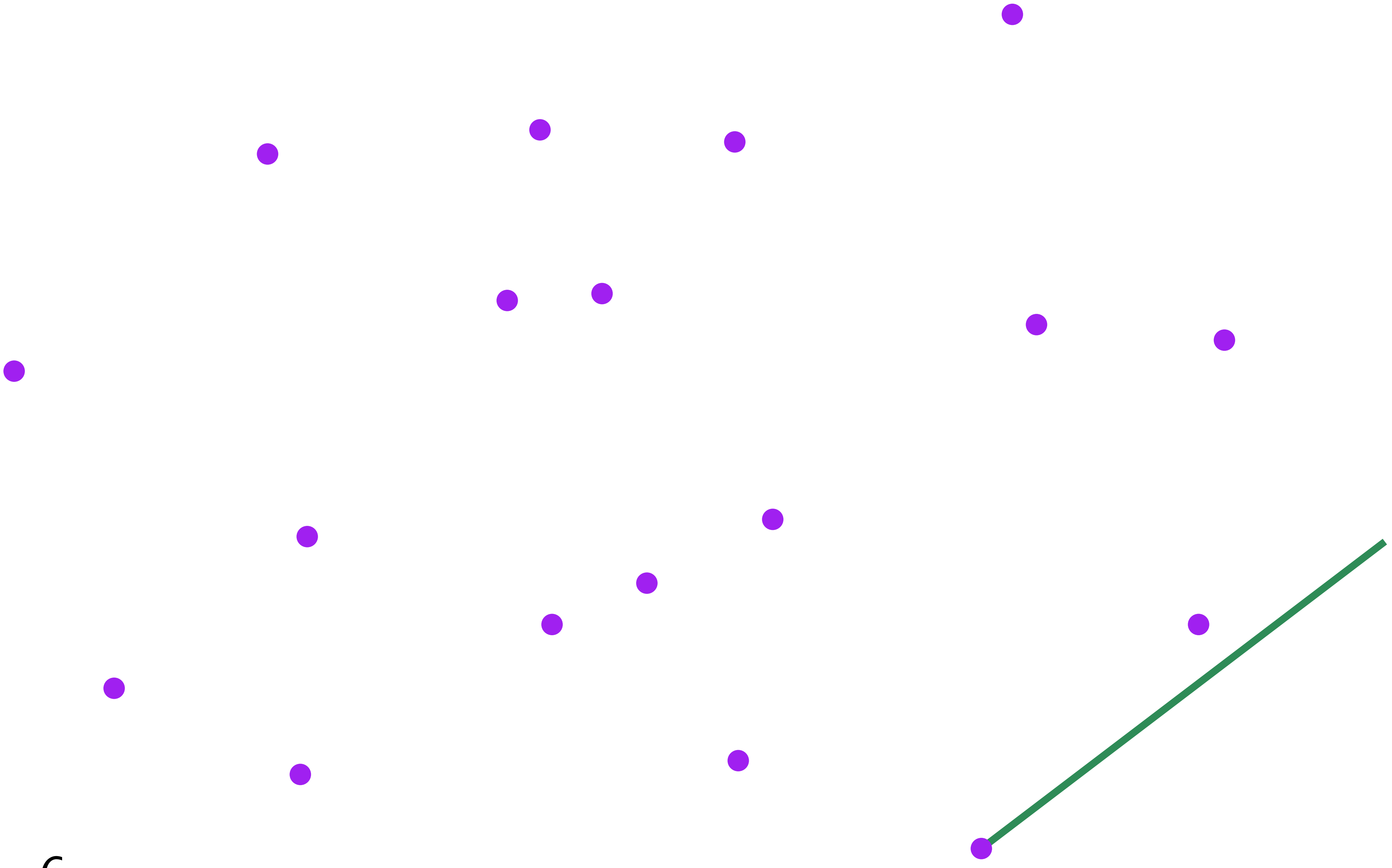
Convex hull

rotate



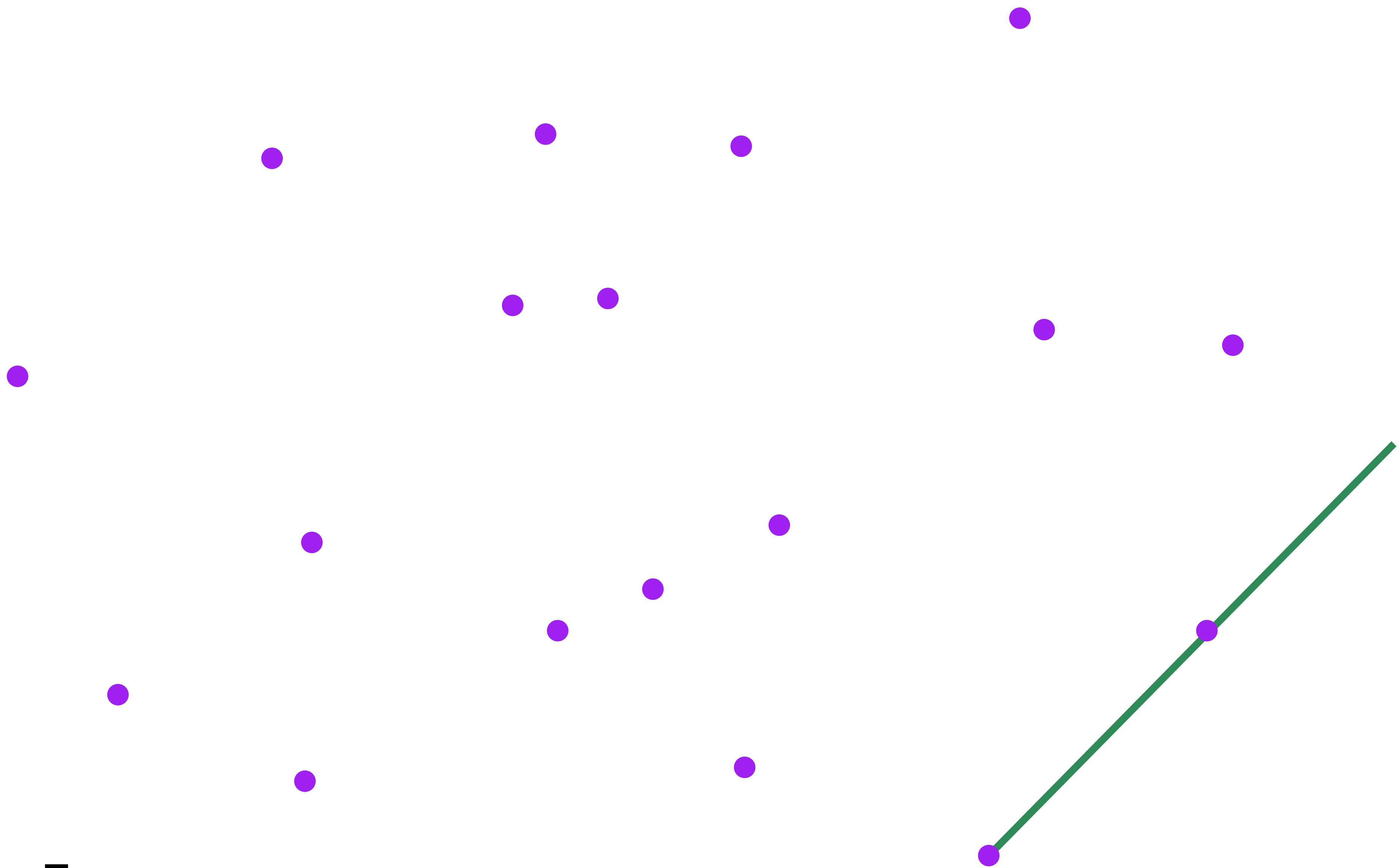
Convex hull

rotate



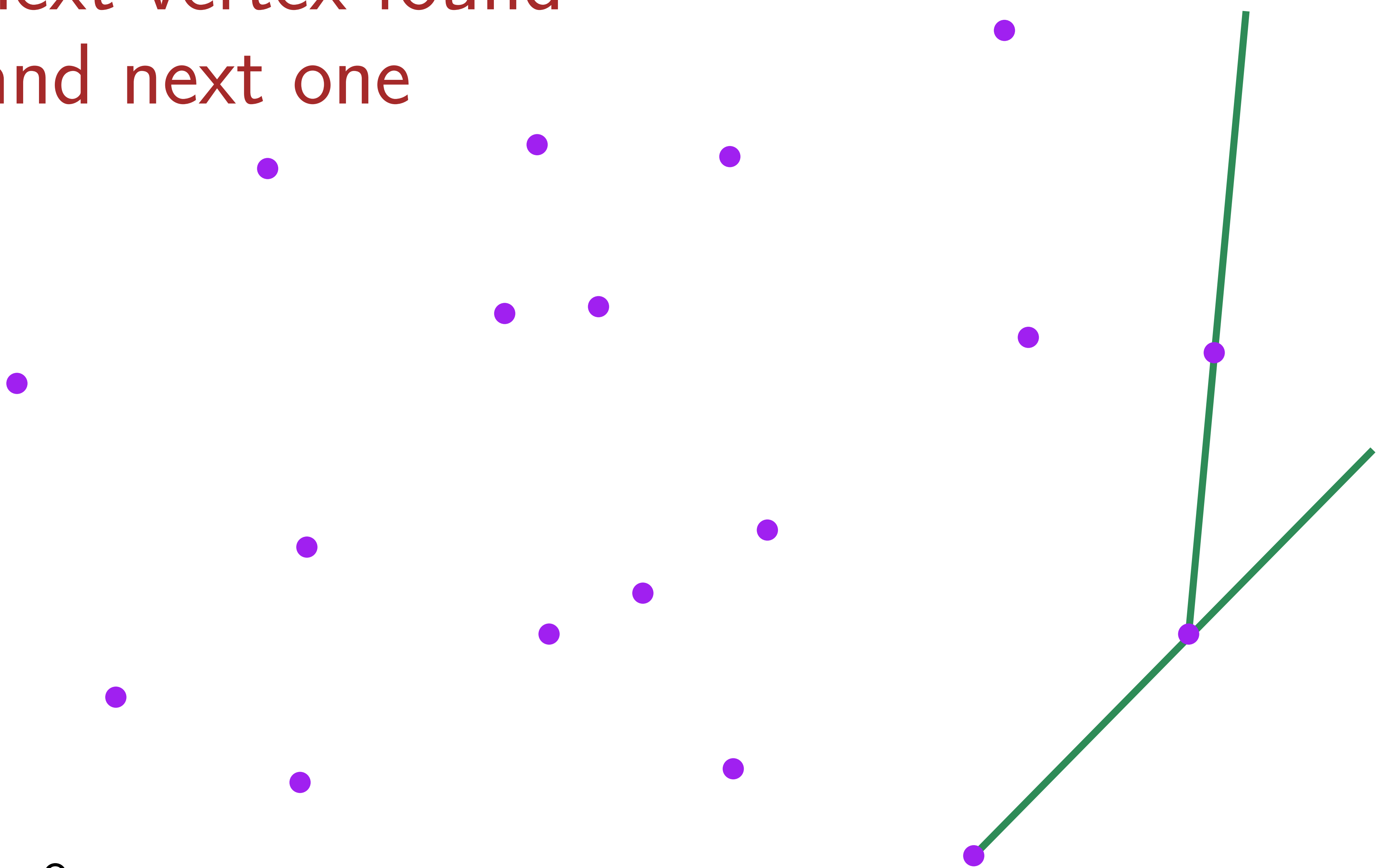
Convex hull

next vertex found



Convex hull

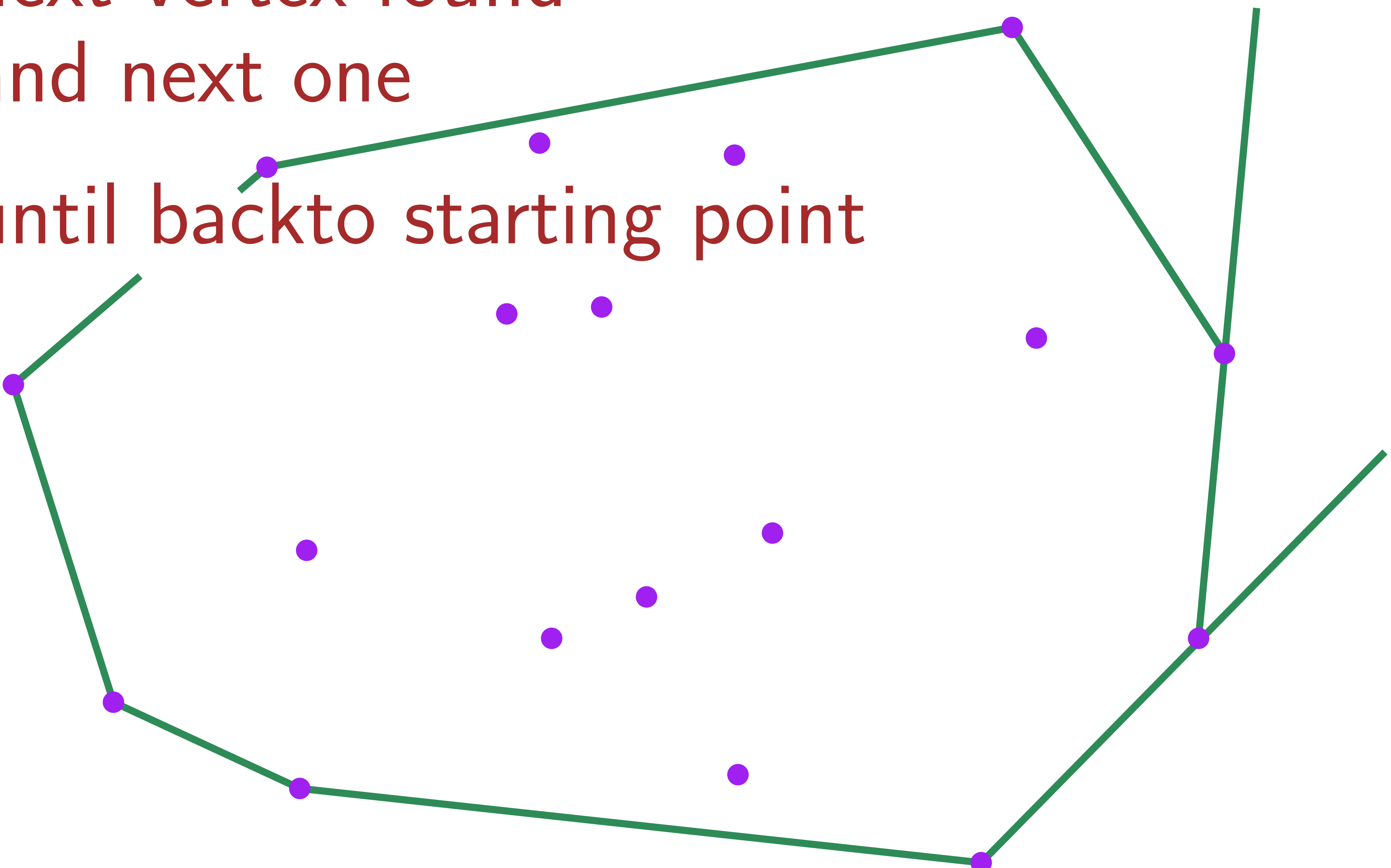
next vertex found
and next one



Convex hull

next vertex found
and next one

until backto starting point



Convex hull

Input : point set S

$u =$ lowest point in S; $min = \infty$

For each $w \in S \setminus \{u\}$

  if $angle(ux, uw) < min$

    then $min = angle(ux, uw)$; $v = w$;

$u.next = v$;

Do

  $S = S \setminus \{v\}$

  For each $w \in S$

    $min = \infty$

    if $angle(v.pred\ v, vw) < min$

     then $min = angle(v.pred\ v, vw)$; $v.next = w$;

  $v = v.next$;

While $v \neq u$

Convex hull

Complexity ?

Jarvis algorithm

Input : point set S

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Convex hull

Jarvis algorithm

Complexity ?

Input : point set S

u = lowest point in S; $min = \infty$

For each $w \in S \setminus \{u\}$ $O(n)$

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While $v \neq u$

$O(n)$

Convex hull

Complexity ?

Jarvis algorithm

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For each $w \in S \setminus \{u\}$

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Do

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For each $w \in S$

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if $angle(v.pred\ v, vw) < min$

then $min = angle(v.pred\ v, vw)$; $v.next = w$;

$v = v.next$;

While $v \neq u$

$O(n^2)$

Convex hull

Jarvis algorithm

Complexity ?

Input : point set S

u = lowest point in S; $min = \infty$

For each $w \in S \setminus \{u\}$

  if $angle(ux, uw) < min$

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    if $angle(v.pred\ v, vw) < min$

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  $v = v.next$;

While $v \neq u$

$O(n^2)$

$O(nh)$

Convex hull

Orientation predicate

Input : point set S

u = lowest point in S; $min = \infty$

For each $w \in S \setminus \{u\}$

if $angle(ux, uw) < min$

then $min = angle(ux, uw)$; $v = w$;

$u.next = v$;

Do

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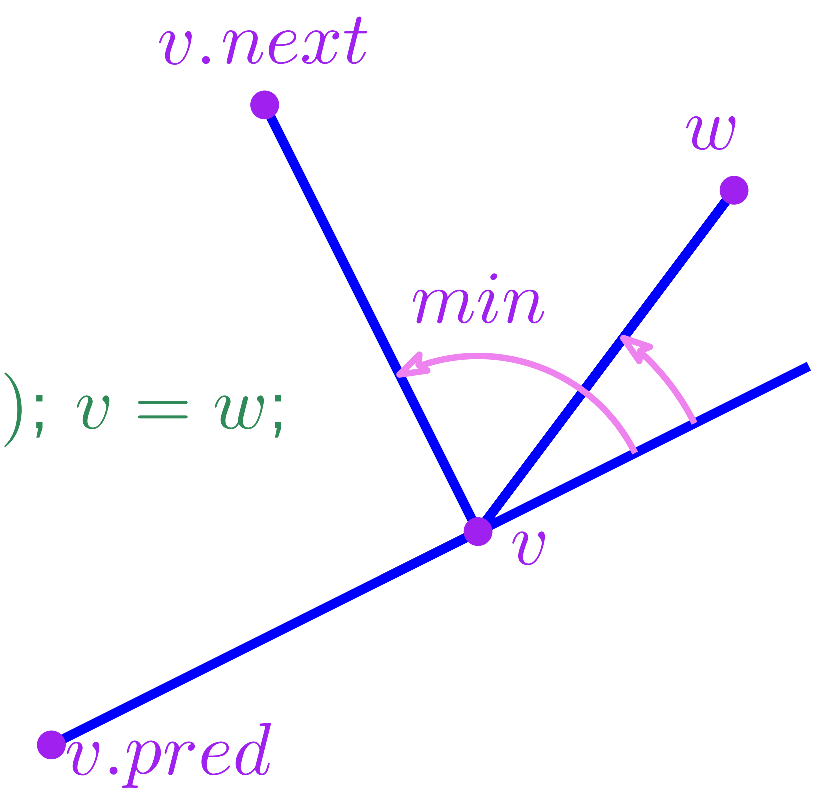
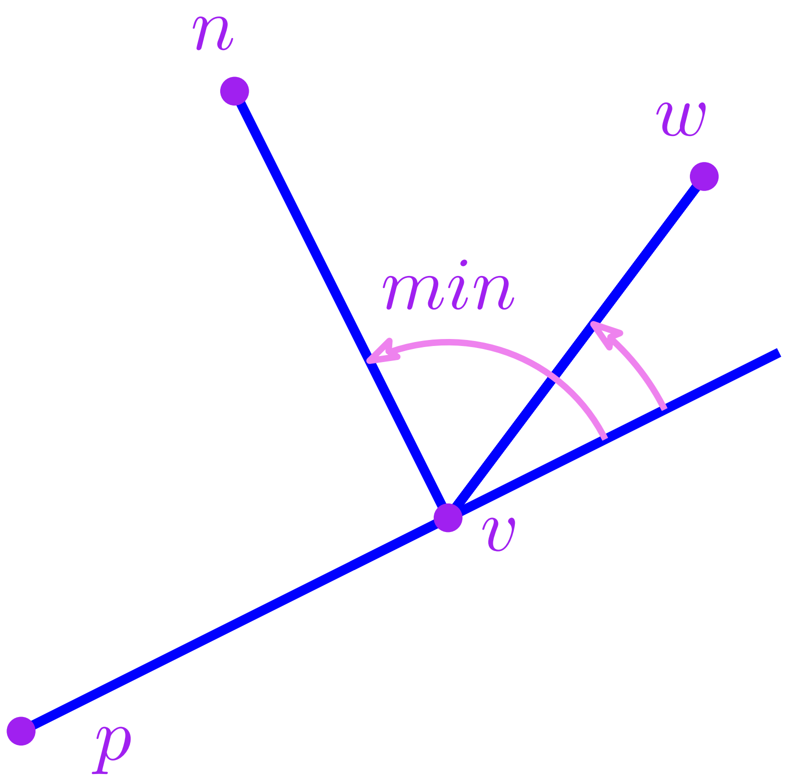
While $v \neq u$



Convex hull

Orientation predicate

if $angle(pv, vw) < min$

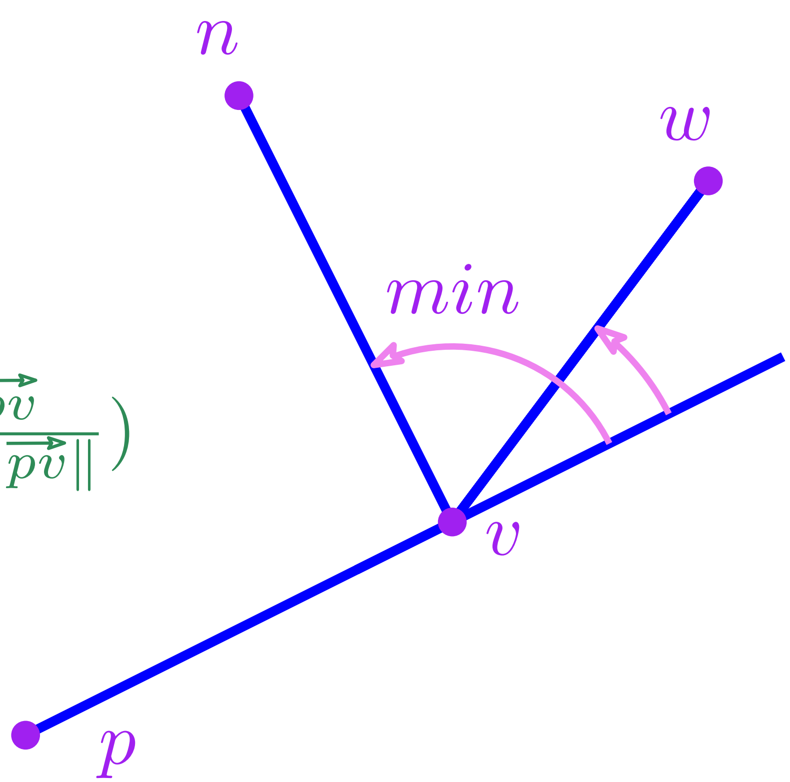


Convex hull

Orientation predicate

if $angle(pv, vw) < min$

$$angle(pv, vw) = \arccos\left(\frac{\overrightarrow{vw} \cdot \overrightarrow{pv}}{\|\overrightarrow{vw}\| \cdot \|\overrightarrow{pv}\|}\right)$$



Convex hull

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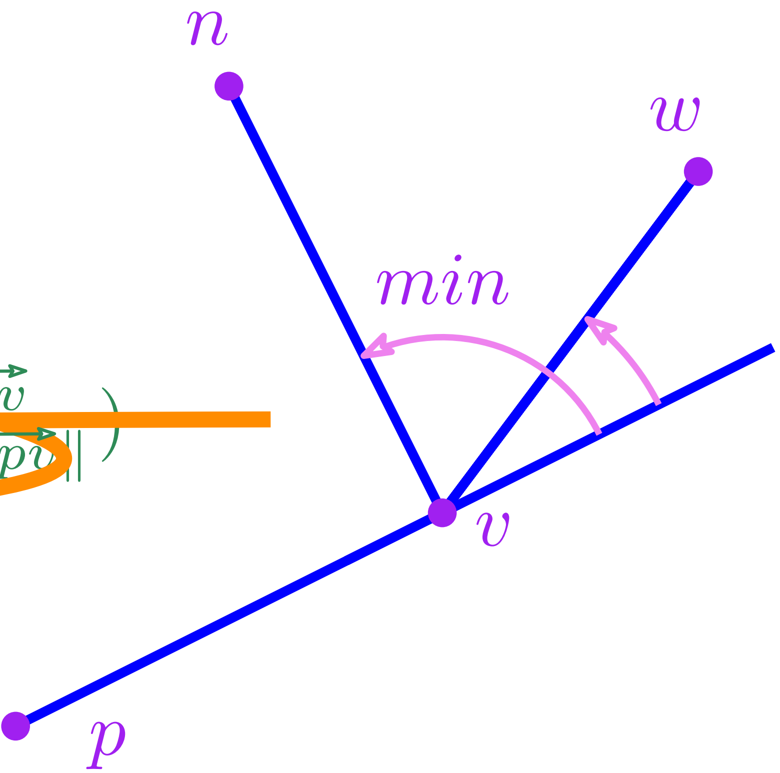

Convex hull

Orientation predicate

if $angle(pv, vw) < min$

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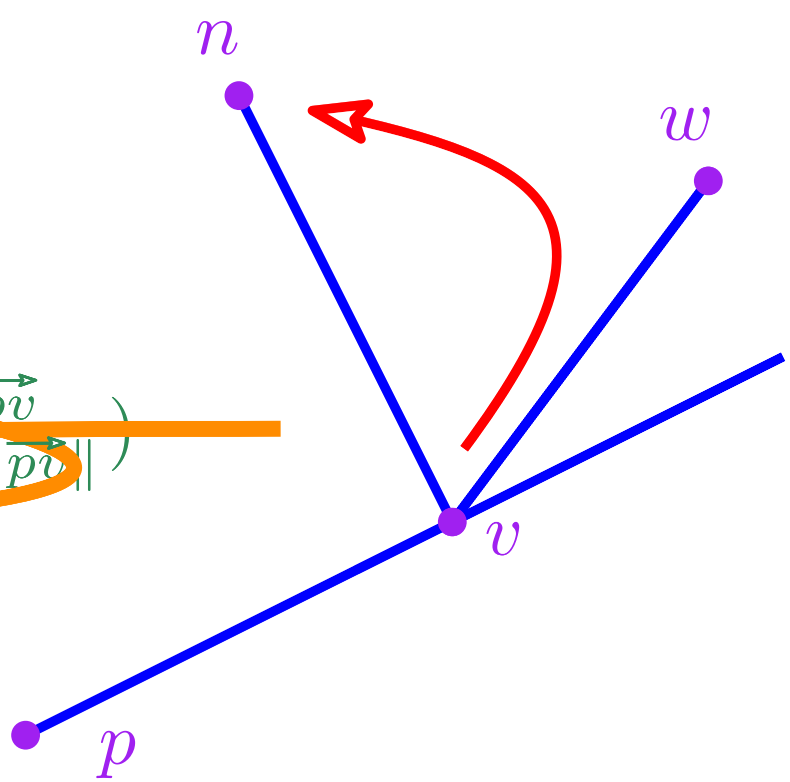
if vwn turn left



Convex hull

Orientation predicate

if $angle(pv, vw) < min$

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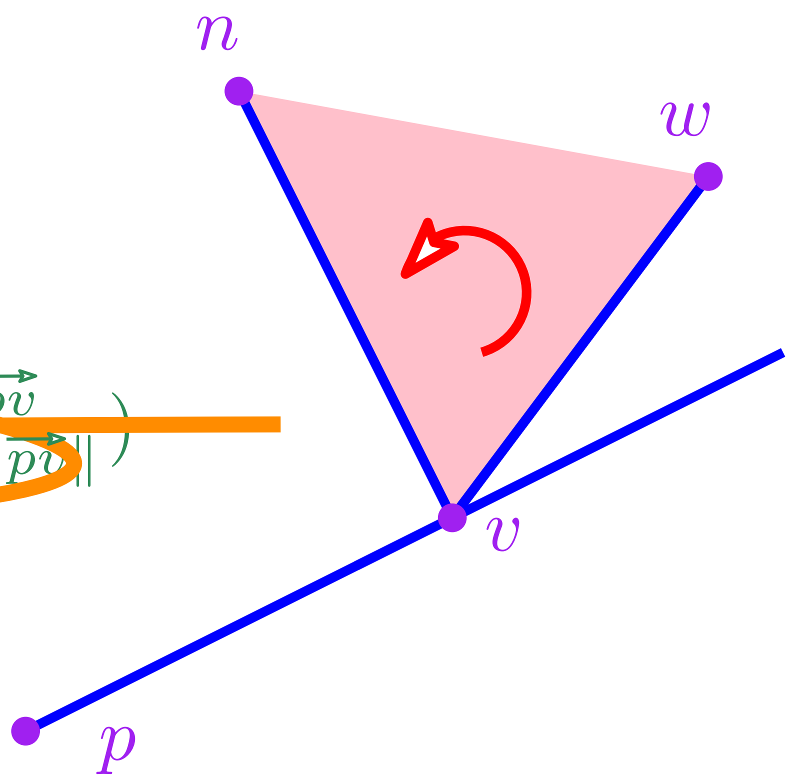


if vwn turn left

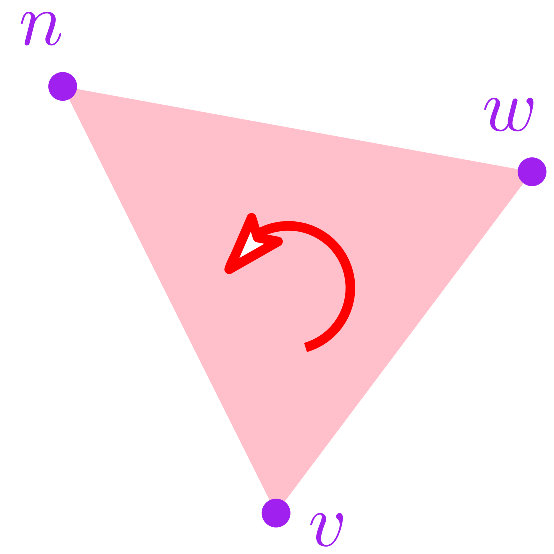
if triangle vwn counterclockwise (ccw)

if triangle vwn positively oriented

Convex hull

Orientation predicate

$vwn + ?$

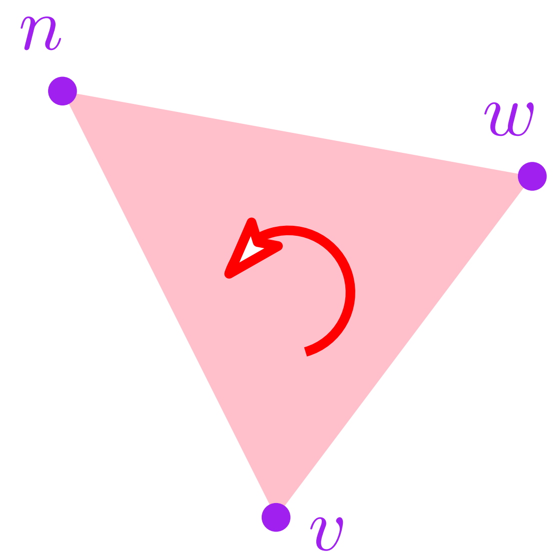


Convex hull

Orientation predicate

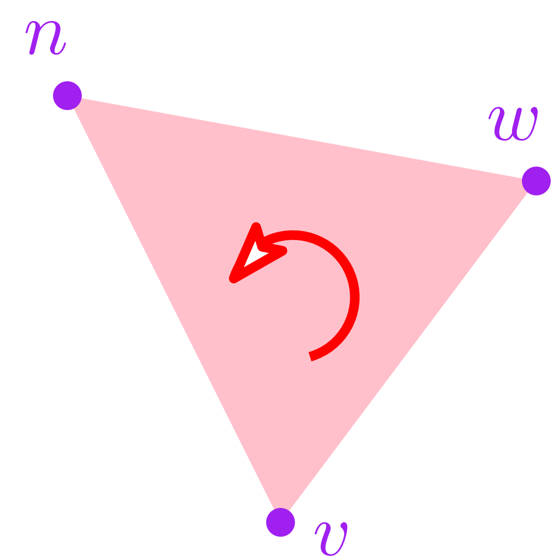
$vwn + ?$

$$\begin{vmatrix} x_w - x_v & x_n - x_v \\ y_w - y_v & y_n - y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} > 0$$



Convex hull

Orientation predicate

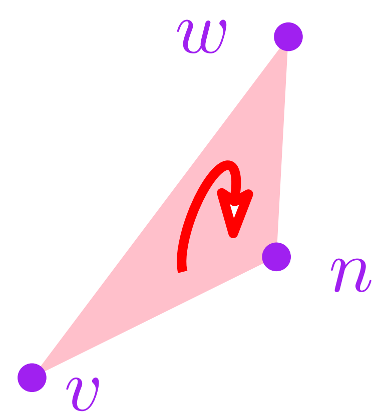


vwn + ?

$$\begin{vmatrix} x_w - x_v & x_n - x_v \\ y_w - y_v & y_n - y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} > 0$$

vwn - ?

$$\begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} < 0$$

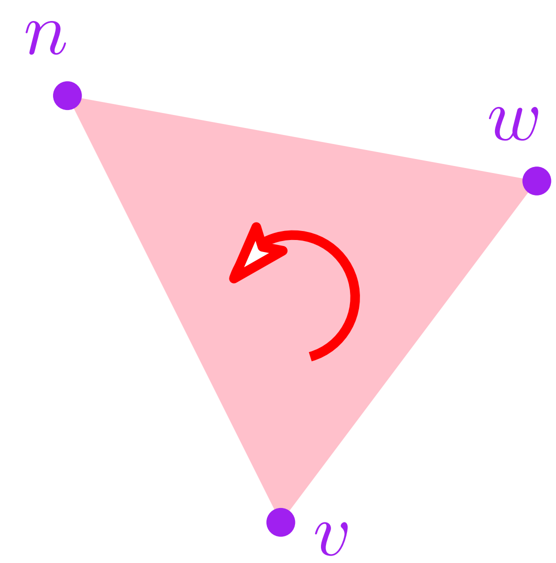


Convex hull

Orientation predicate

vwn + ?

$$\begin{vmatrix} x_w - x_v & x_n - x_v \\ y_w - y_v & y_n - y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} > 0$$



vwn - ?

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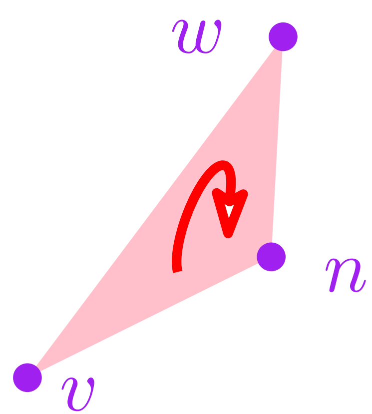
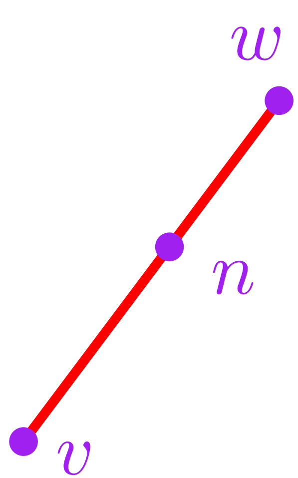


vwn 0 ?

$$\begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} = 0$$



Convex hull

Orientation predicate

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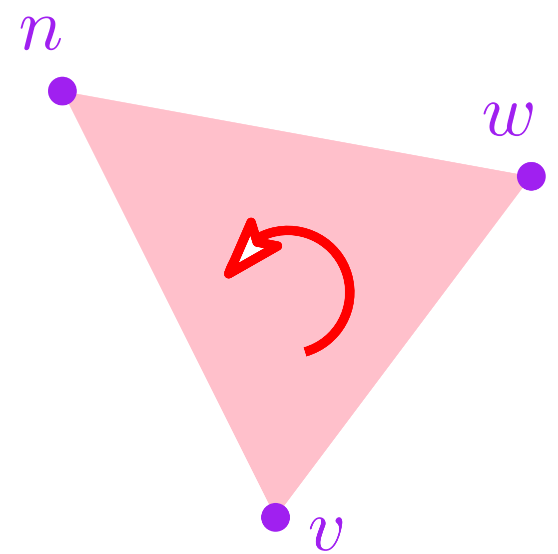
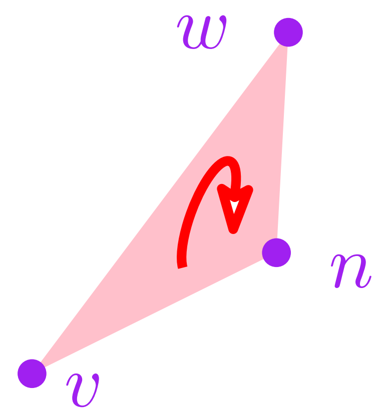


vwn - ?

$$\begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} < 0$$



vwn 0 ?

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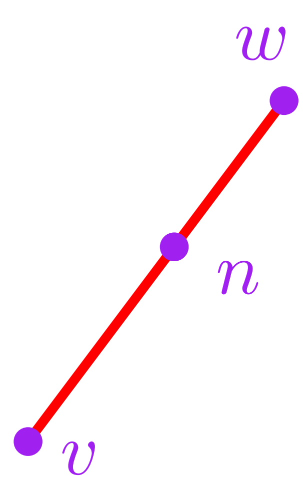


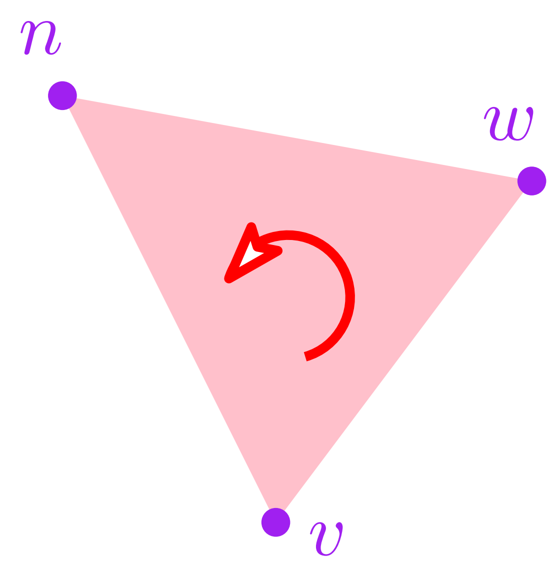
degenerate case

Convex hull

Orientation predicate

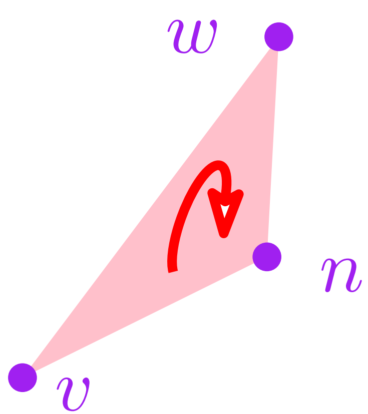
vwn + ?

$$\begin{vmatrix} x_w - x_v & x_n - x_v \\ y_w - y_v & y_n - y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} > 0$$



vwn - ?

$$\begin{vmatrix} 1 & 1 & 1 \\ x_v & x_w & x_n \\ y_v & y_w & y_n \end{vmatrix} < 0$$



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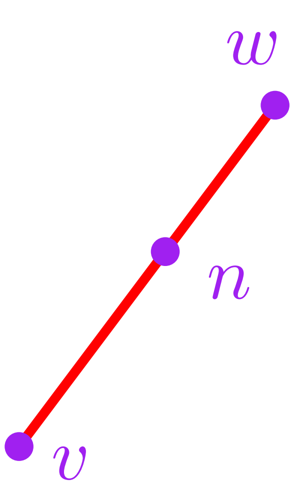


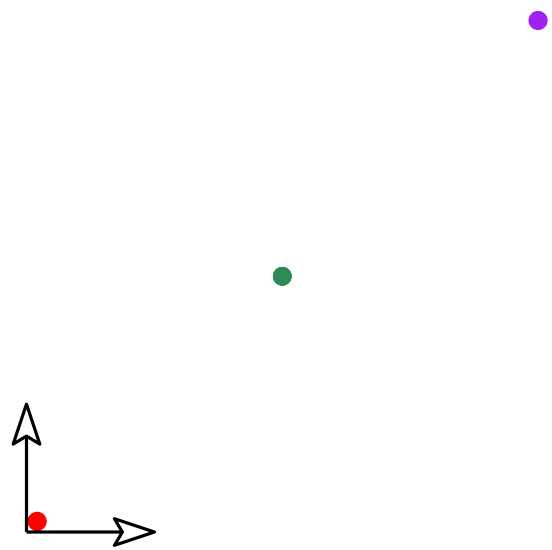
rounding errors

?

Convex hull

Orientation predicate

Rounding errors possible



$p = (\frac{1}{2} + x.u, \frac{1}{2} + y.u)$

$0 \leq x, y \leq 256, \ u = 2^{-53}$

$q = (12, 12)$

$r = (24, 24)$

Teaser robustness lecture

$orientation(p, q, r)$

evaluated with double

Convex hull

Orientation predicate

Rounding errors possible

$p = (\frac{1}{2} + x.u, \frac{1}{2} + y.u)$

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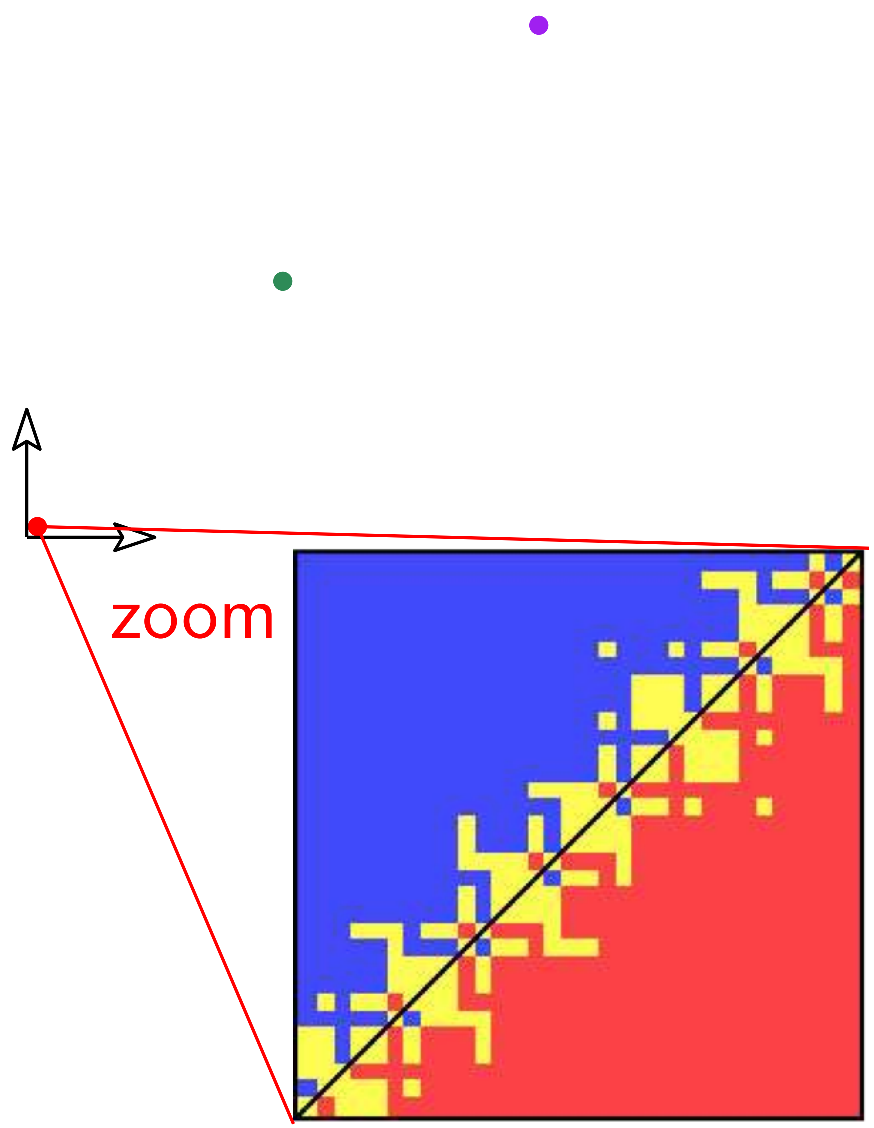
Teaser robustness lecture



$orientation(p, q, r)$

evaluated with double

≤ 0

0

≥ 0

Convex hull

Teaser robustness lecture

Buggy degenerate example (single precision)

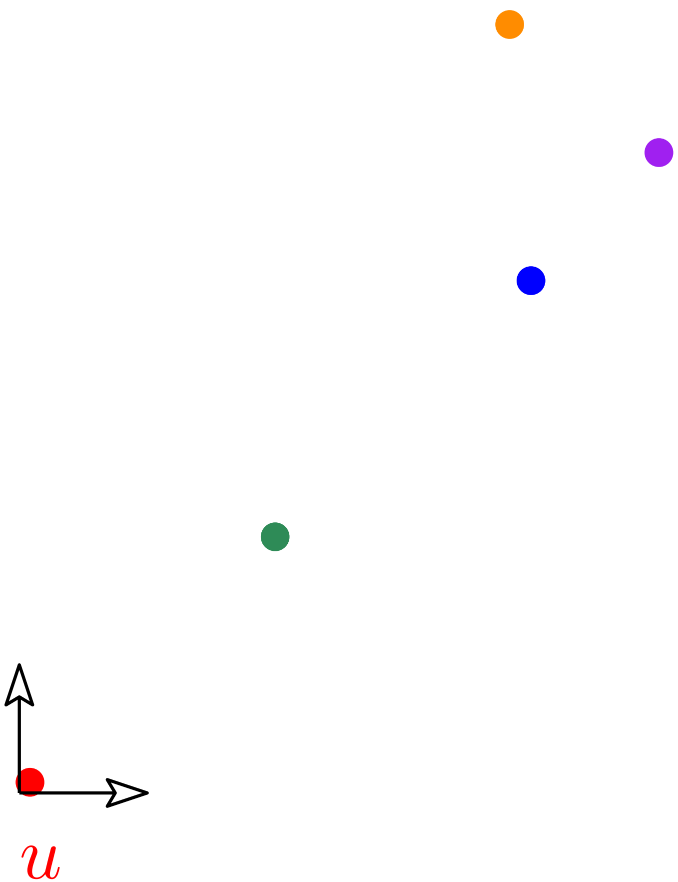
$w_1 = (12, 12)$

$w_2 = (24, 24)$

$w_3 = (30, 30.000001)$

$w_4 = (23, 36)$

$w_5 = (0.5000029, 0.5000027)$



Convex hull

Teaser robustness lecture

Buggy degenerate example
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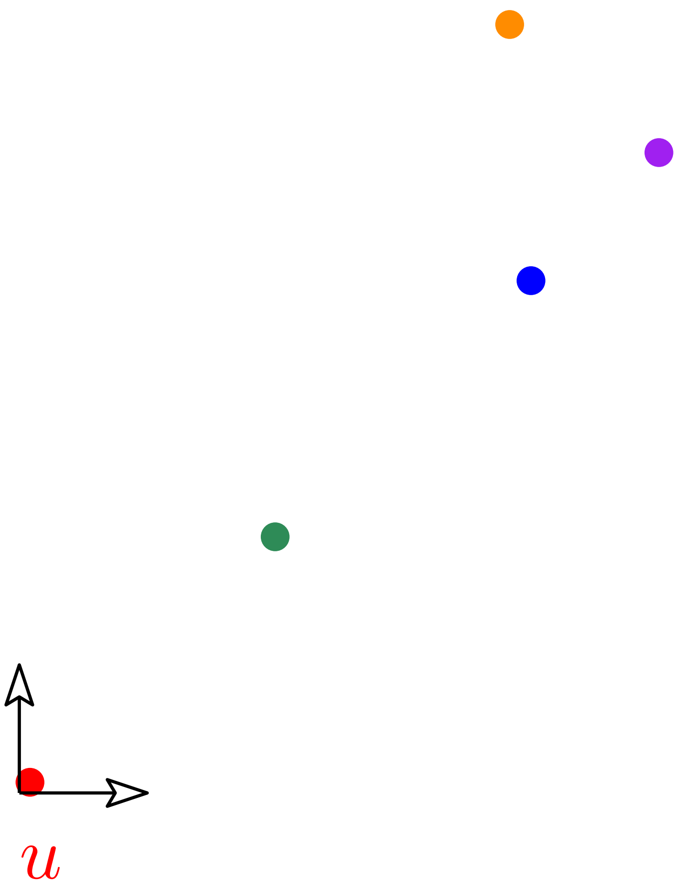


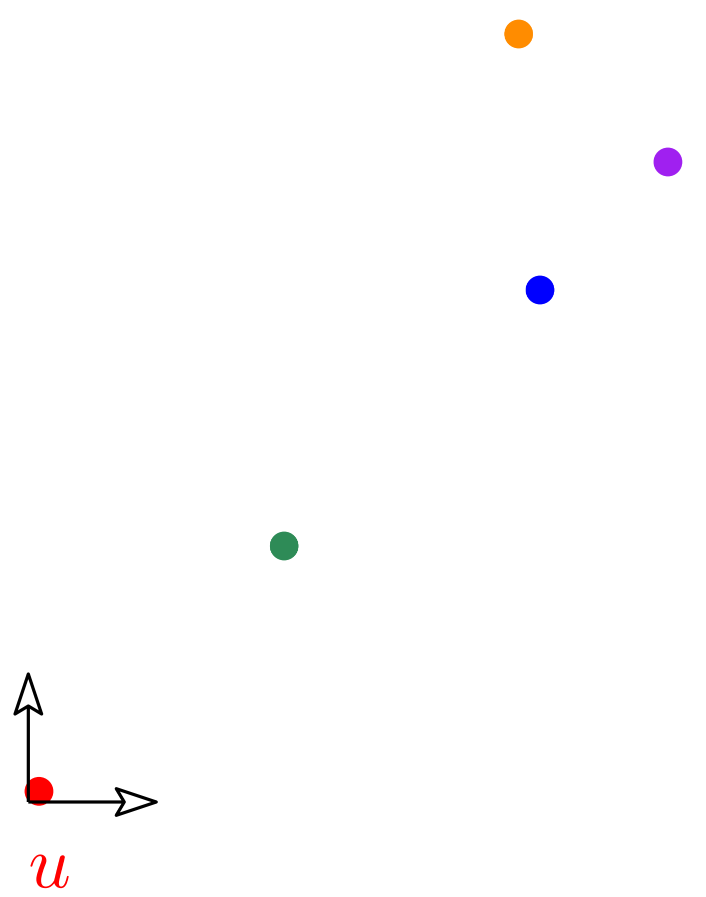
Input : point set S
$u = v =$ lowest point in S;

Jarvis

Convex hull

Teaser robustness lecture

Buggy degenerate example
(single precision)



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$w_3 = (30, 30.000001)$

$w_4 = (23, 36)$

$w_5 = (0.5000029, 0.5000027)$

Do
  $n =$ first in S;
  For each $w \in S$
    if vwn positive
      then $n = w$;
  $v.next = n$; $v = n$;
  $S = S \setminus \{v\}$
While $v \neq u$

Convex hull

Teaser robustness lecture

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(single precision)

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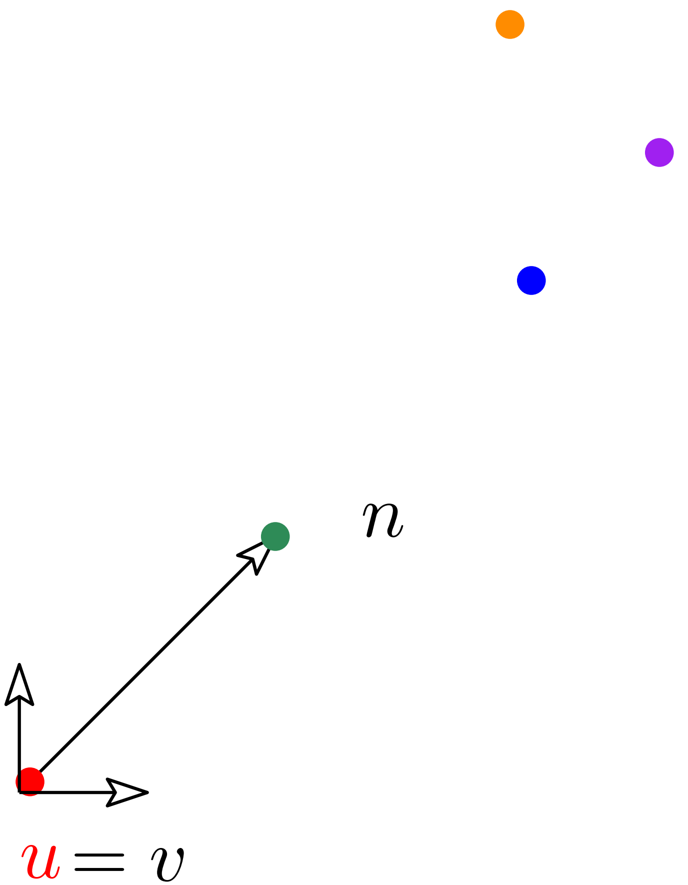

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Convex hull

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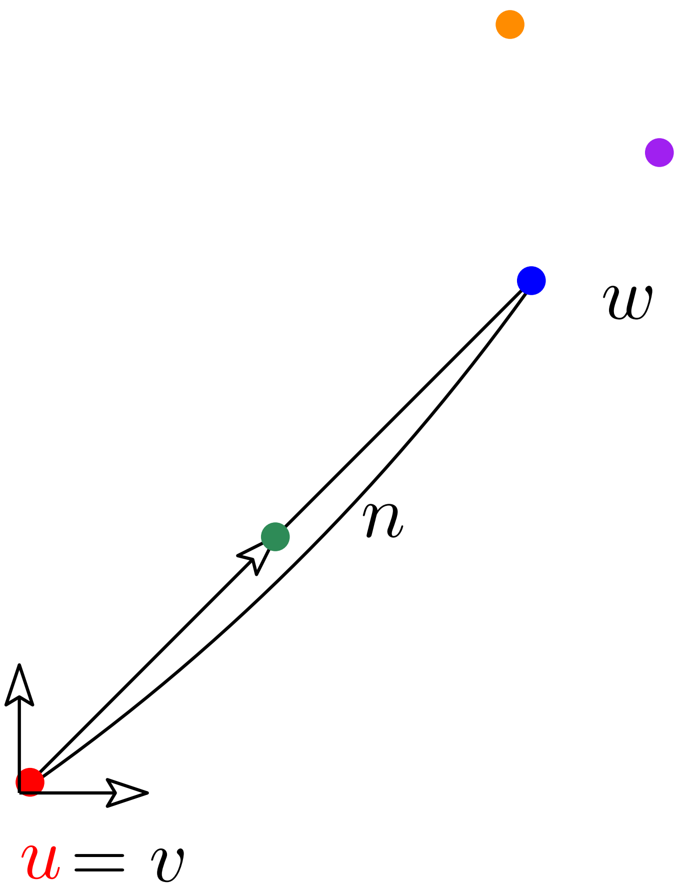

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While $v \neq u$

Convex hull

Teaser robustness lecture

Buggy degenerate example
(single precision)

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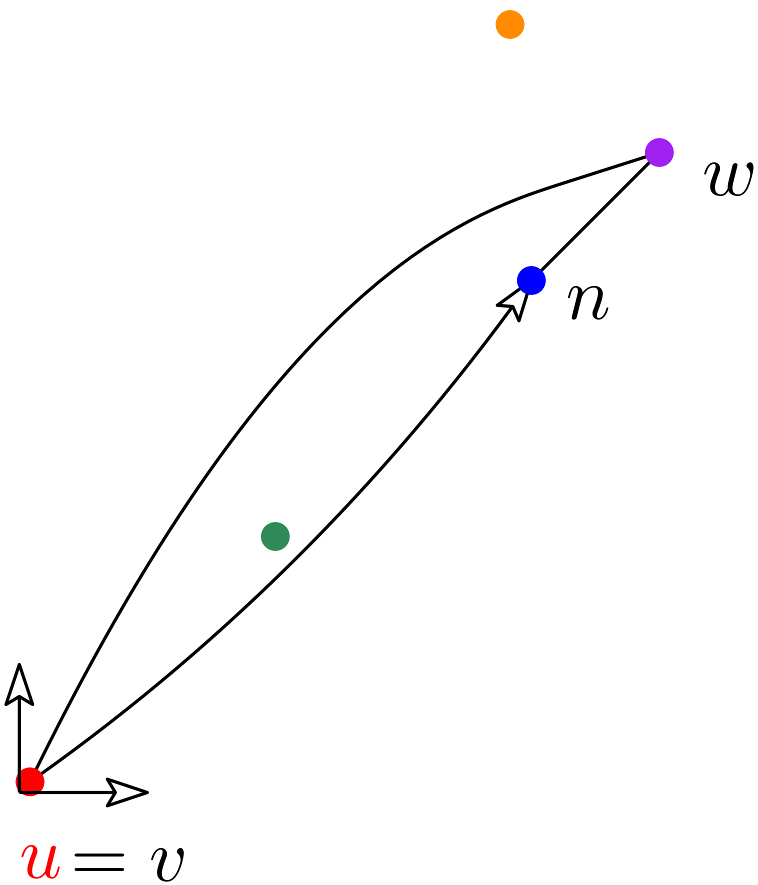

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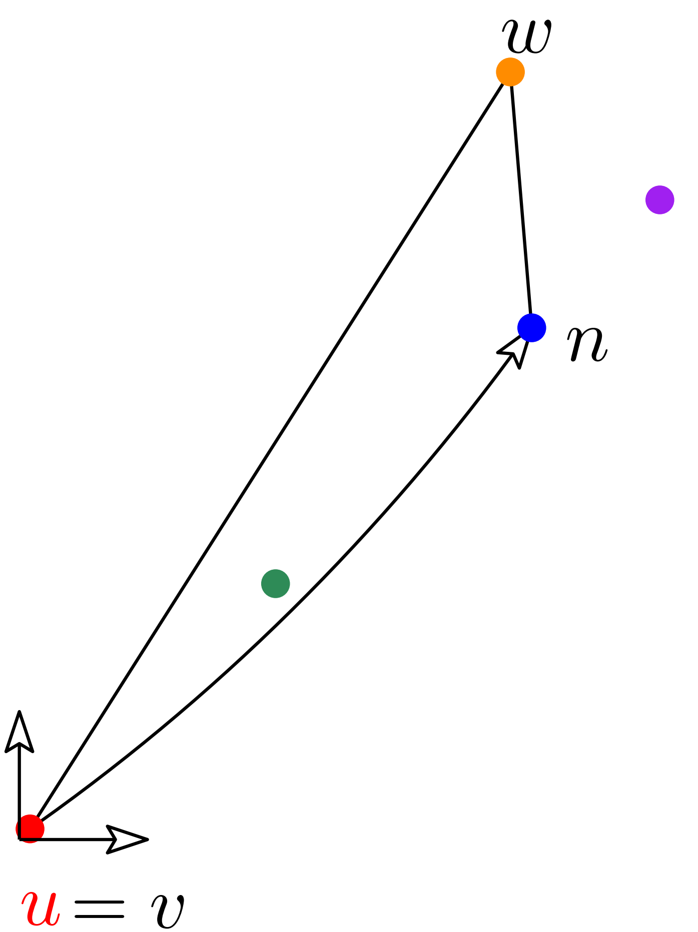

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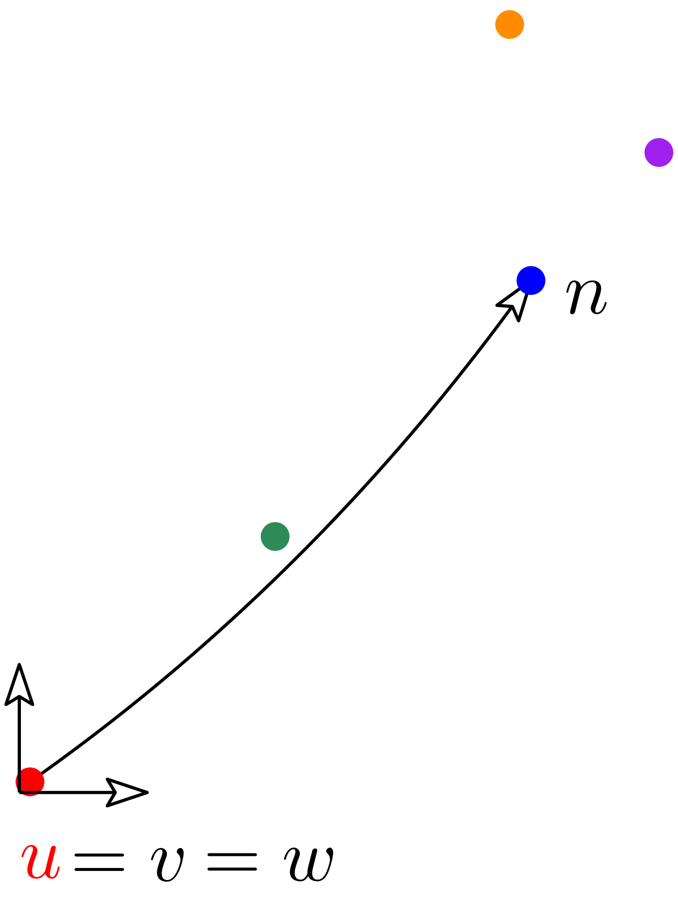

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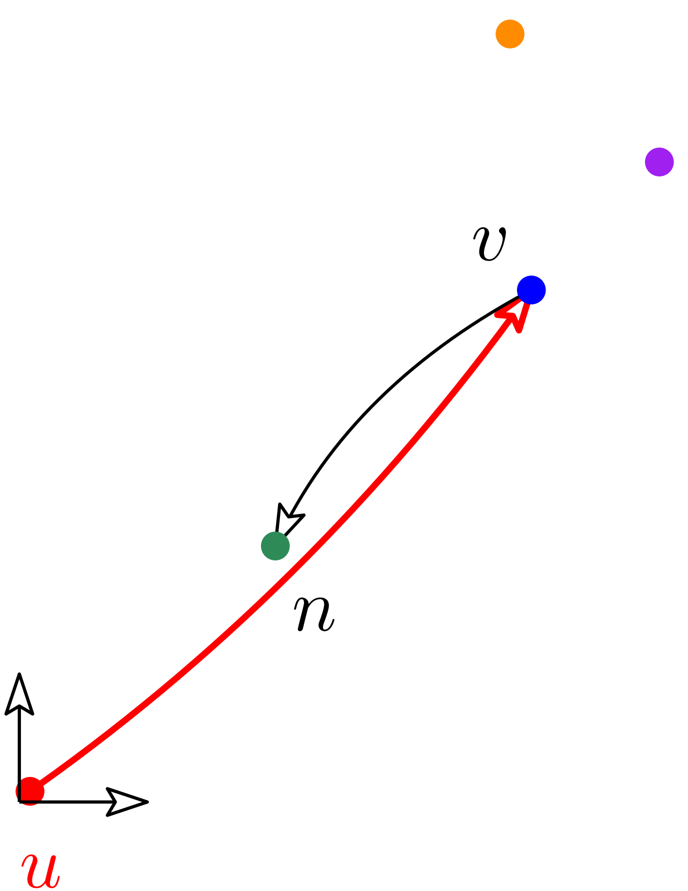

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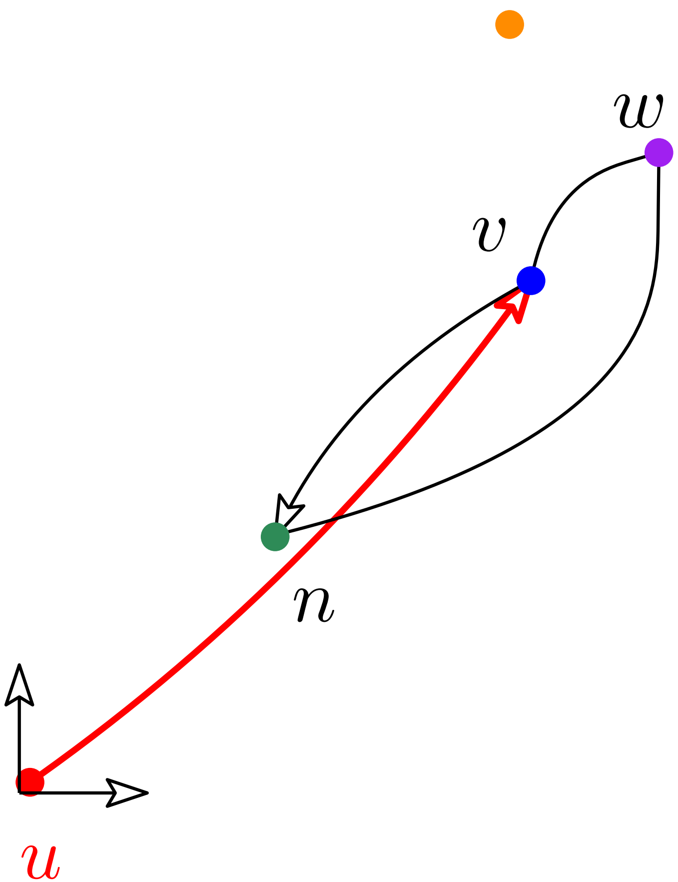

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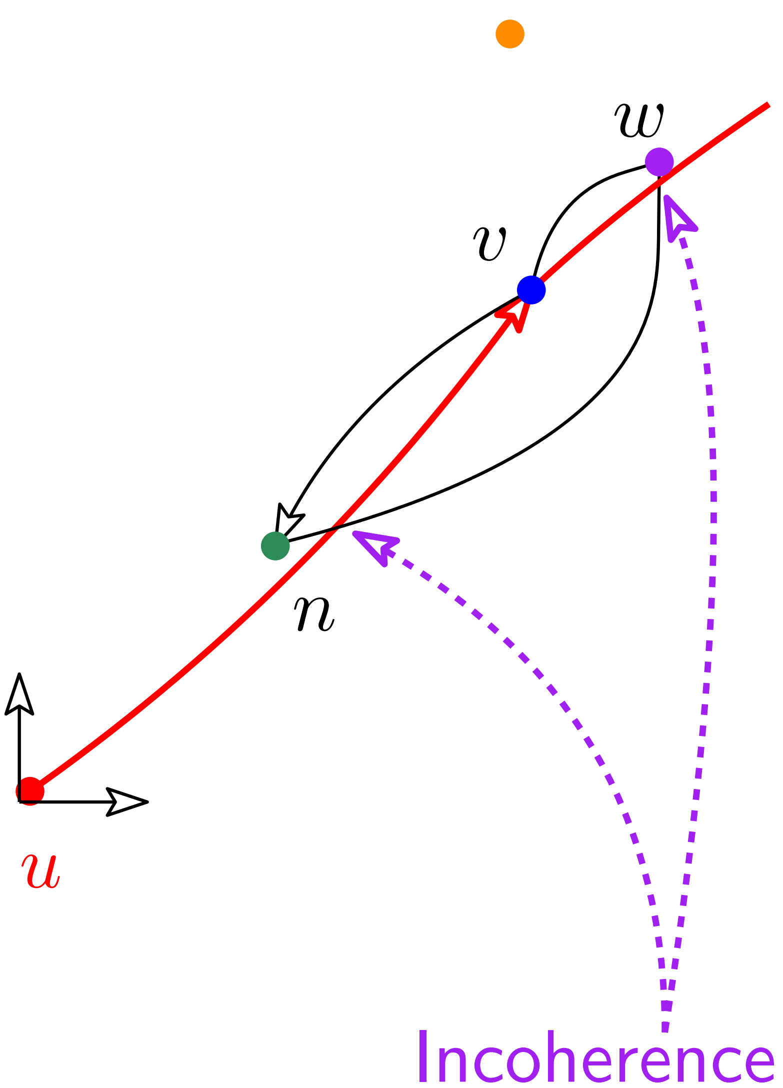

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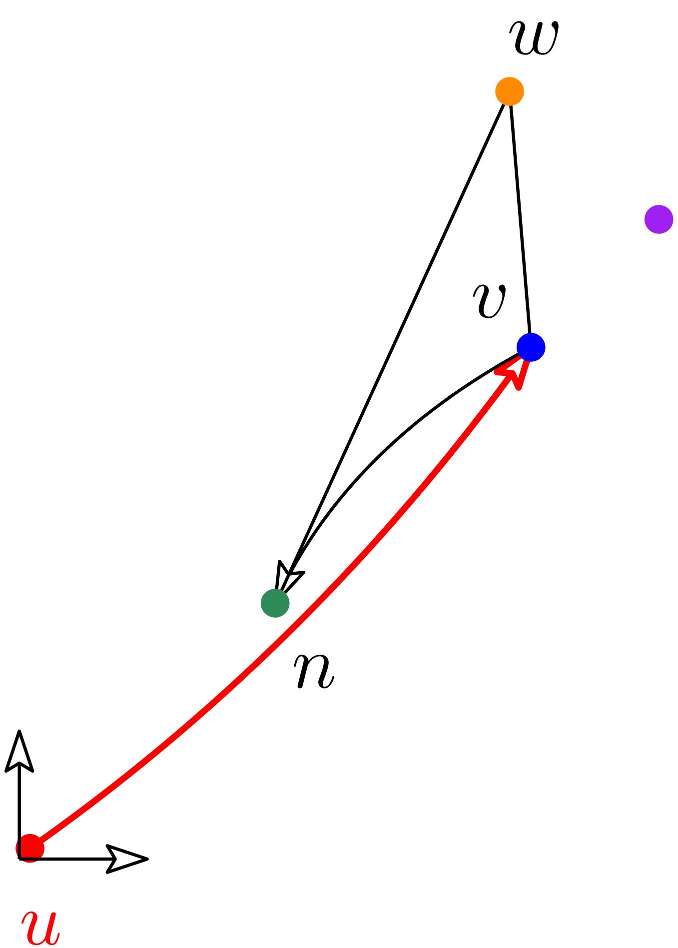

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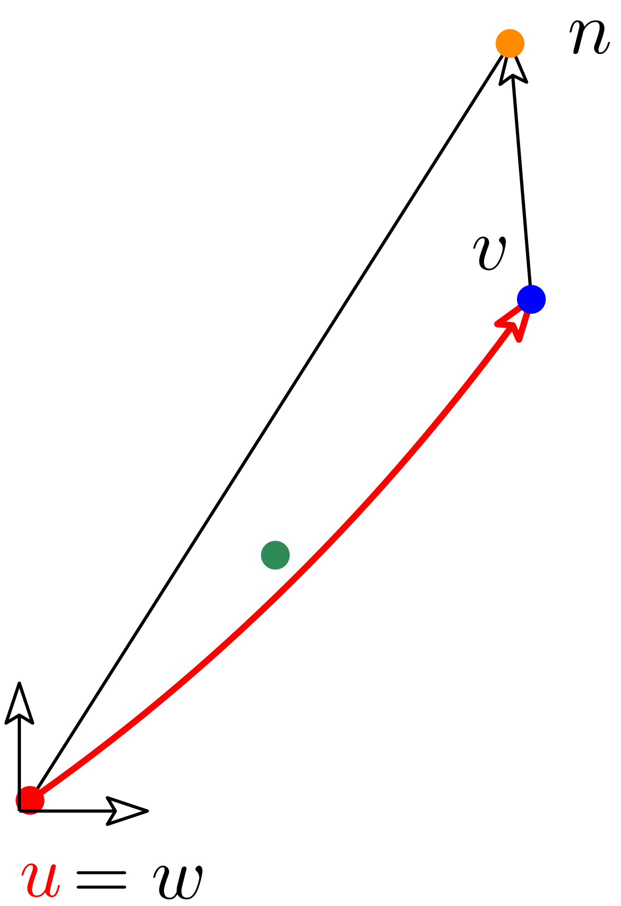

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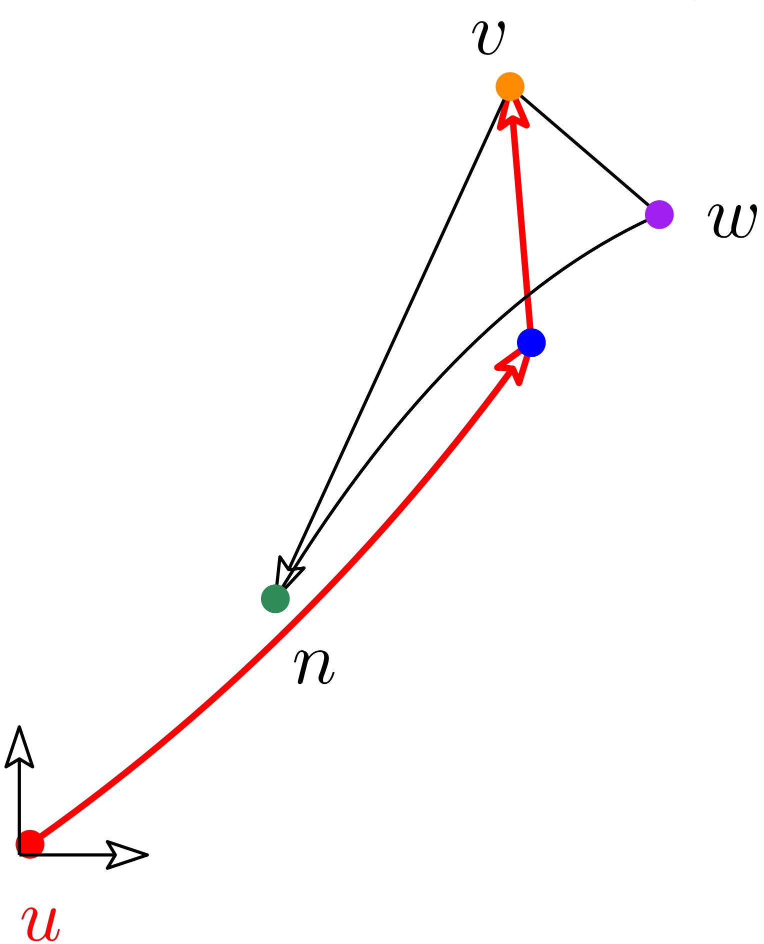

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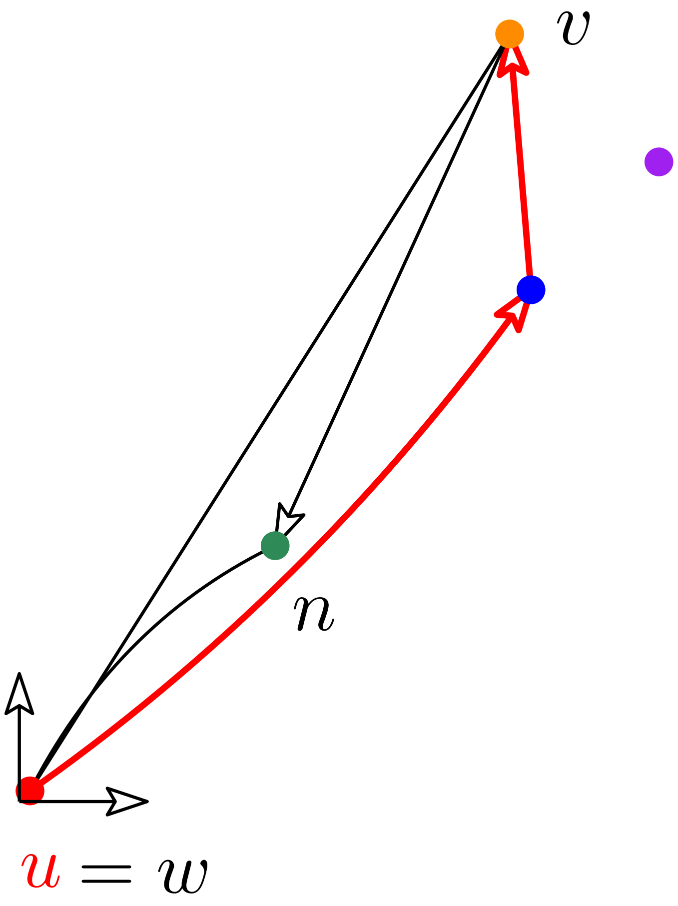

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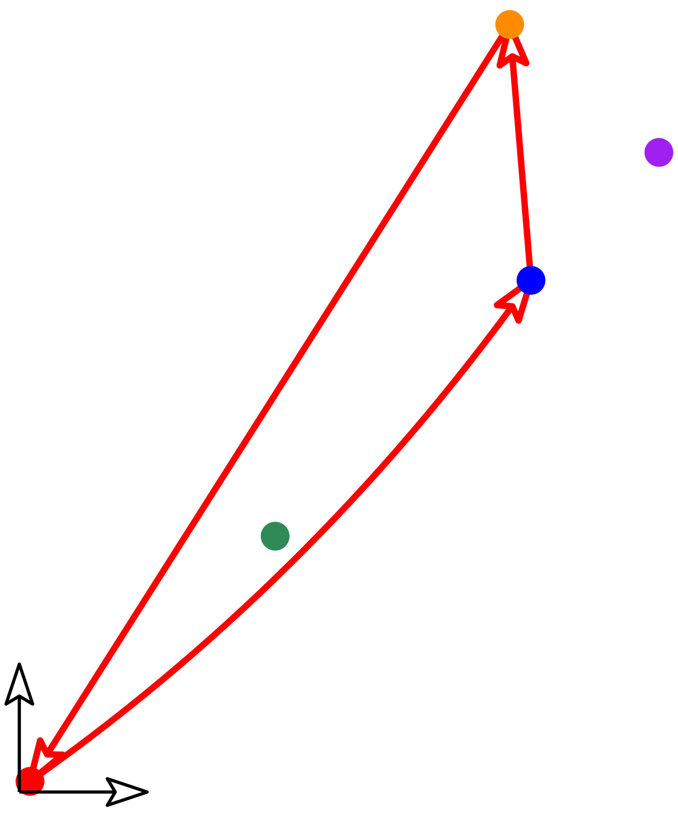
Teaser robustness lecture



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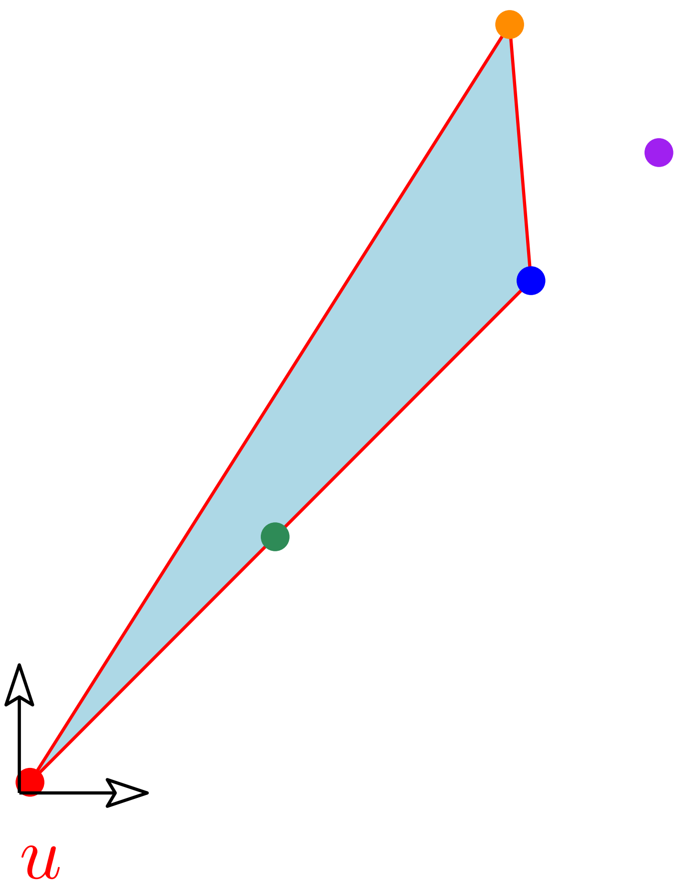

Result is really wrong

Convex hull

Real Random Access Memory model

Assume exact computation on real numbers

constant time for single operations: $+, -, \sqrt{\ }, \sin \ldots$

Convex hull

Real Random Access Memory model

Assume exact computation on real numbers

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General position hypotheses

Predicate: Input $\longmapsto \{-1, 0, 1\}$

Convex hull

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General position hypotheses

Predicate: Input $\longmapsto \{-1, 0, 1\}$

2D convex hull: no three points colinear

Convex hull

Real RAM model and general position hypothesis

Real Random Access Memory model

Assume exact computation on real numbers

constant time for single operations: $+, -, \sqrt{}, \sin \ldots$

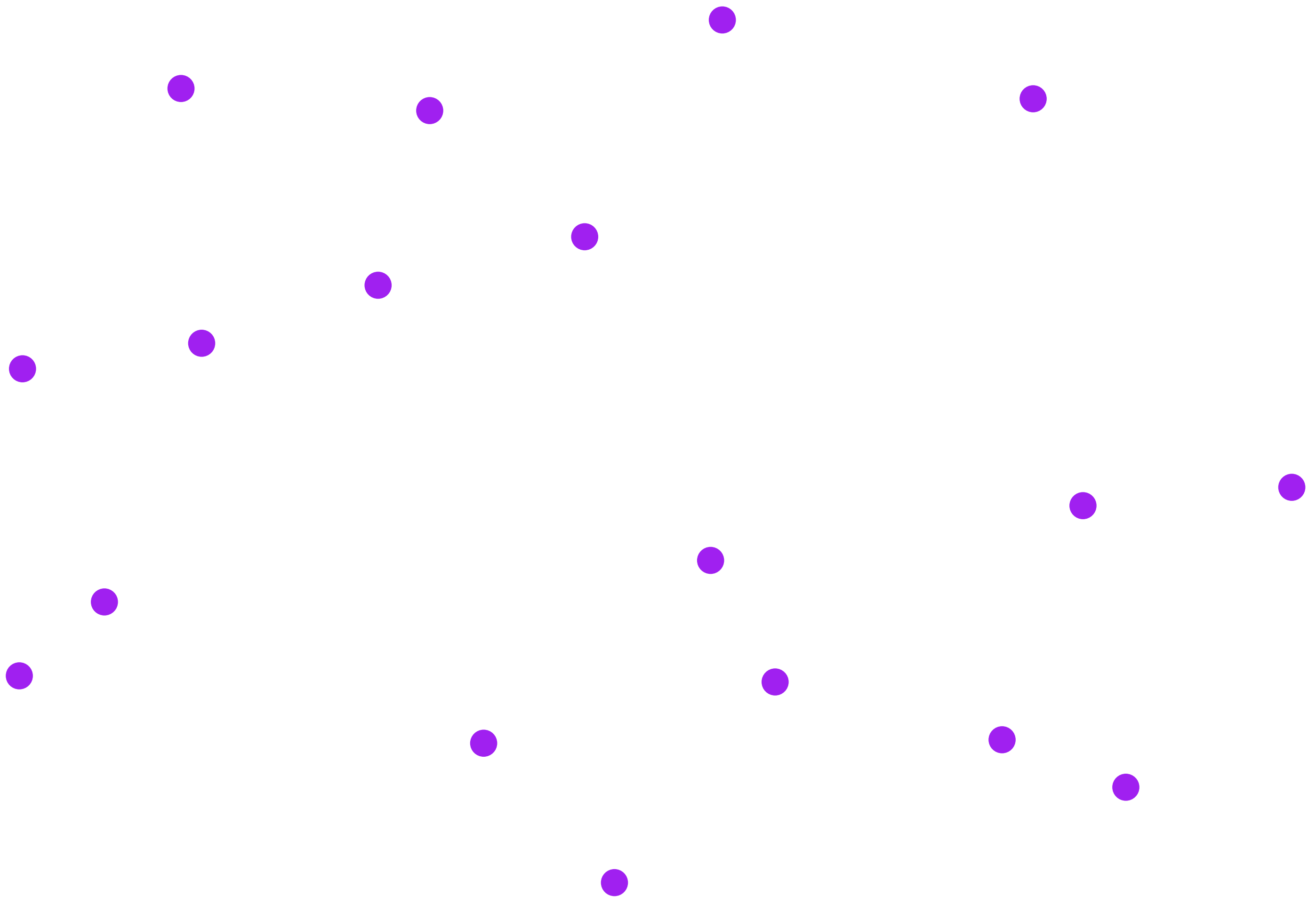
General position hypotheses

Predicate: Input $\longmapsto \{-1, 0, 1\}$

2D convex hull: no three points colinear

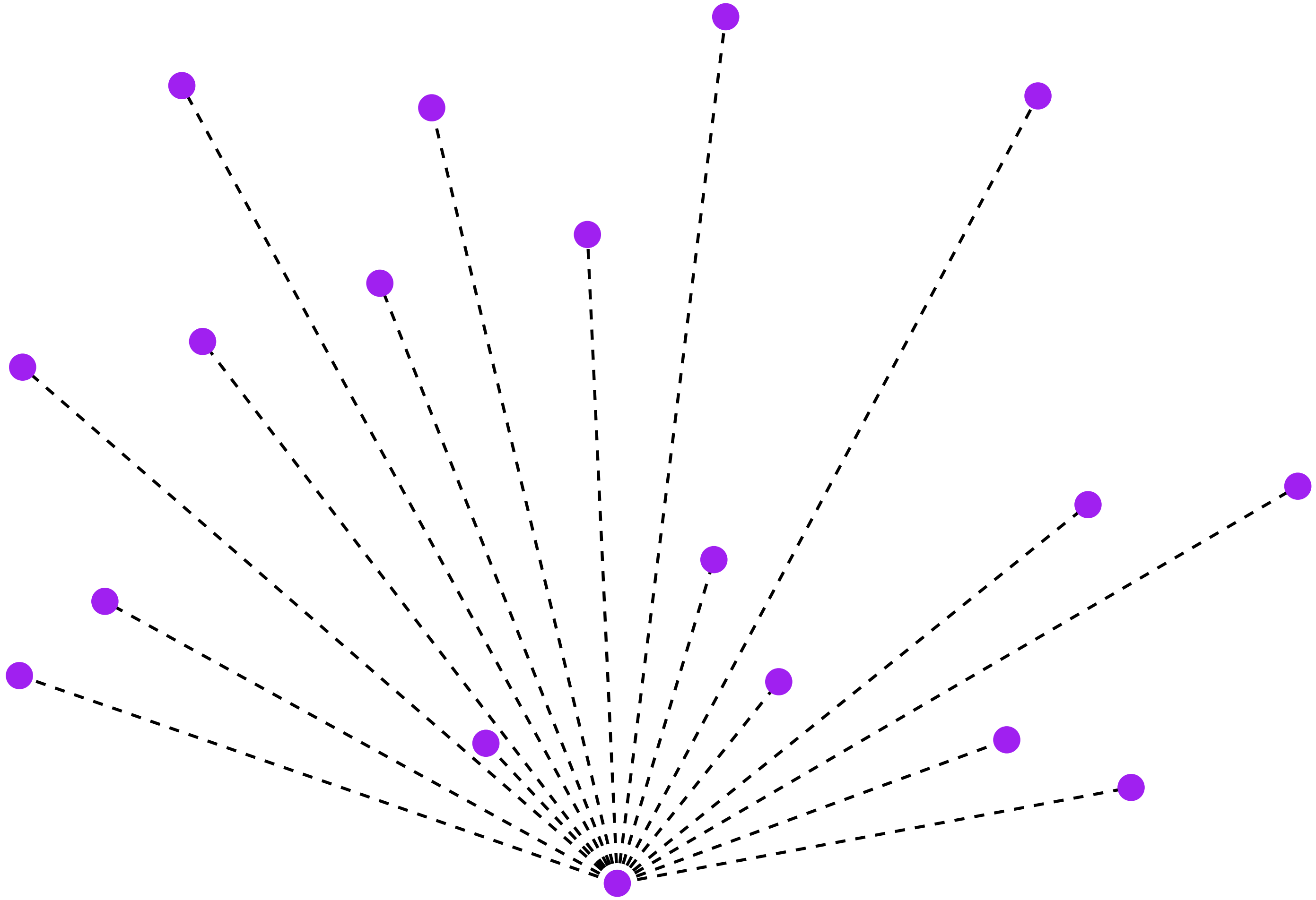
possibly: no 2 points with same x

Convex hull



Convex hull

sort around a point (e.g. lowest point)



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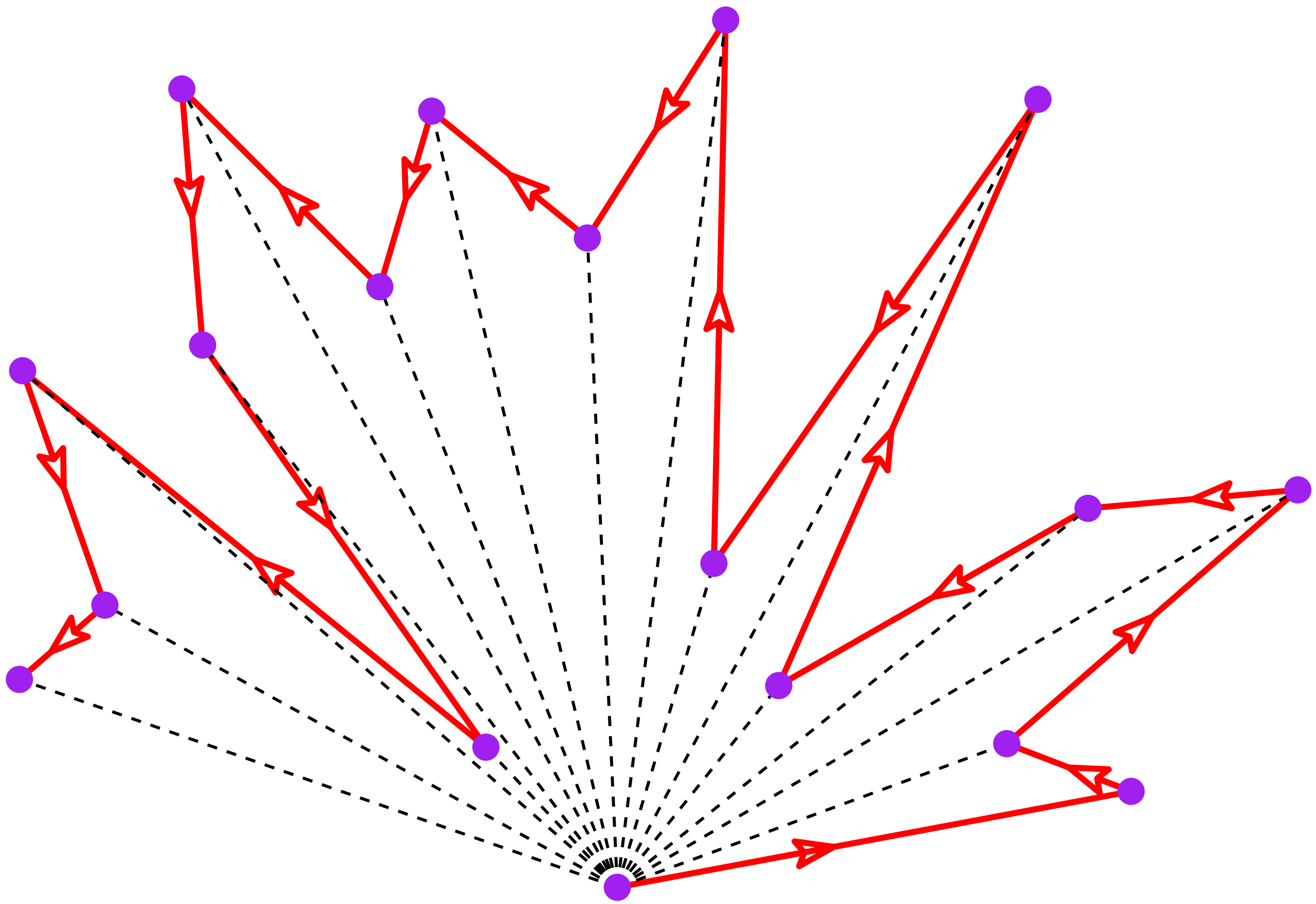

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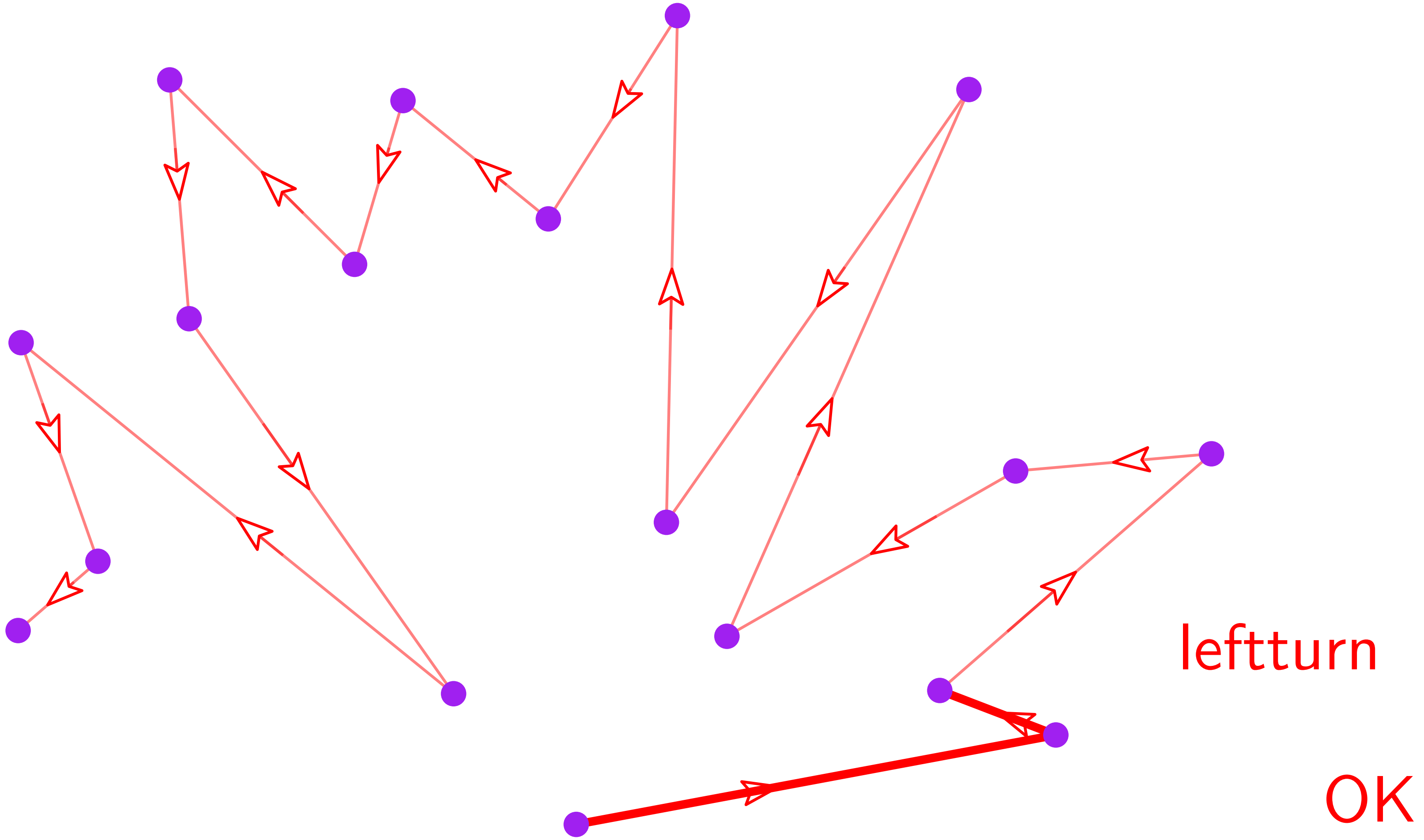
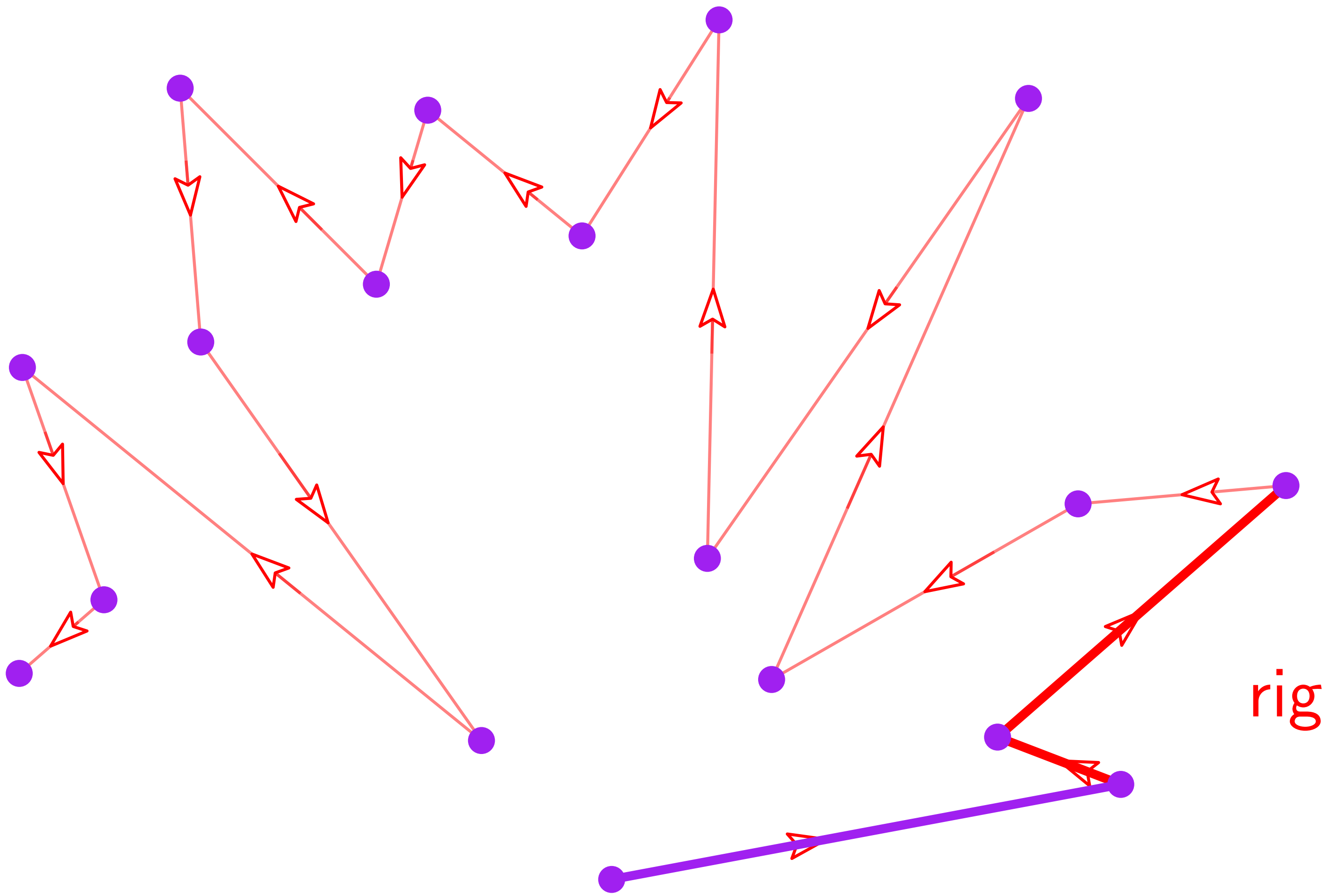


Convex hull

Graham algorithm



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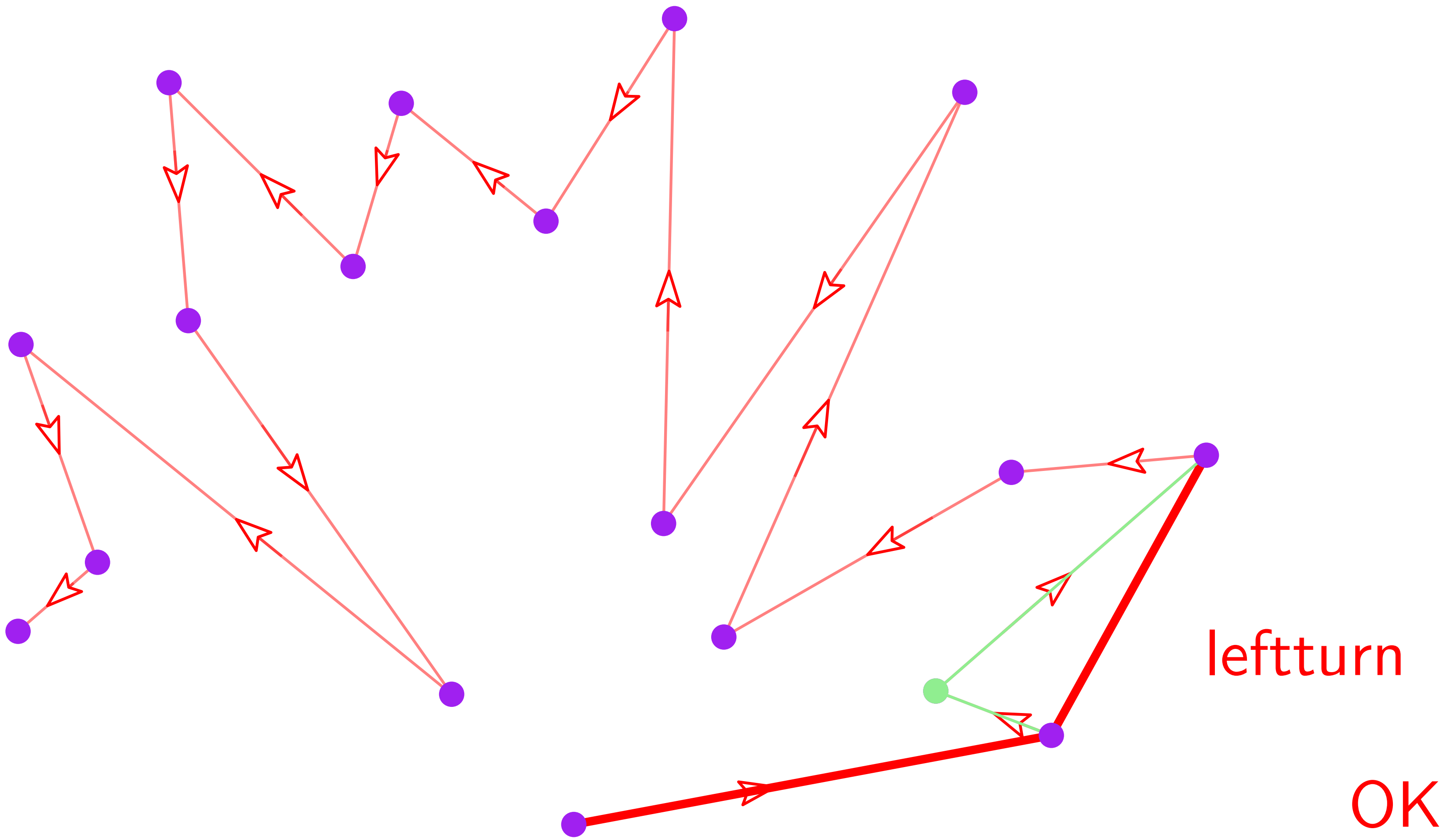

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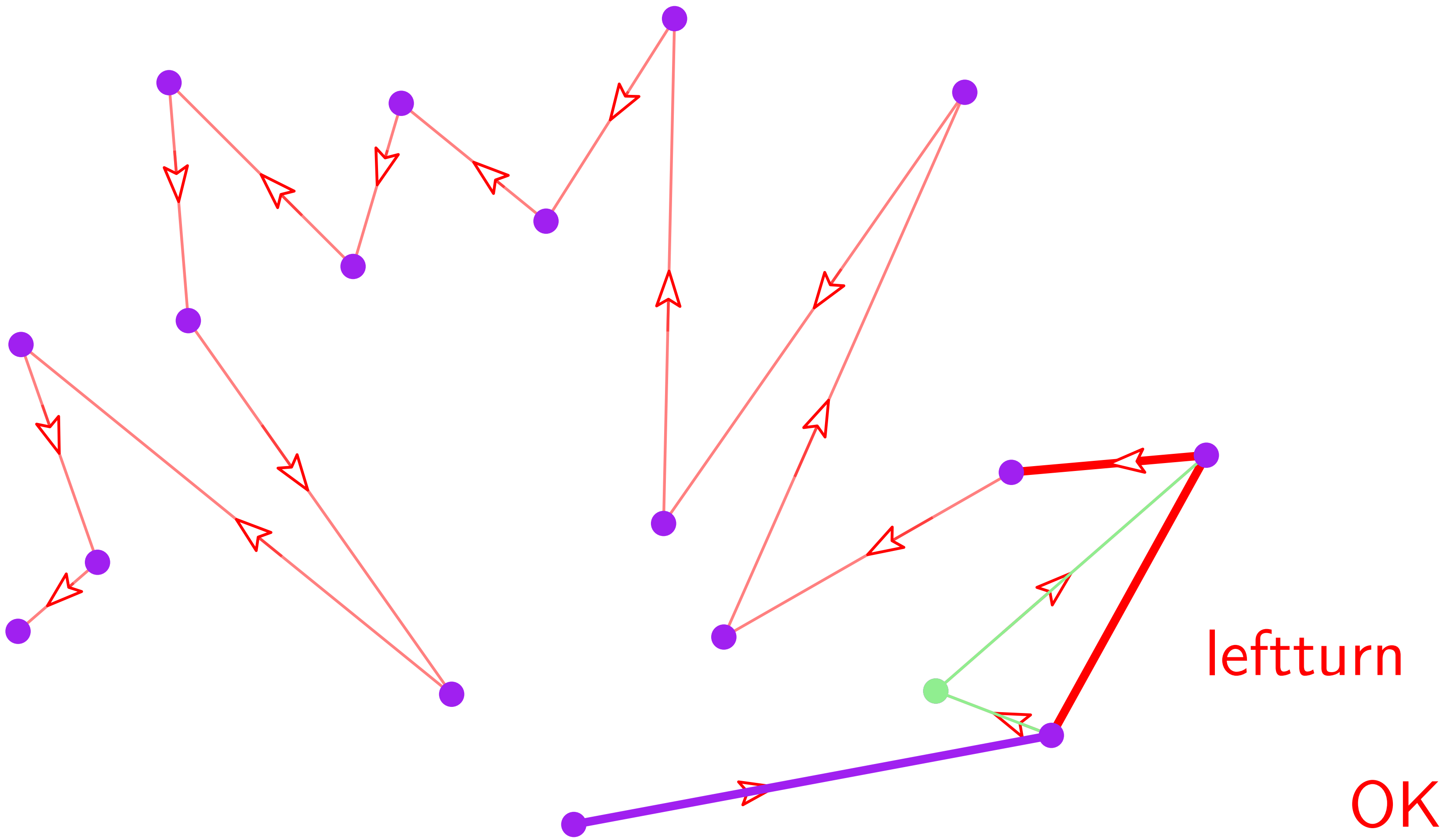

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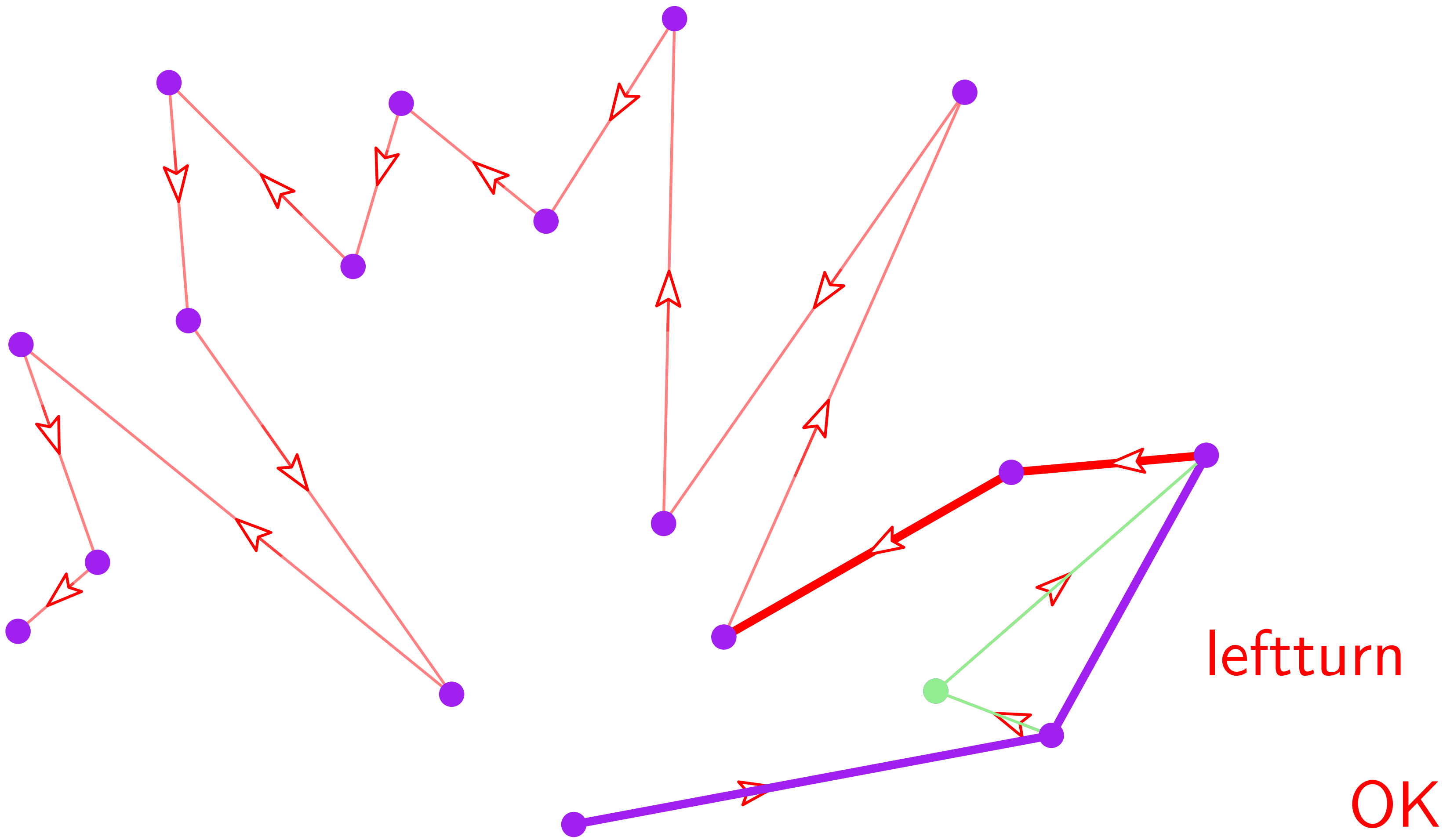

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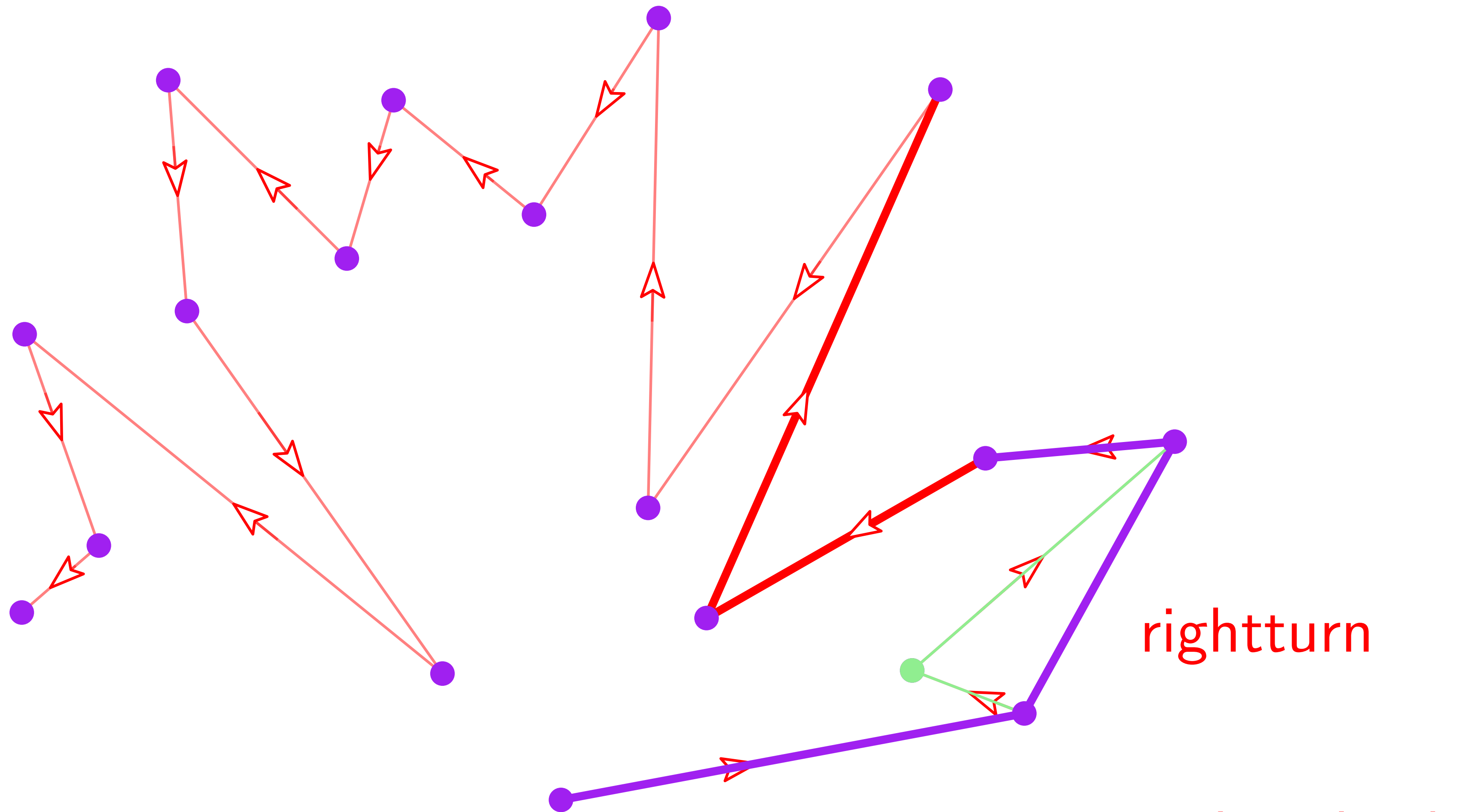

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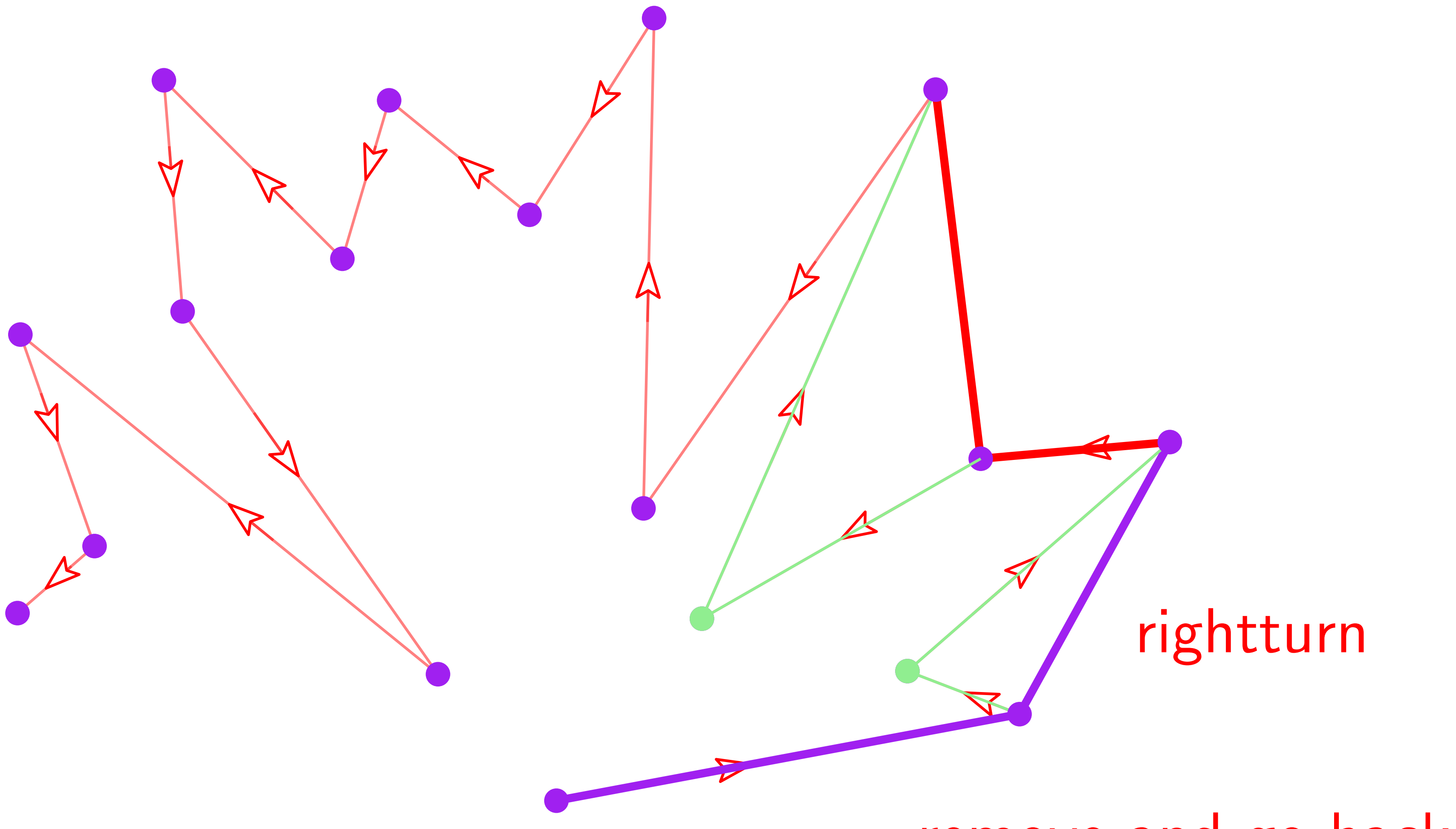

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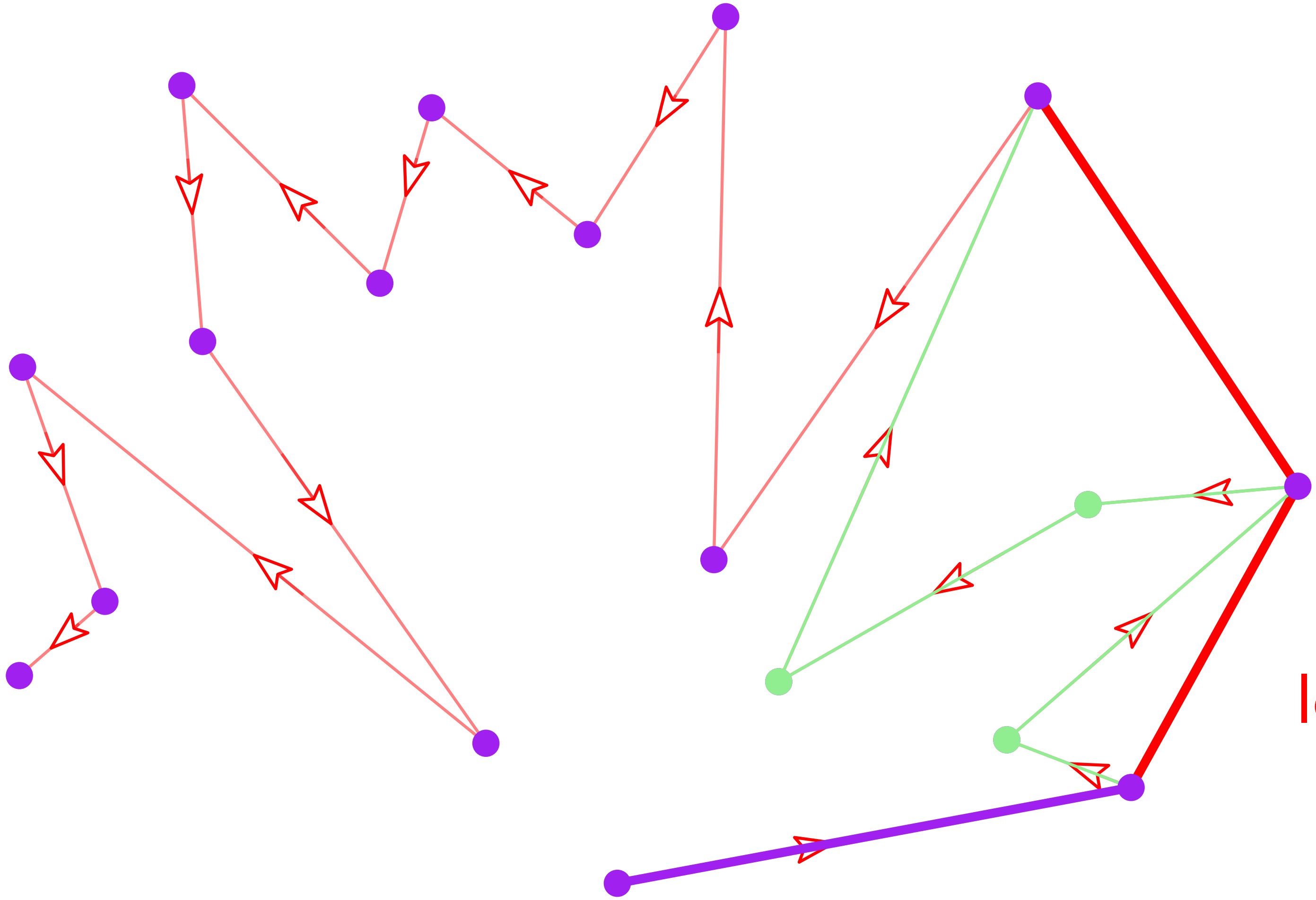
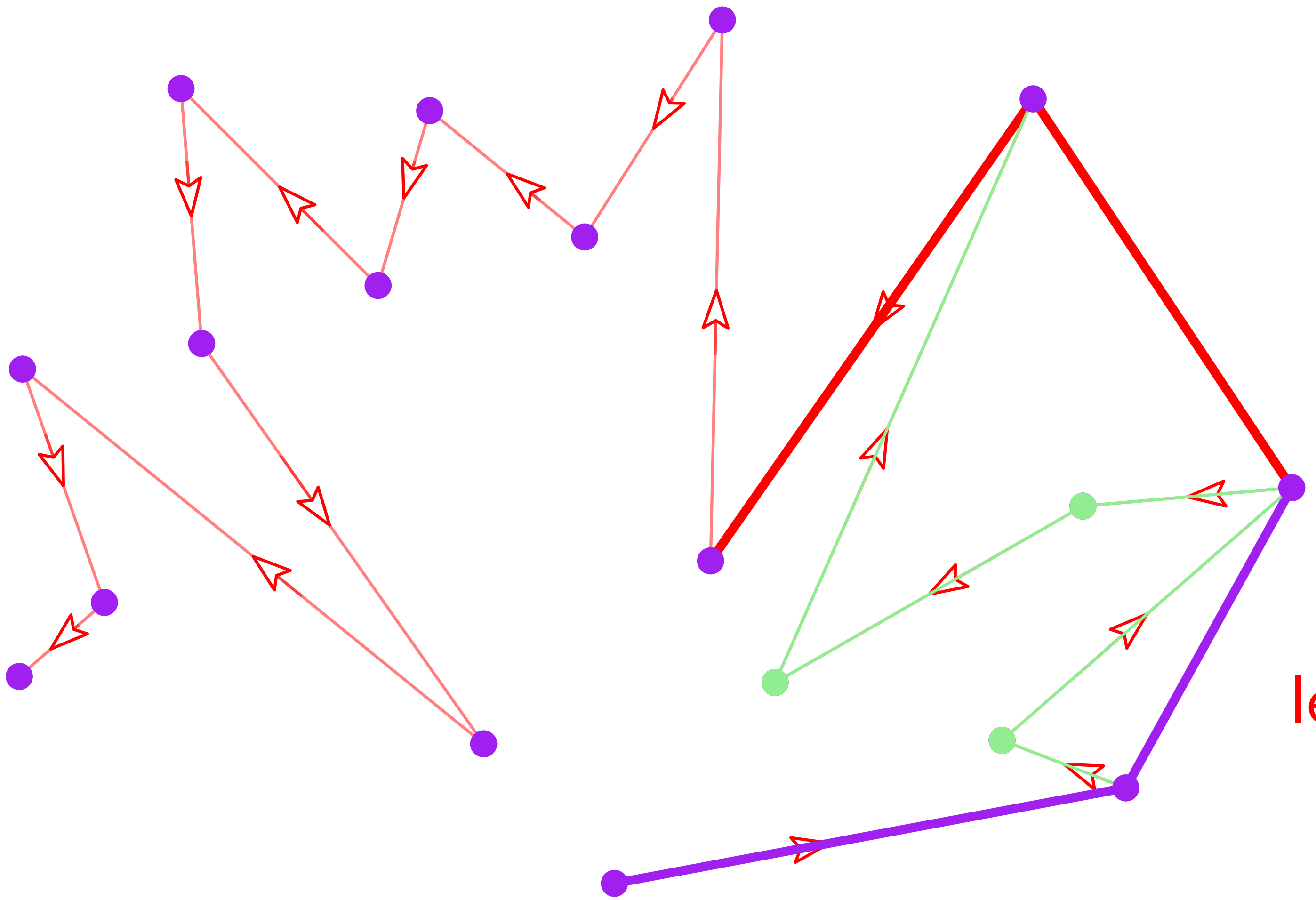


leftturn

OK

Convex hull

Graham algorithm



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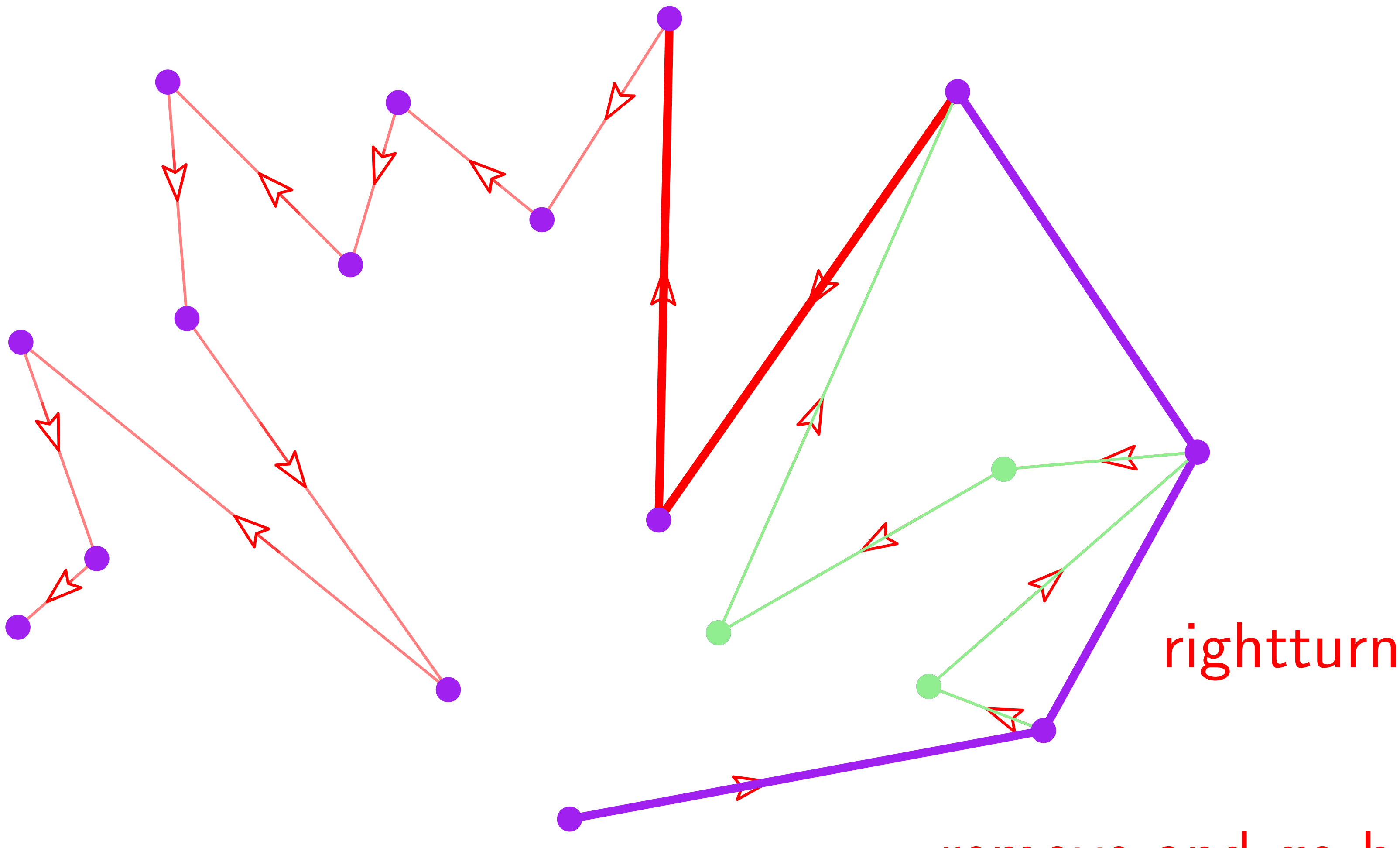

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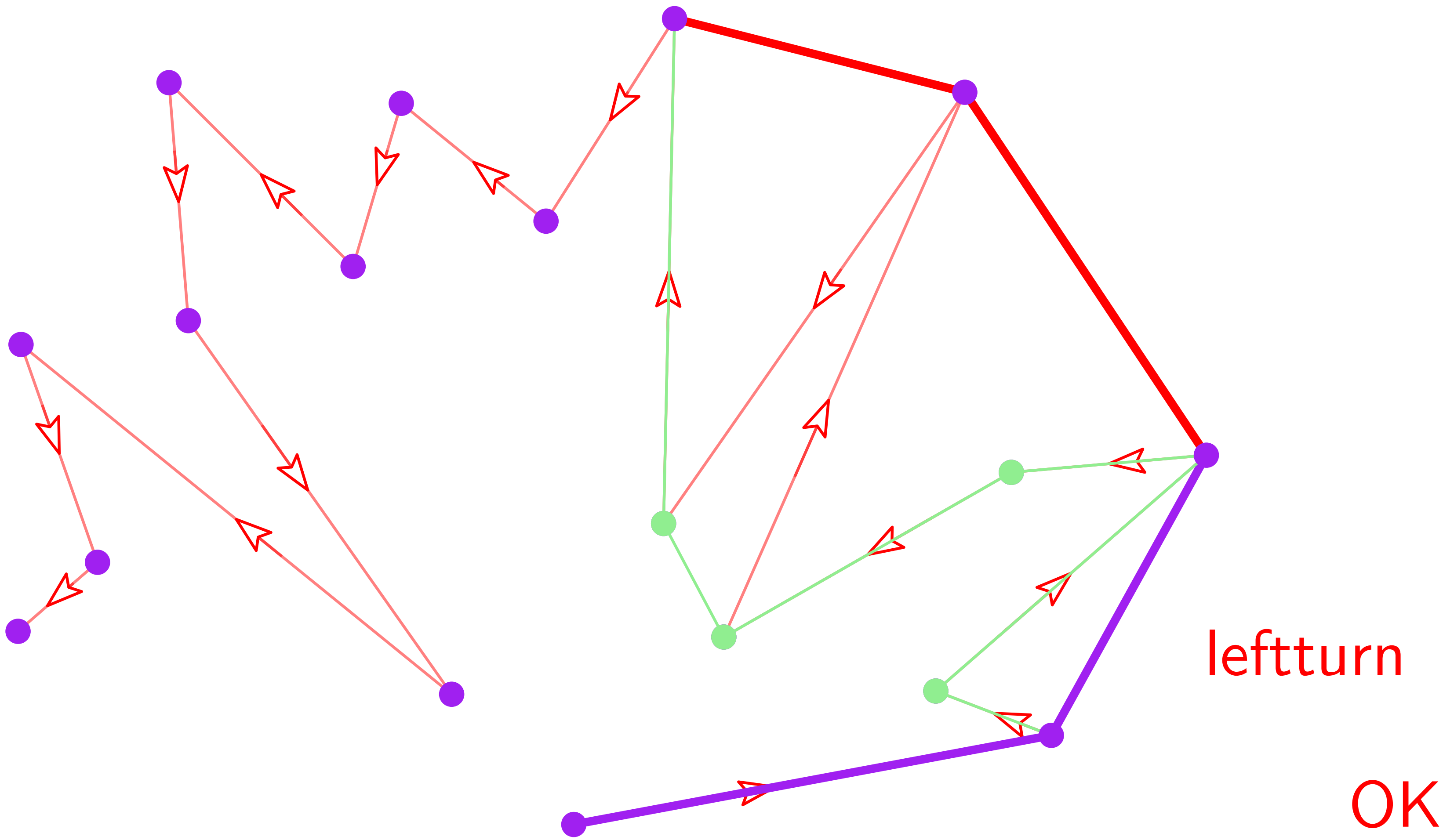

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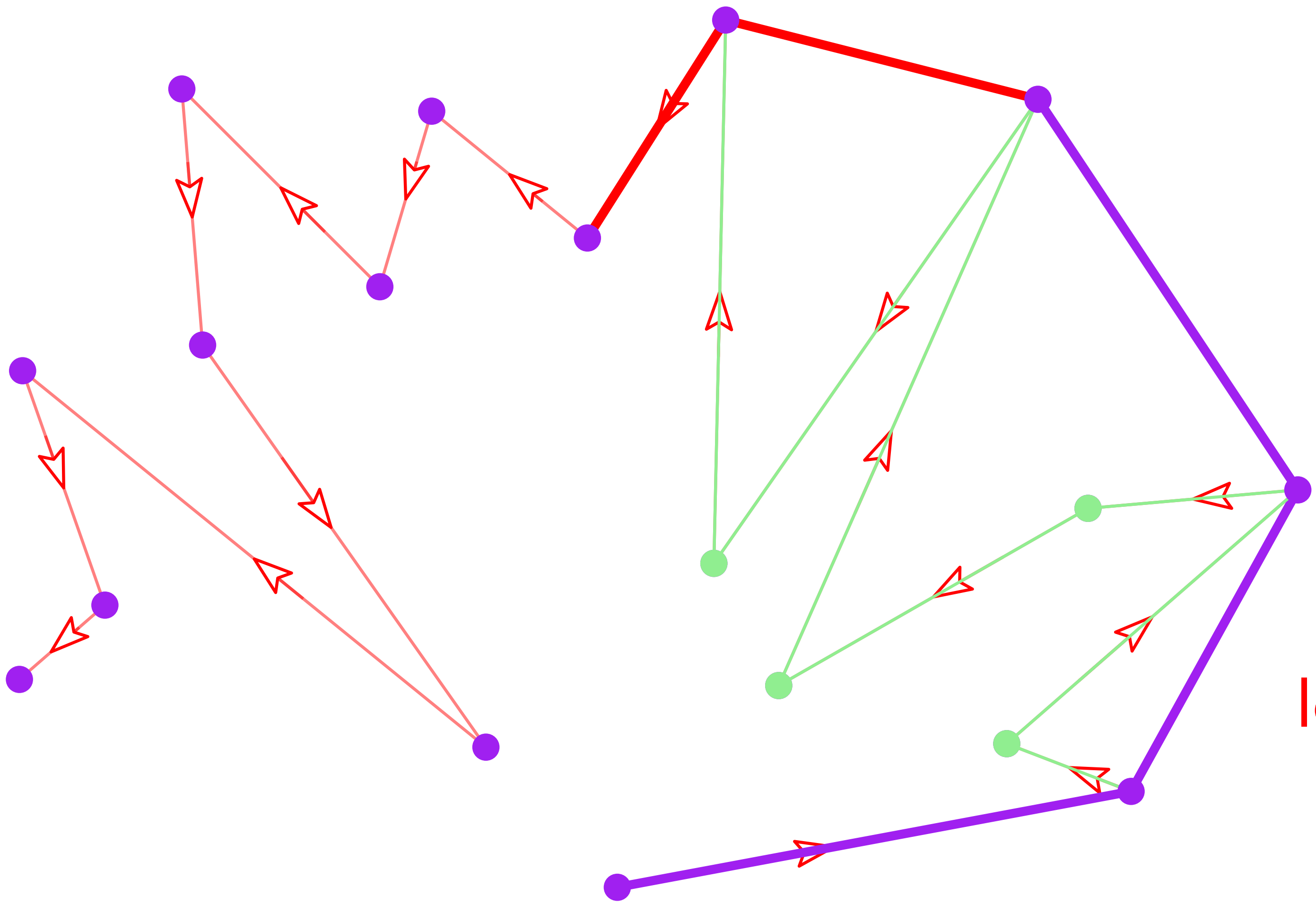

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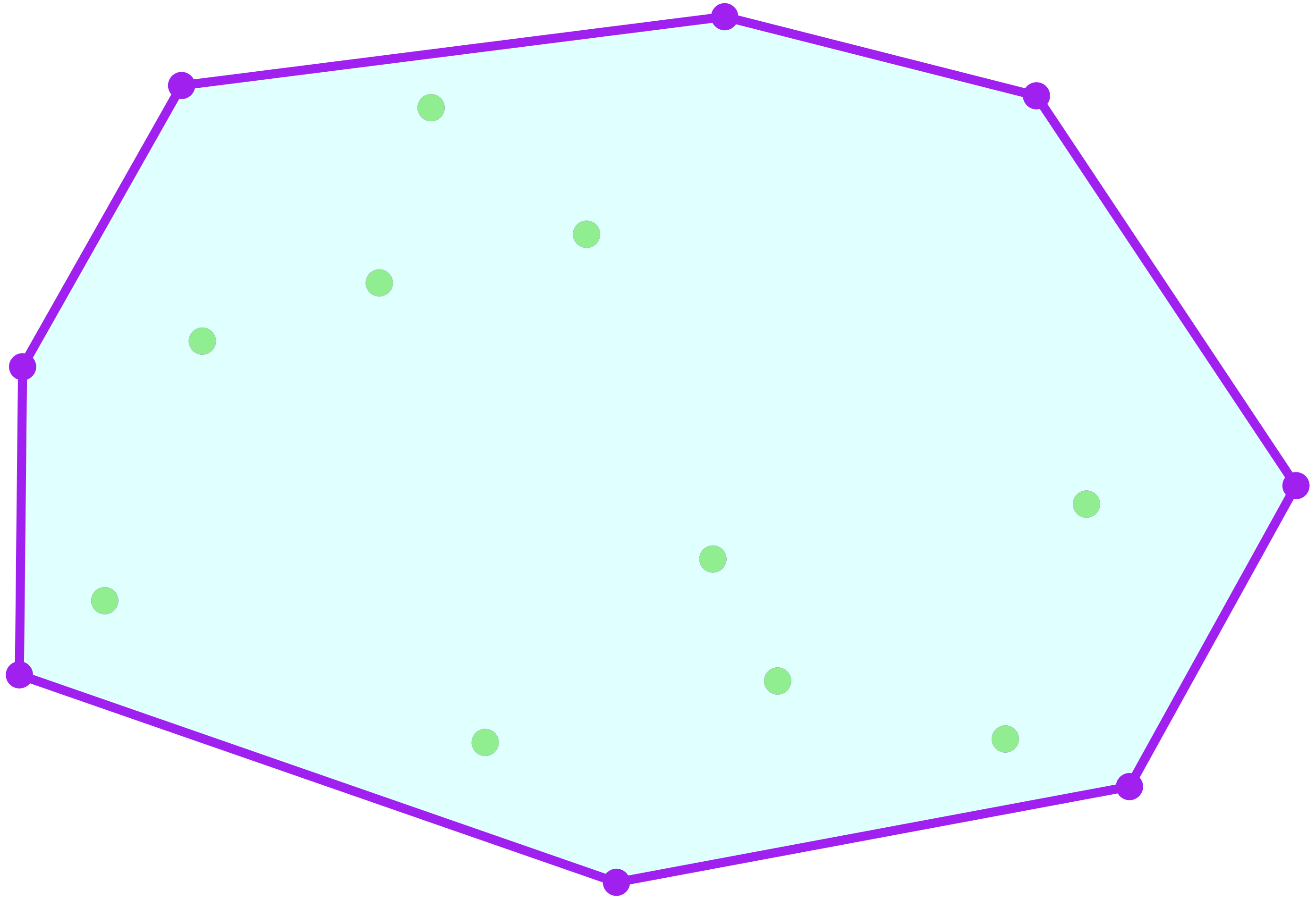
OK

Convex hull



Convex hull

Complexity

Input: point set S
u lowest point of S;
sort S around u in a circular list including u;
$v = u$;

while $v.next \neq u$
    if $(v, v.next, v.next.next)$ ccw
        $v = v.next$;
    else
        $v.next = v.next.next$; $v.next.previous = v$;
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Convex hull

Complexity

Graham algorithm

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$O(n)$

Convex hull

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$O(n \log n)$

Convex hull

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delete one point

at most n times

Convex hull

Complexity

Graham algorithm

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distance to u decreases

at most n times

delete one point

at most n times

Convex hull

Complexity

Graham algorithm

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sort S around u in a circular list including u;
$v = u$;

$O(n)$

while $v.next \neq u$
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distance to u decreases
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delete one point
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Graham algorithm

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$O(n \log n)$

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Convex hull

Problem lower bound is $\Omega(f(n))$

Iff there is NO algorithm

solving all size n problems

using less than $Cf(n)$ operations

$\forall n$

C constant independent of n

Sorting

Input: n real (positive) numbers

Sorting

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Output: sorting permutation

Sorting

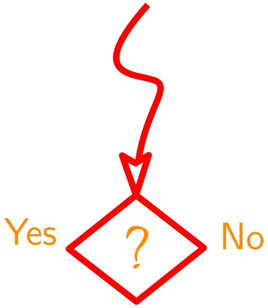
Input: n real (positive) numbers

Output: sorting permutation

Monitoring execution

Sorting

Input: n real (positive) numbers



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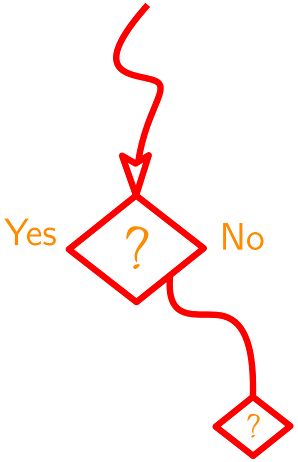

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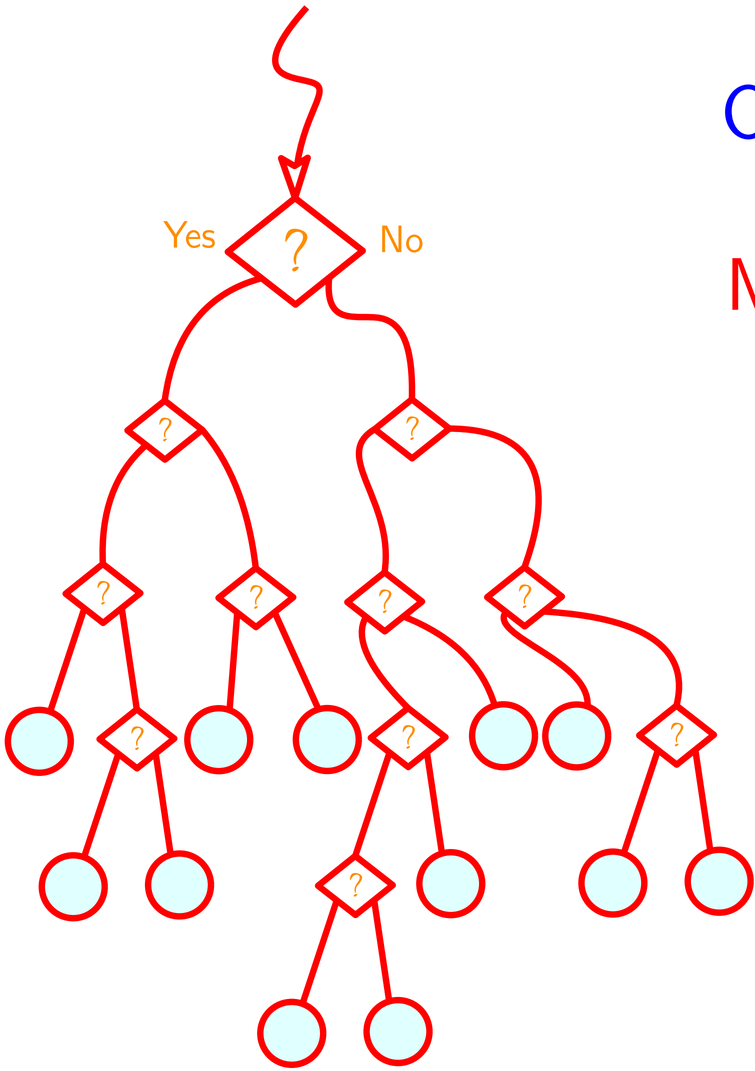

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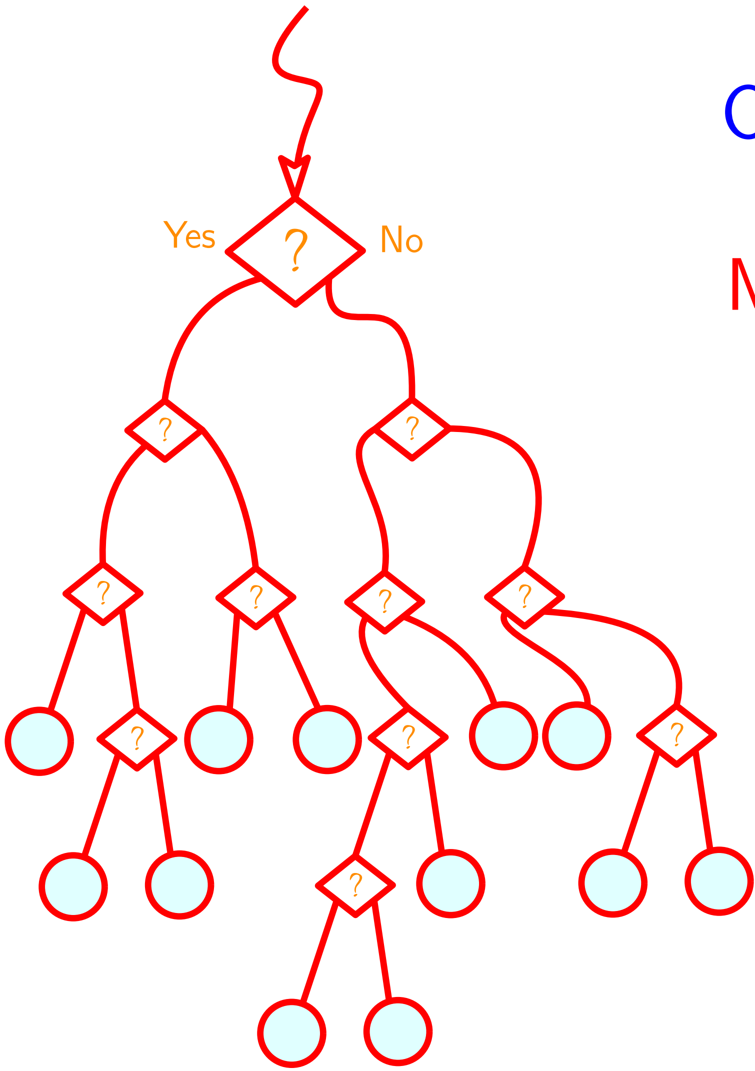

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Monitoring execution

$\sharp$ leaves $\geq$ $\sharp$ permutations

Sorting

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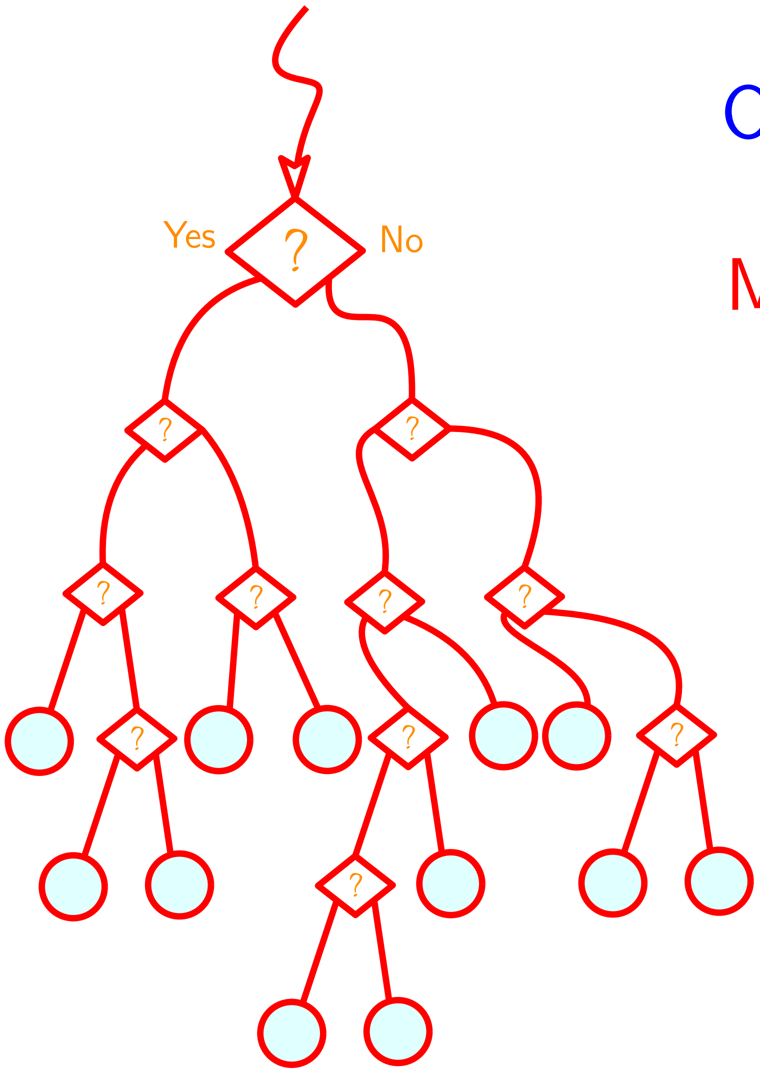


Output: sorting permutation

Monitoring execution

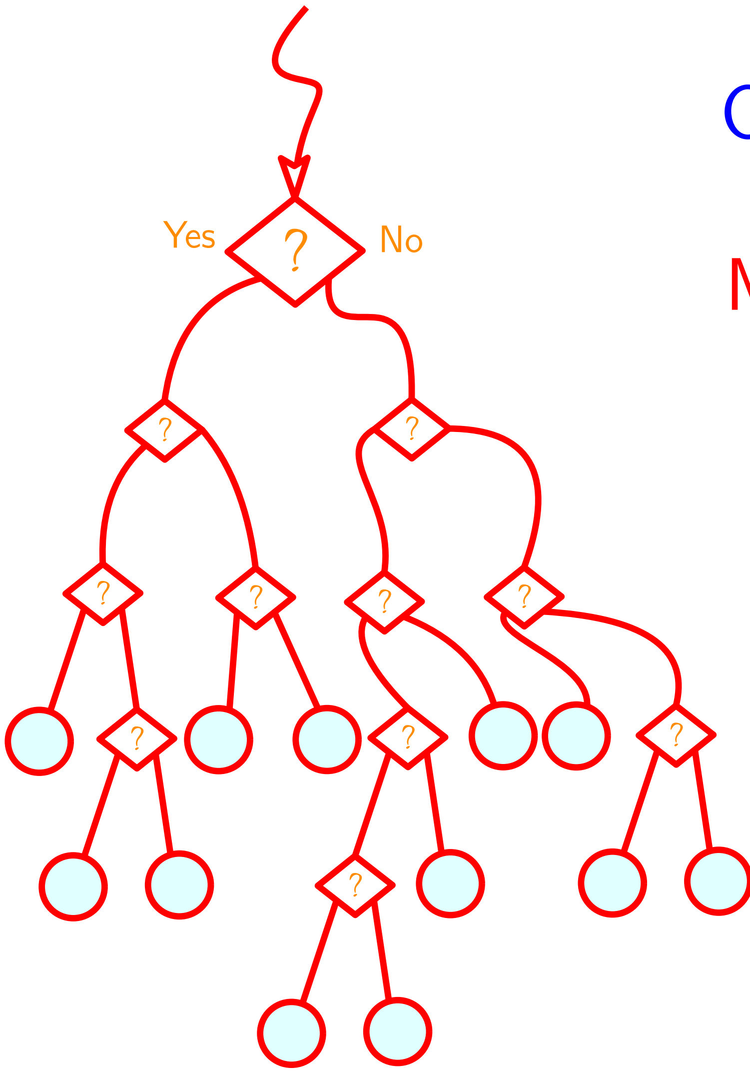
$\sharp$ leaves $\geq$ $\sharp$ permutations

There are $n!$ permutations

Sorting

Lower bound

Input: n real (positive) numbers



Output: sorting permutation

Monitoring execution

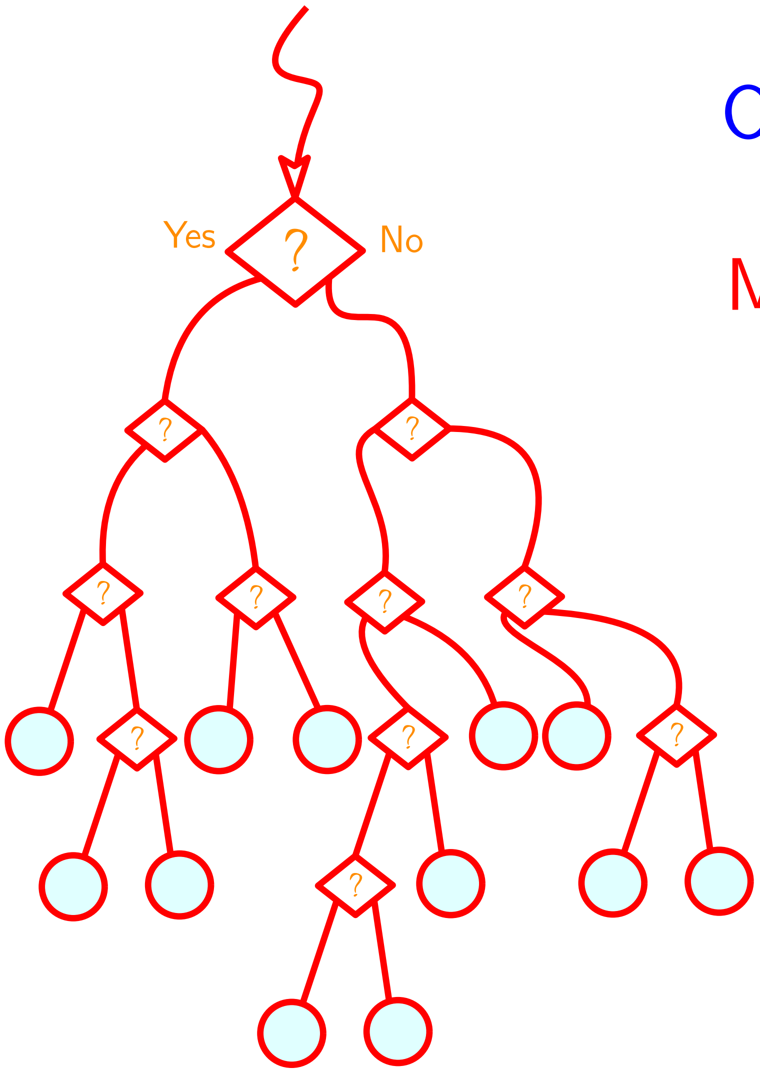
$\sharp$ leaves $\geq$ $\sharp$ permutations

There are $n!$ permutations

Tree height is at least $\log_2 \sharp$ leaves

Sorting

Input: n real (positive) numbers



Output: sorting permutation

Monitoring execution

$\sharp$ leaves $\geq$ $\sharp$ permutations

There are $n!$ permutations

Tree height is at least $\log_2 \sharp$ leaves

$\sharp$ comparisons $\leq \log_2 n! \simeq n \log_2 n$

Convex hull

Input: n 2D points (real coordinates)

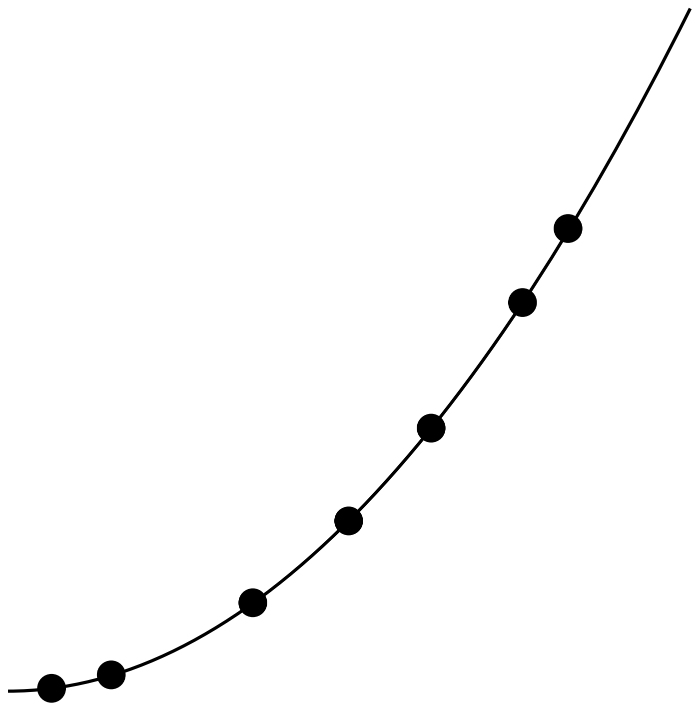
Output: list of points along the convex hull

Convex hull

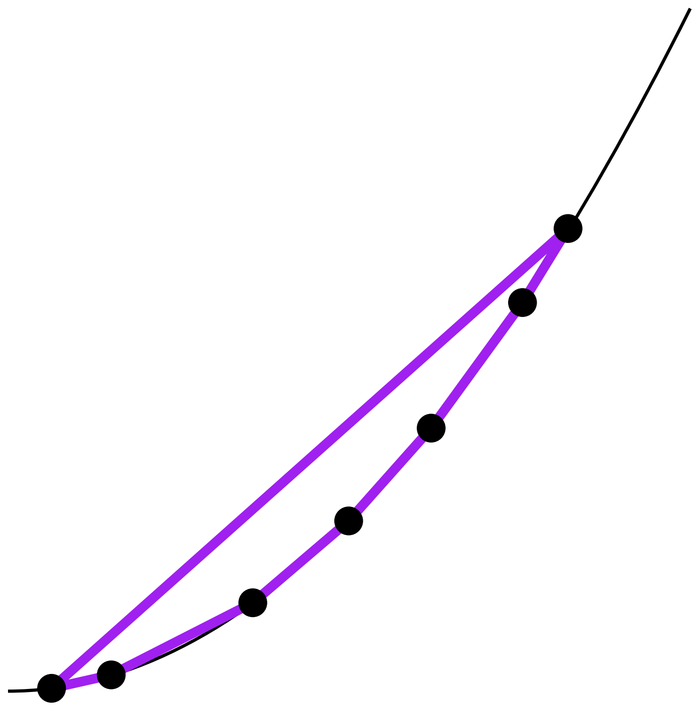
A stupid algorithm for sorting numbers

Convex hull

project on parabola



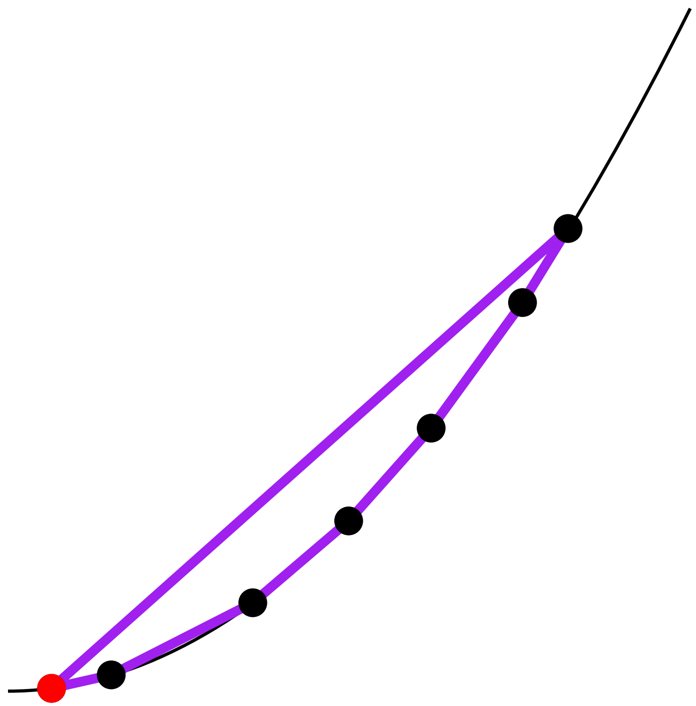
Convex hull



project on parabola

compute convex hull

Convex hull

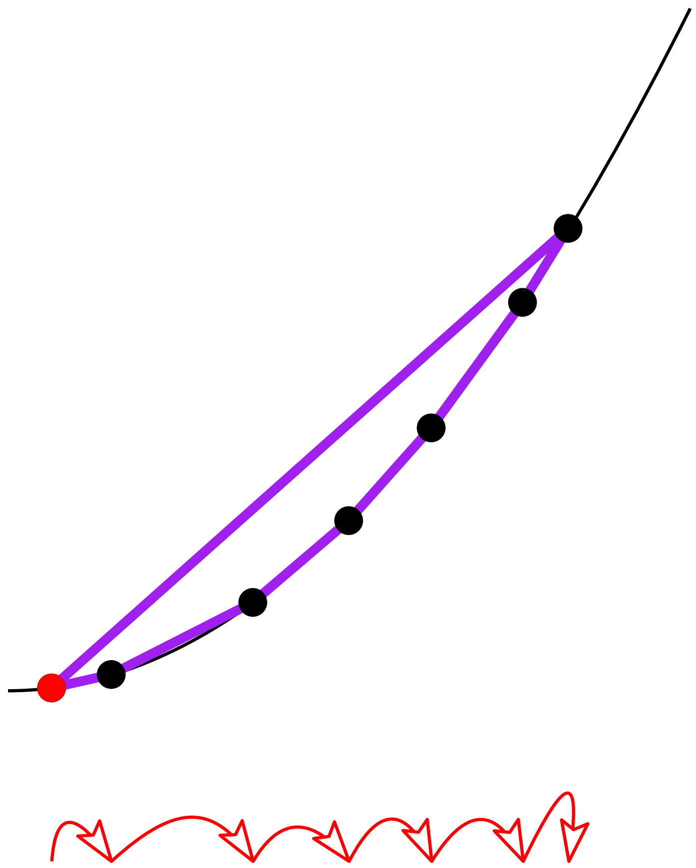


project on parabola

compute convex hull

find lowest point

Convex hull



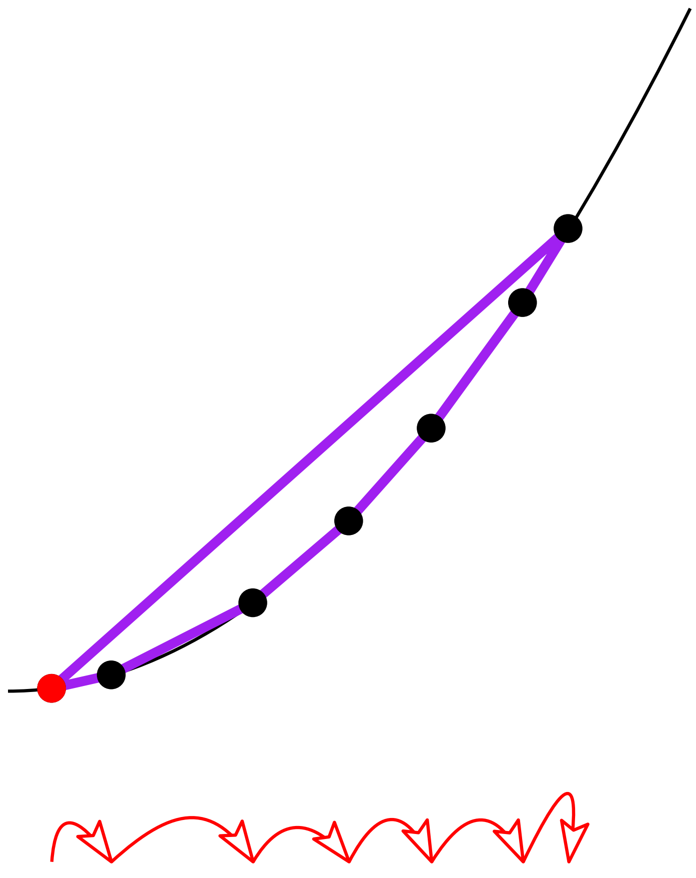
project on parabola

compute convex hull

find lowest point

enumerate x coordinates
in ccw CH order

Convex hull



$O(n)$ project on parabola

$f(n)$ compute convex hull

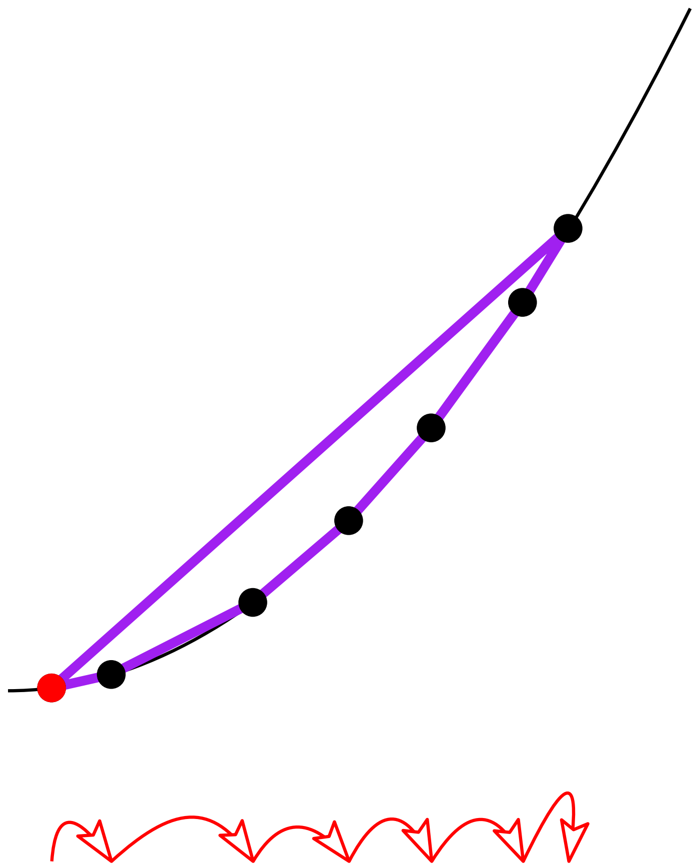
$O(n)$ find lowest point

$O(n)$ enumerate x coordinates in ccw CH order

Lower bound on sorting

$\Longrightarrow f(n) + O(n) \geq \Omega(n \log n)$

Convex hull



$O(n)$ project on parabola

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Lower bound on sorting

$\Longrightarrow f(n) + O(n) \geq \Omega(n \log n)$

Convex hull

Euler relation

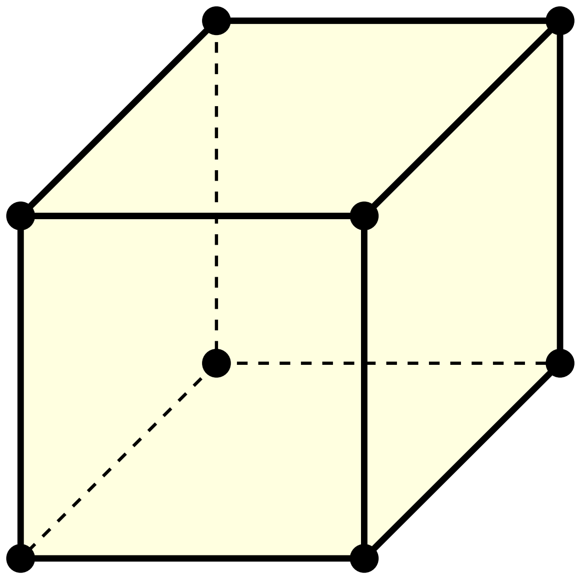
Polytope boundary

Vertices Edges Faces

Convex hull

Euler relation

Polytope boundary

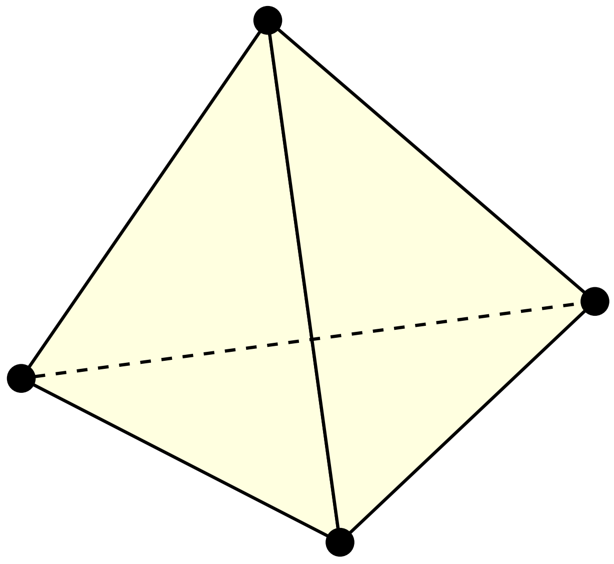


$$8 \quad - \quad 12 \quad + \quad 6 \qquad = 2$$

Convex hull

Euler relation

Polytope boundary

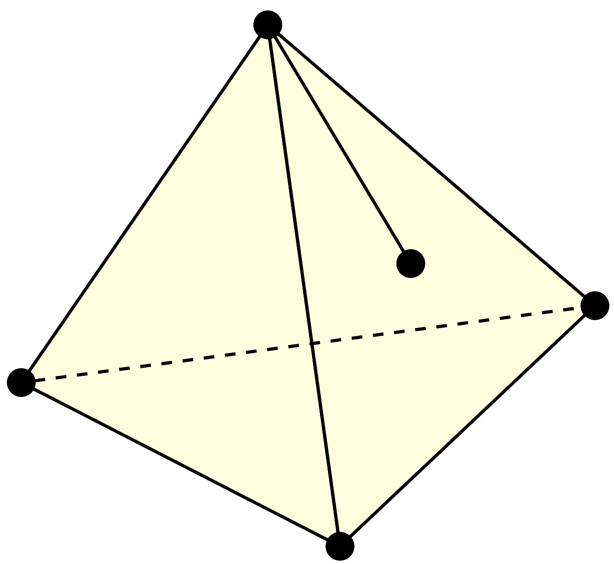


Vertices		Edges		Faces	
8	$-$	12	$+$	6	$=2$
4	$-$	6	$+$	4	$=2$

Convex hull

Euler relation

Polytope boundary

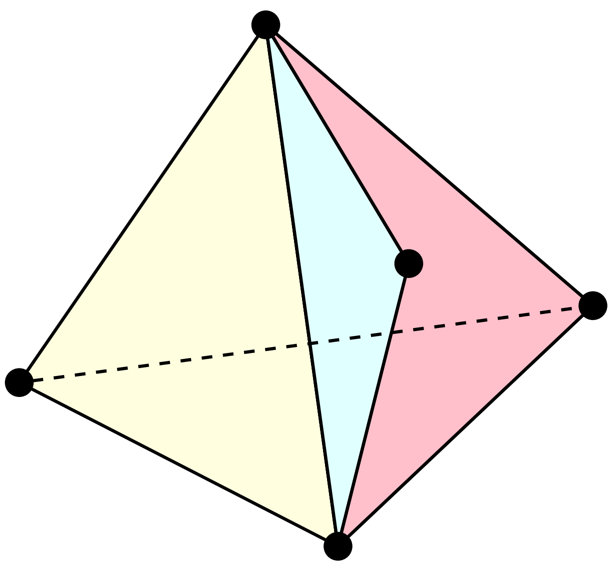


Vertices		Edges		Faces	
8	$-$	12	$+$	6	$=2$
4	$-$	6	$+$	4	$=2$
$+1$	$-$	$+1$	$+$	0	$=+0$

Convex hull

Euler relation

Polytope boundary

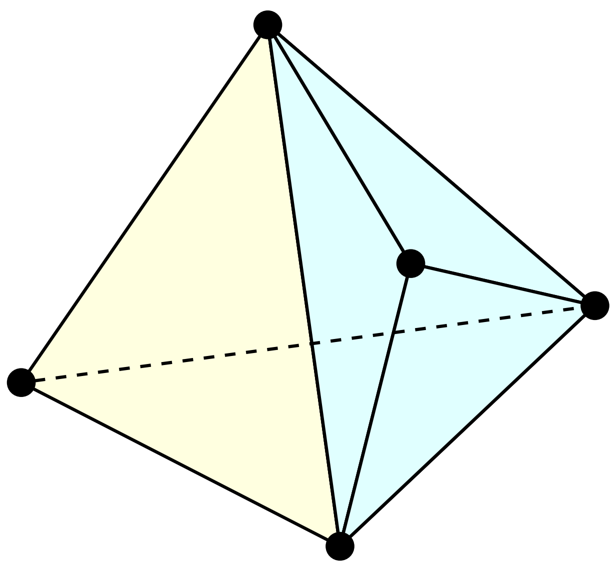


Vertices		Edges		Faces	
8	$-$	12	$+$	6	$= 2$
4	$-$	6	$+$	4	$= 2$
$+1$	$-$	$+1$	$+$	0	$= +0$
0	$-$	$+1$	$+$	$+1$	$= +0$

Convex hull

Euler relation

Polytope boundary



Vertices		Edges		Faces	
8	$-$	12	$+$	6	$=2$
4	$-$	6	$+$	4	$=2$
$+1$	$-$	$+1$	$+$	0	$=+0$
0	$-$	$+1$	$+$	$+1$	$=+0$

Convex hull

Euler relation

Polytope boundary

Vertices Edges Faces

$$n \quad - \quad e \quad + \quad f \qquad = 2$$

Convex hull

Euler relation

Polytope boundary

Vertices		Edges		Faces	
n	$-$	e	$+$	f	$= 2$

triangular faces $3f = 2e$

Convex hull

Euler relation

Polytope boundary

Vertices		Edges		Faces	
n	$-$	e	$+$	f	$= 2$

triangular faces $3f = 2e$

$$f = 2n - 4$$

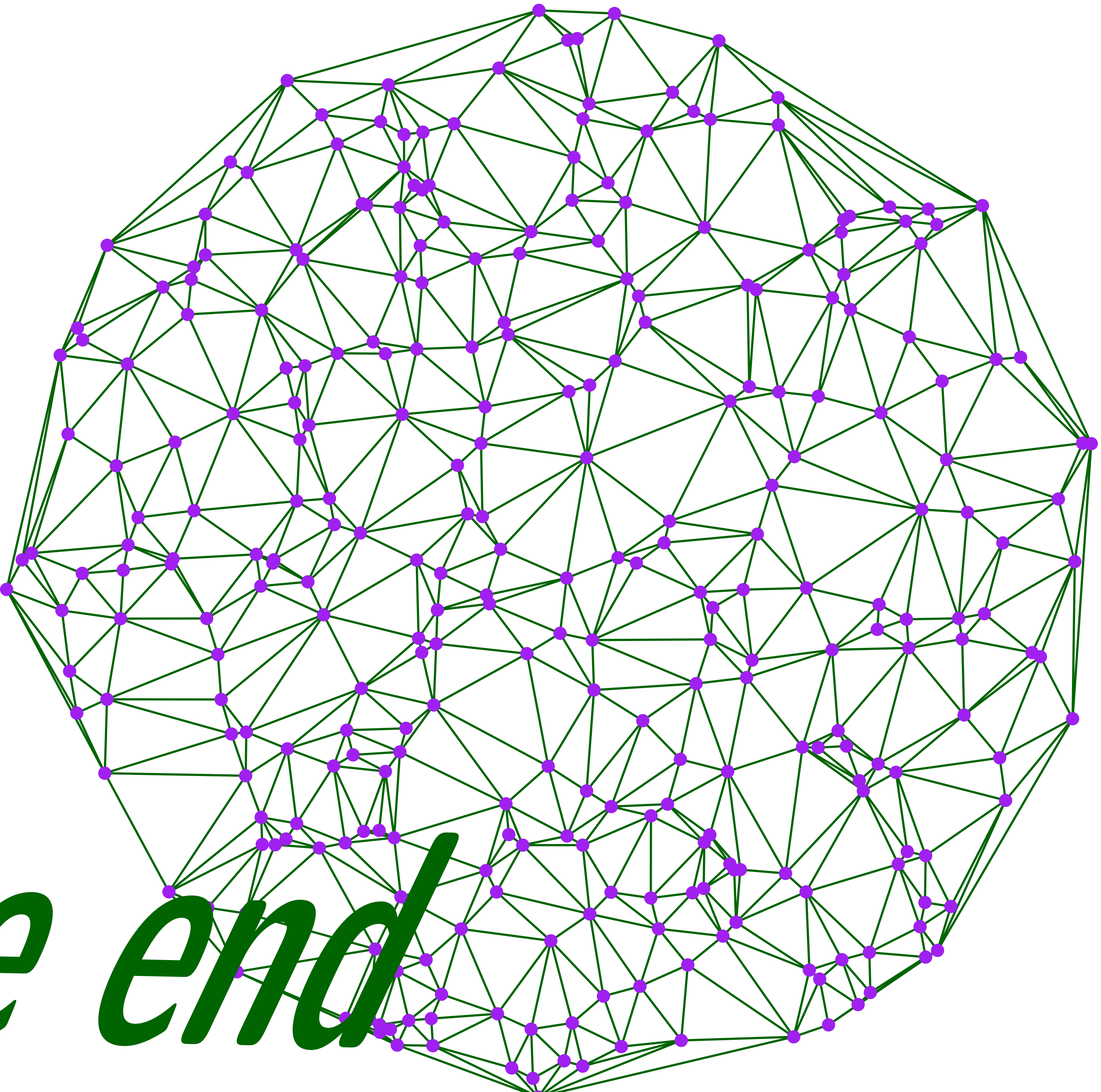
$$e = 3n - 6$$

Convex hull

Linear size

$O(n \log n)$ divide and conquer algorithm

$O(nh)$ gift wraping algorithm



The end