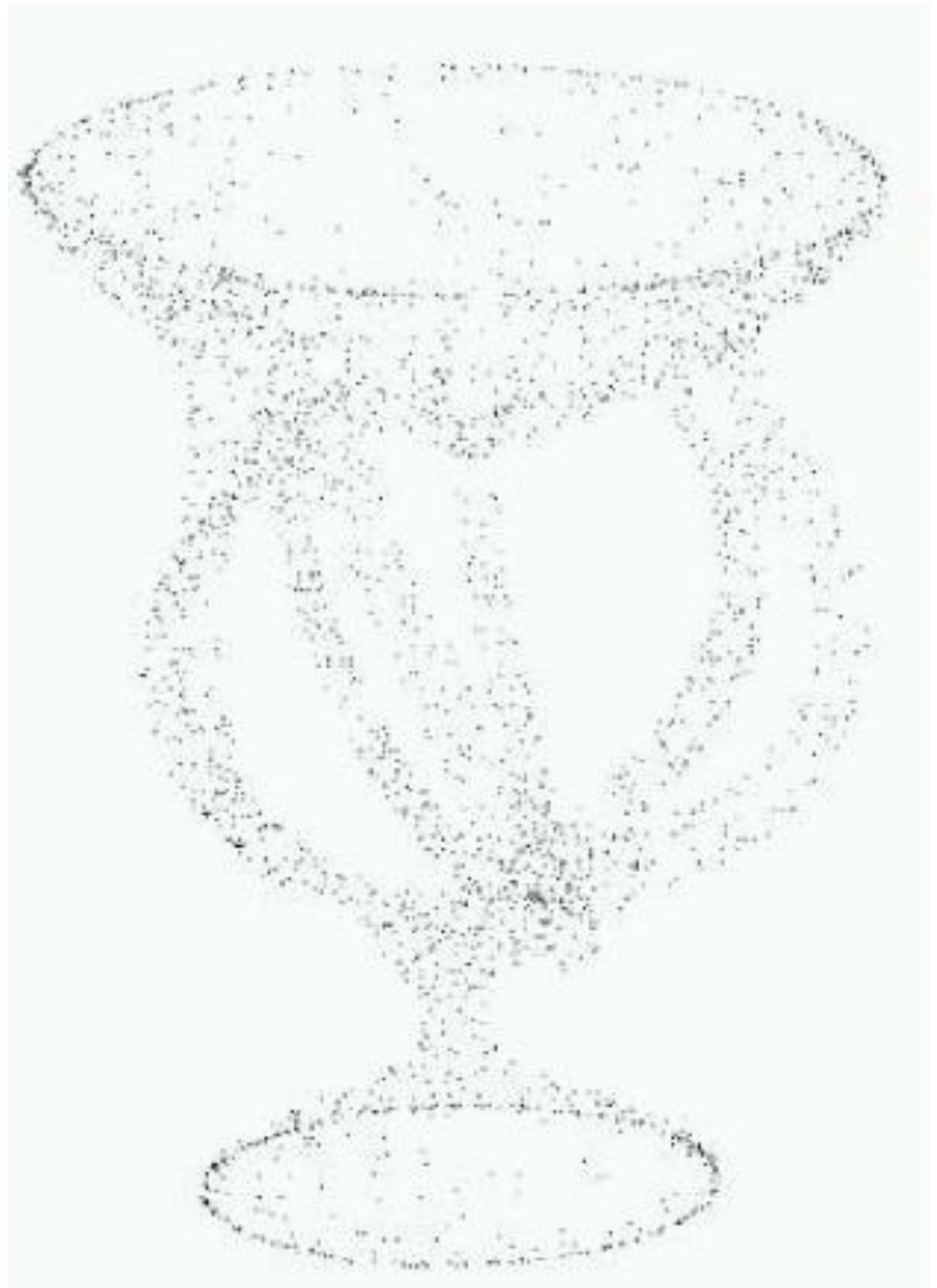


Reconstruction

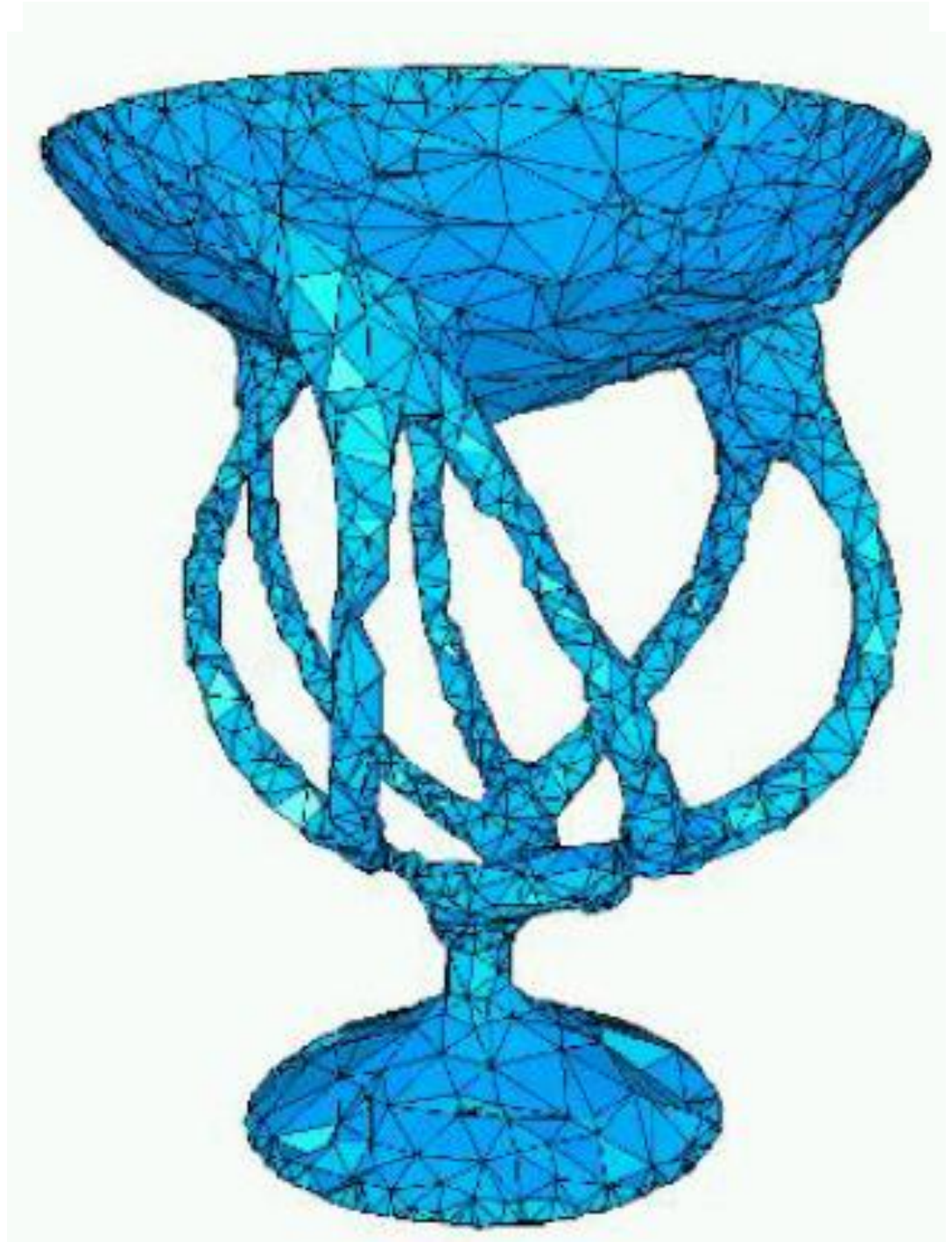
From points



Reconstruction

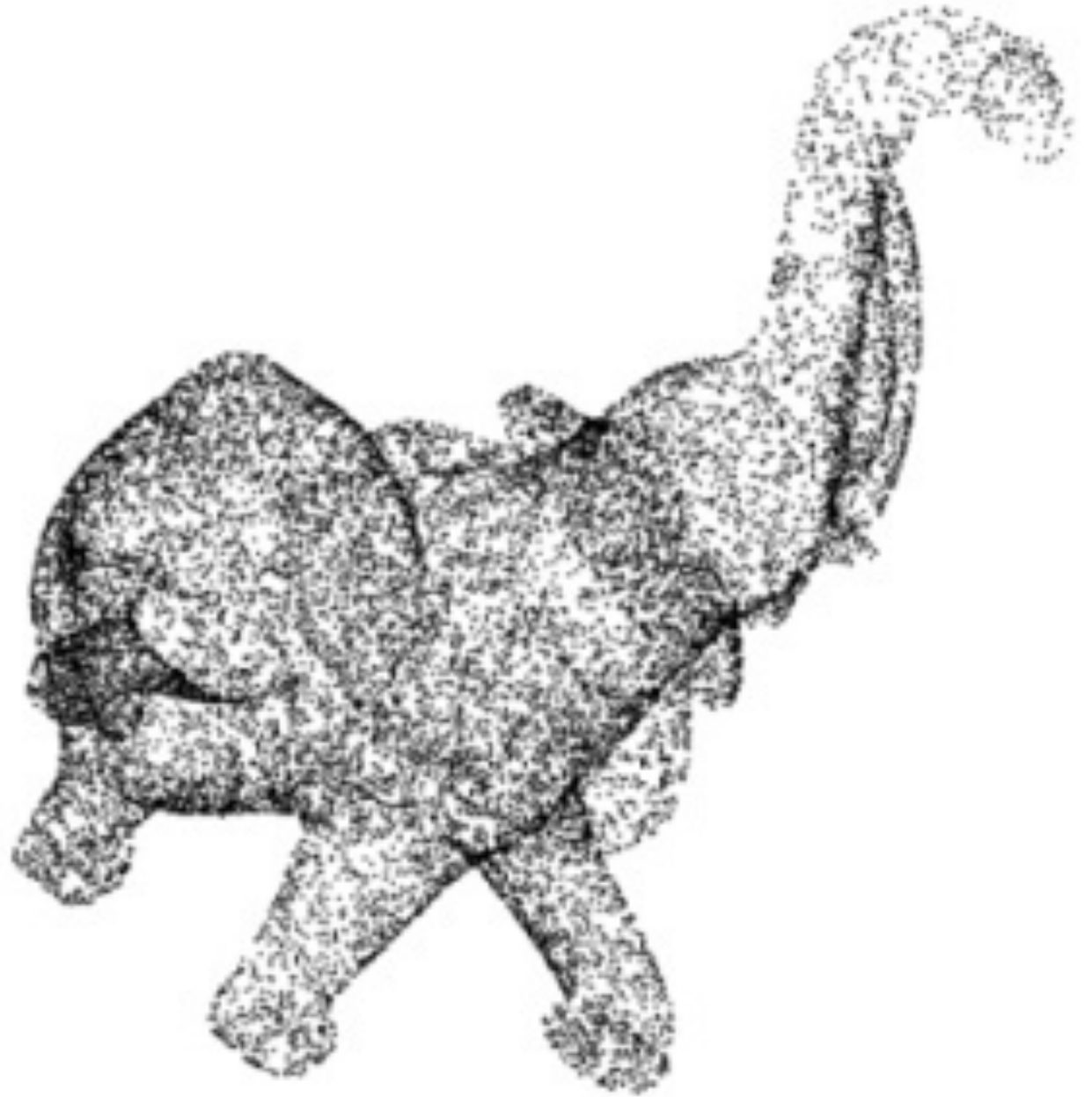
From points

to shape



Reconstruction

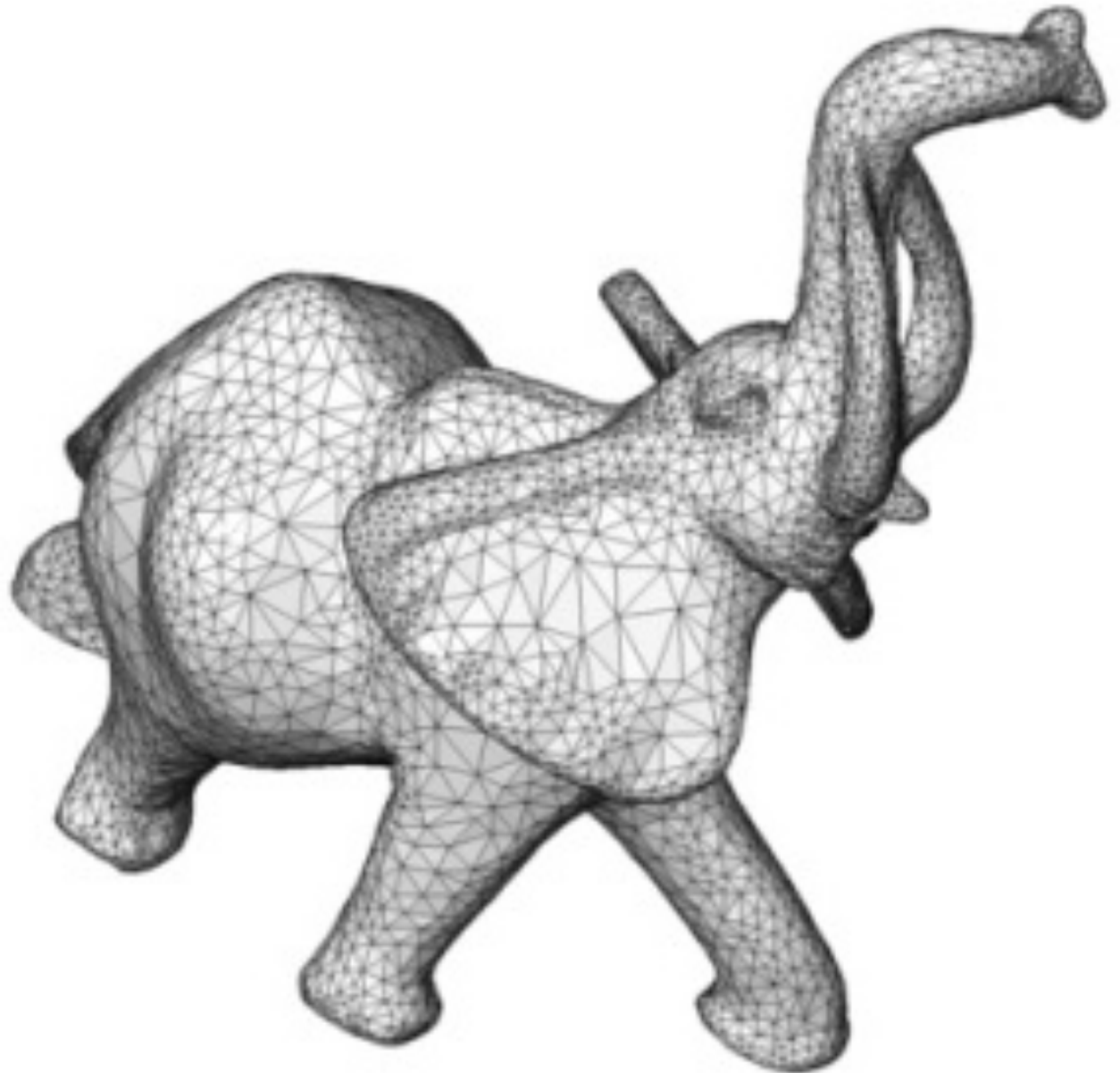
From points



Reconstruction

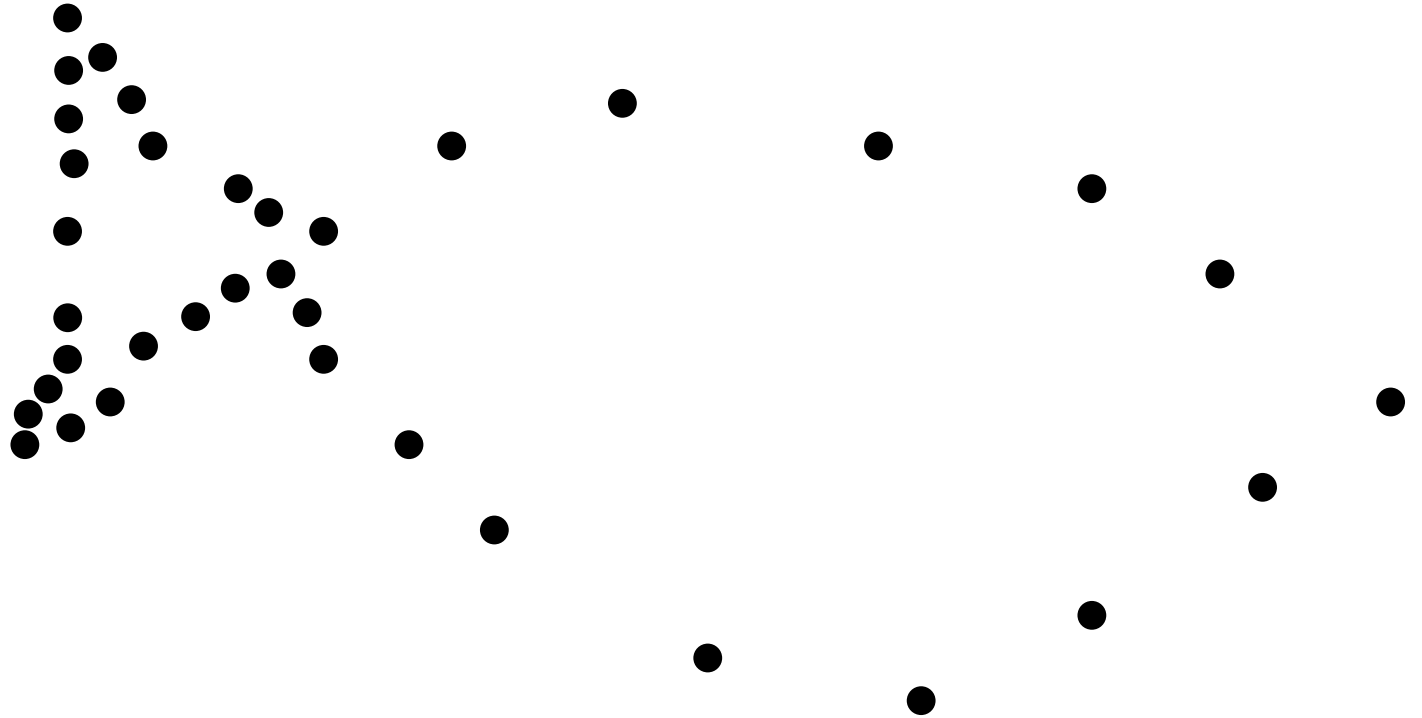
From points

to shape



Reconstruction

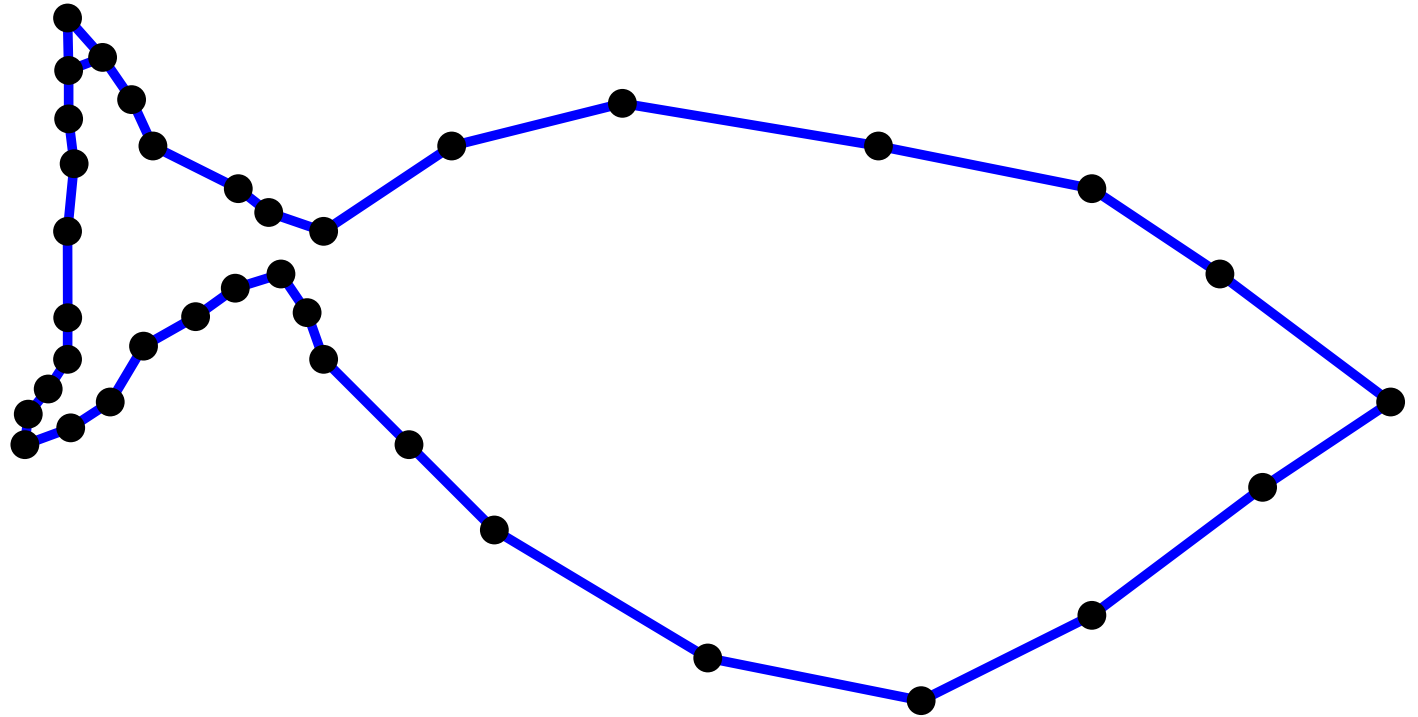
From points



Reconstruction

From points

to shape



Reconstruction

Context

Delaunay is a good start (wanted result \subset Delaunay)

Crust 2D

Algorithm

0.4 sample \Rightarrow wanted result \subset crust

0.25 sample \Rightarrow crust \subset wanted result

3D

Reconstruction

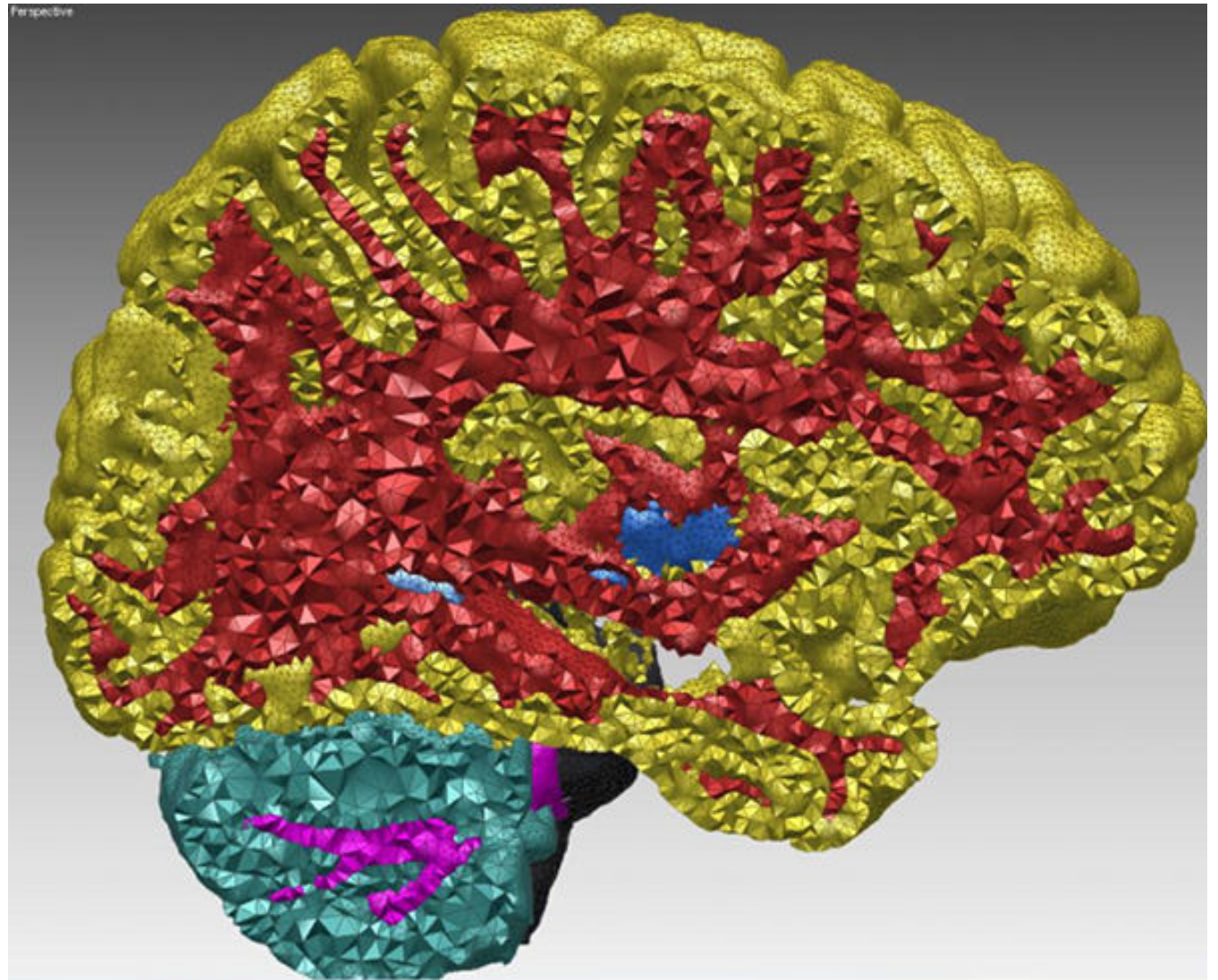
Context

Sensor \longrightarrow Point set (no structure or unknown)

Reconstruction

Context

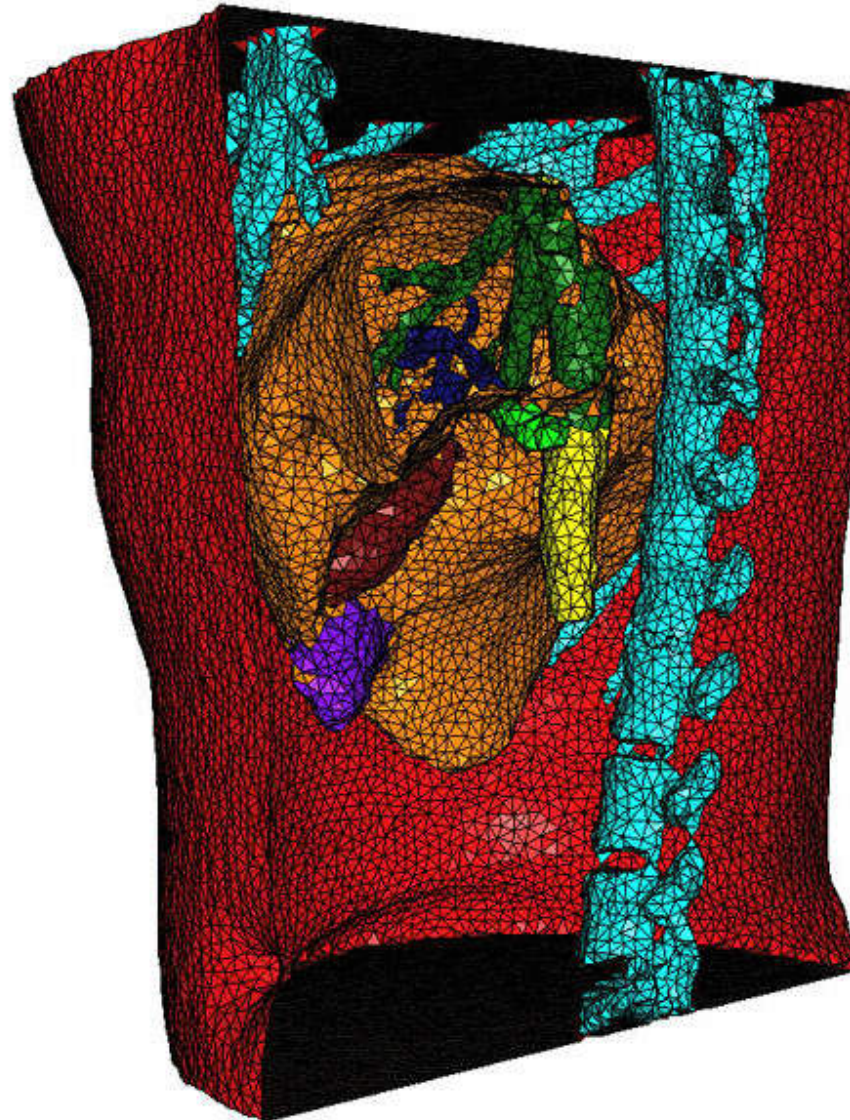
Medical Images



Reconstruction

Medical Images

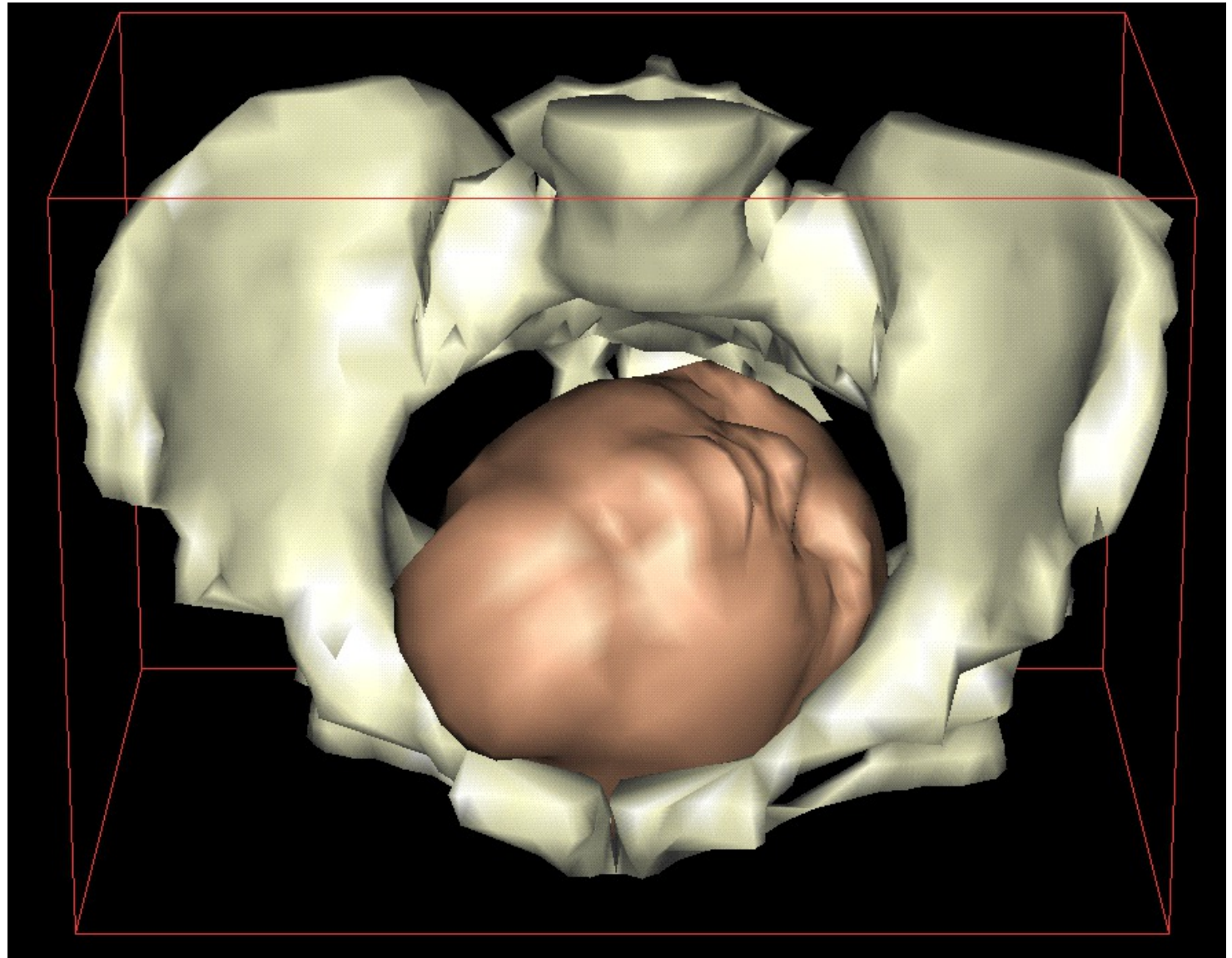
Context



Reconstruction

Context

Childbirth simulation



Reconstruction

Context

Childbirth simulation

Surgery planning

Radiotherapy planing

Endoscopy simulation



Reconstruction

Context

Sensor



Point set (no structure or unknown)

Scanner



Reconstruction

Context

Sensor



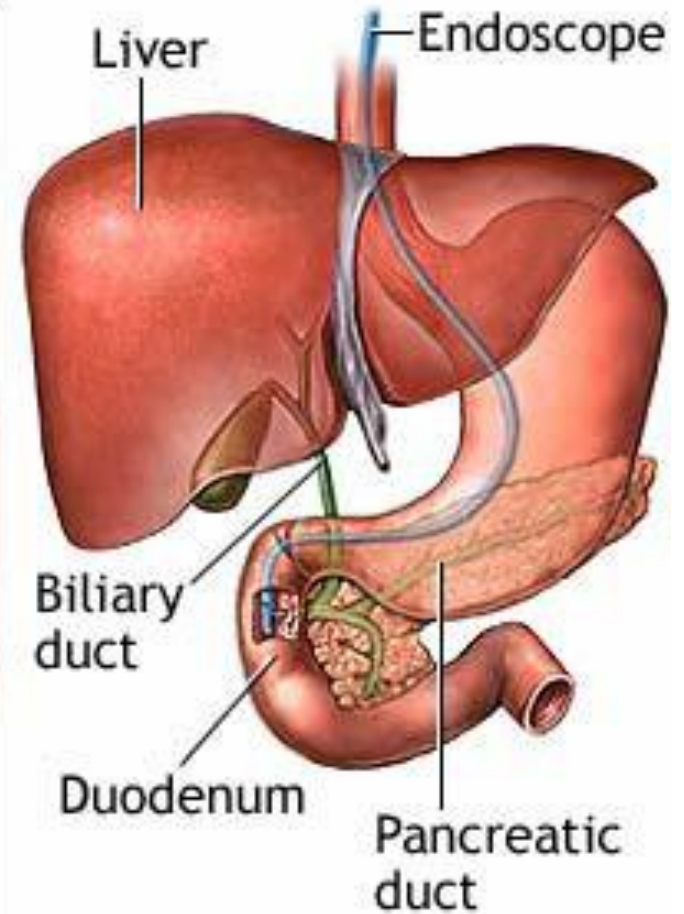
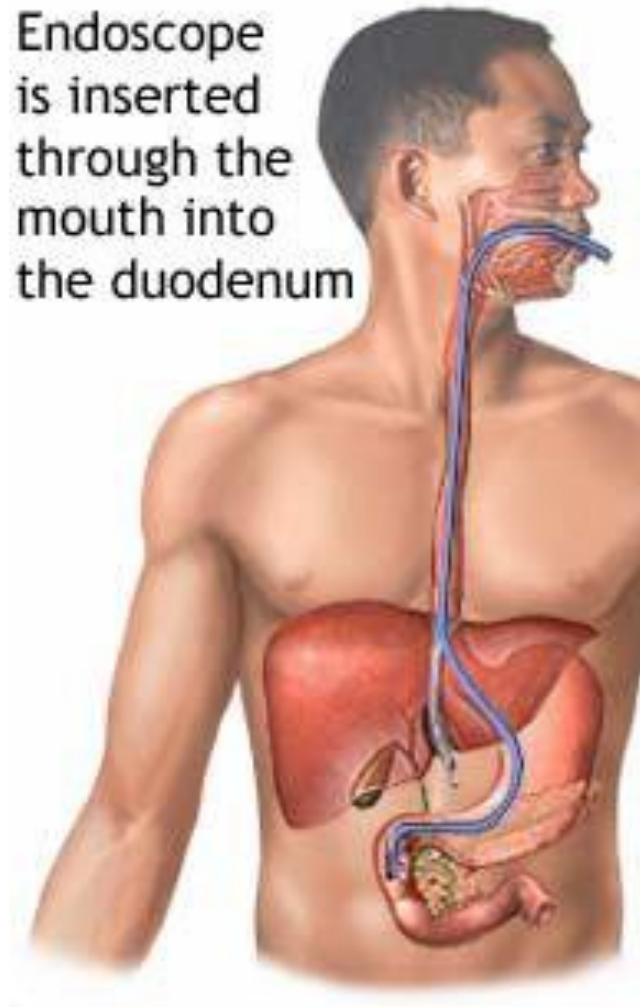
Point set (no structure or unknown)

Scanner

Endoscope



Endoscope is inserted through the mouth into the duodenum



Reconstruction

Context

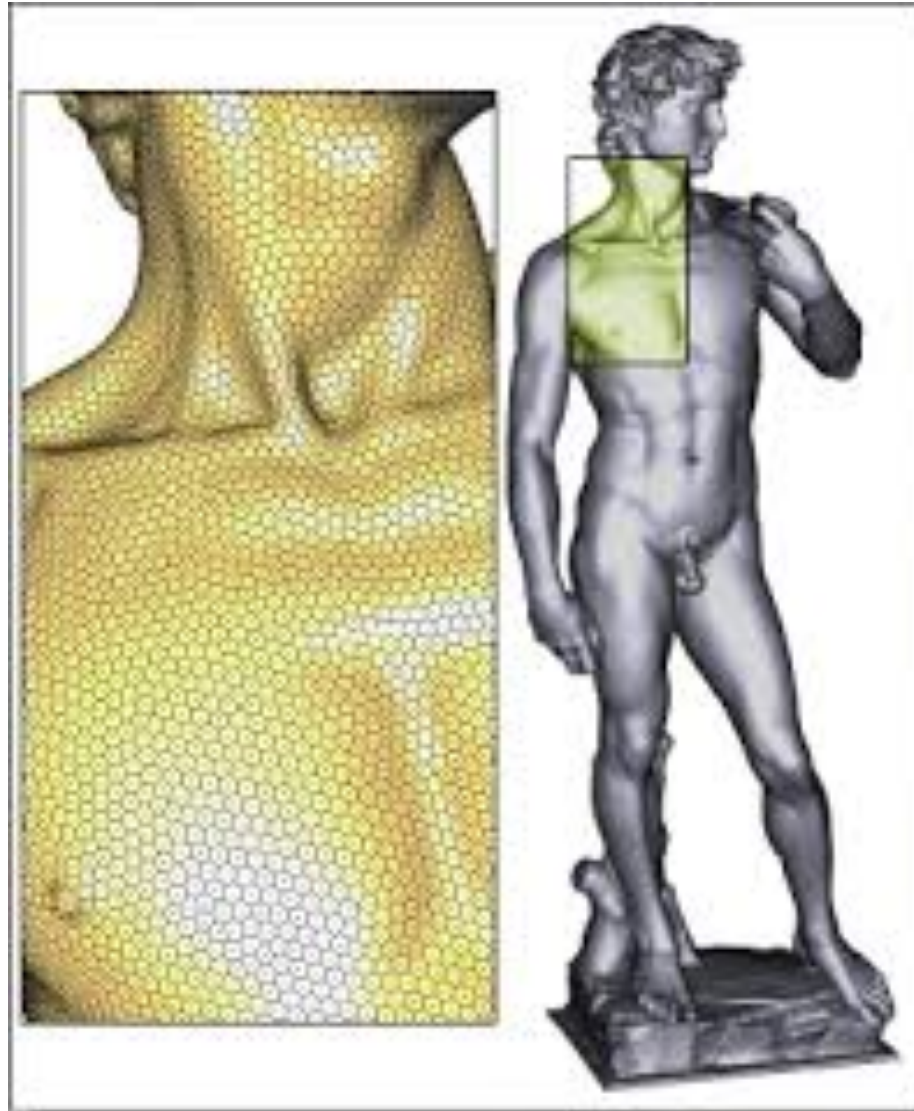
Cultural heritage



Reconstruction

Context

Cultural heritage



Reconstruction

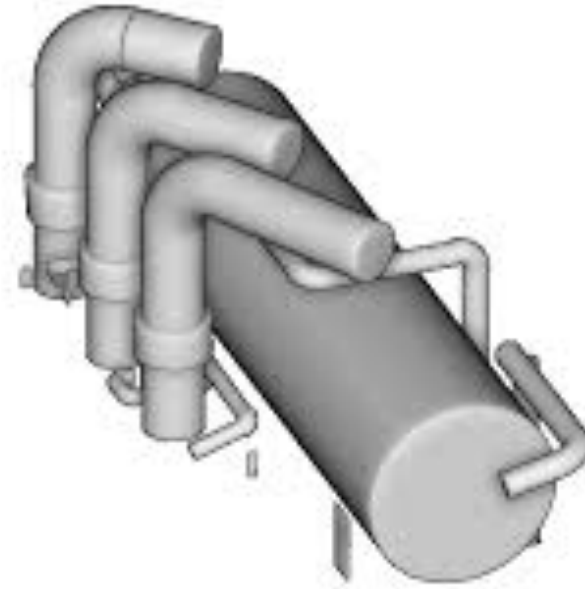
Context



Reconstruction

Context

Reverse engineering



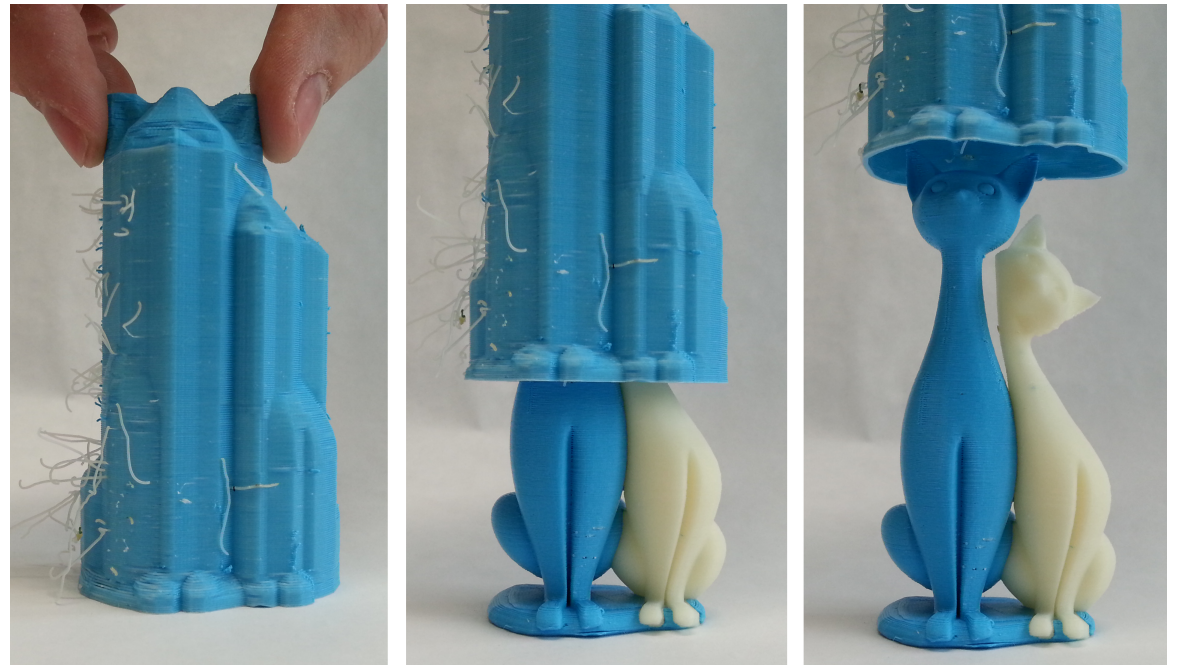
Reconstruction

Context

Reverse engineering

Prototyping (3D print)

Quality control



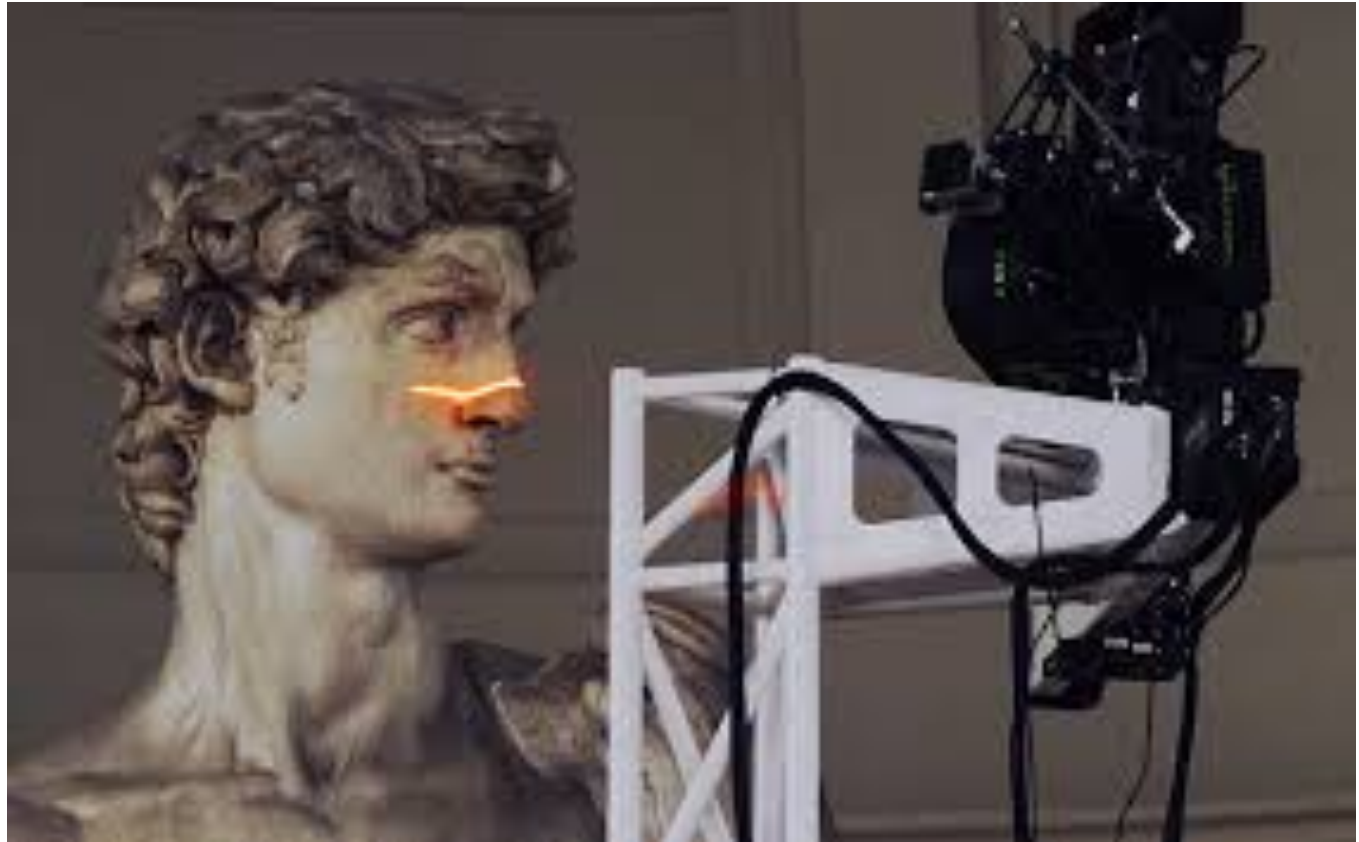
Reconstruction

Context

Sensor



Point set (no structure or unknown)



Reconstruction

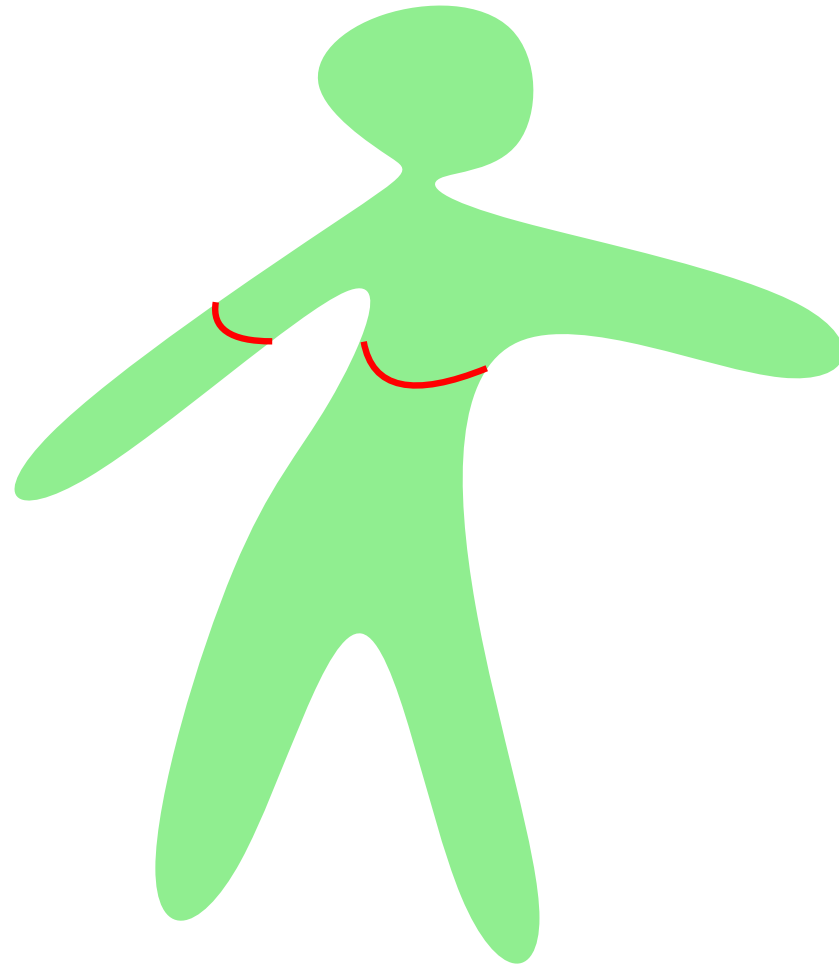
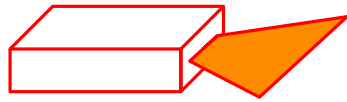
Context

Sensor



Point set (no structure or unknown)

Laser illuminate in a plane



Reconstruction

Context

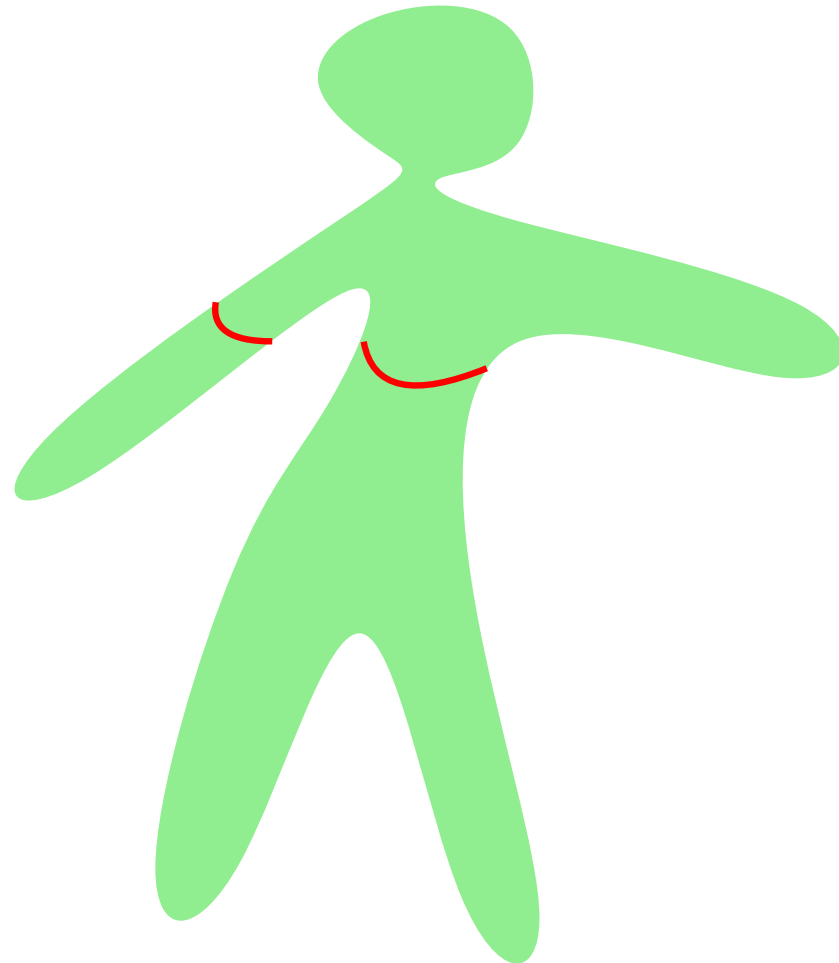
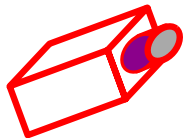
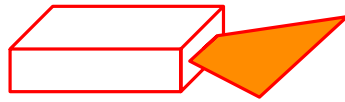
Sensor



Point set (no structure or unknown)

Laser illuminate in a plane

Camera



Reconstruction

Context

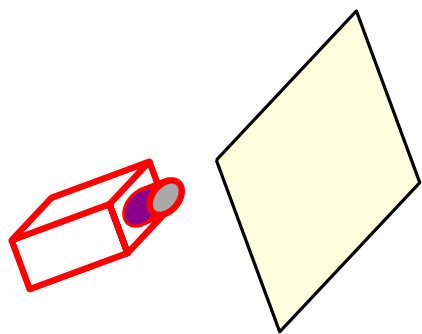
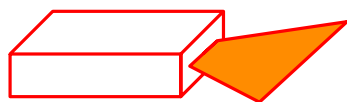
Sensor



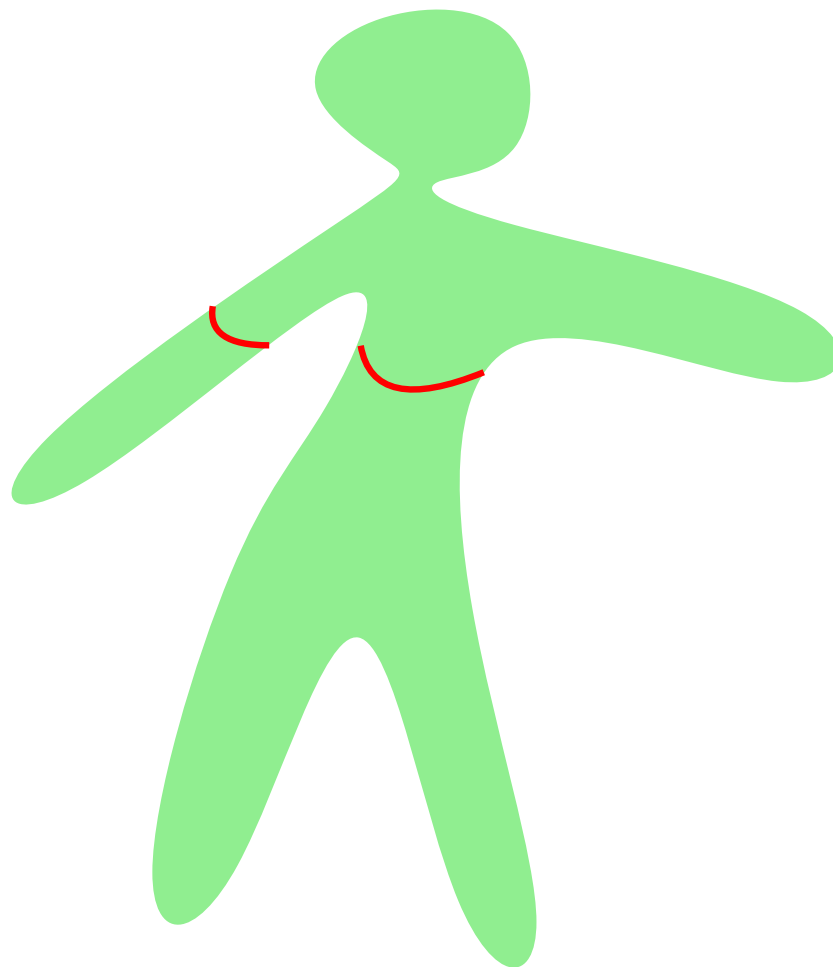
Point set (no structure or unknown)

Laser illuminate in a plane

Camera



Image



Reconstruction

Context

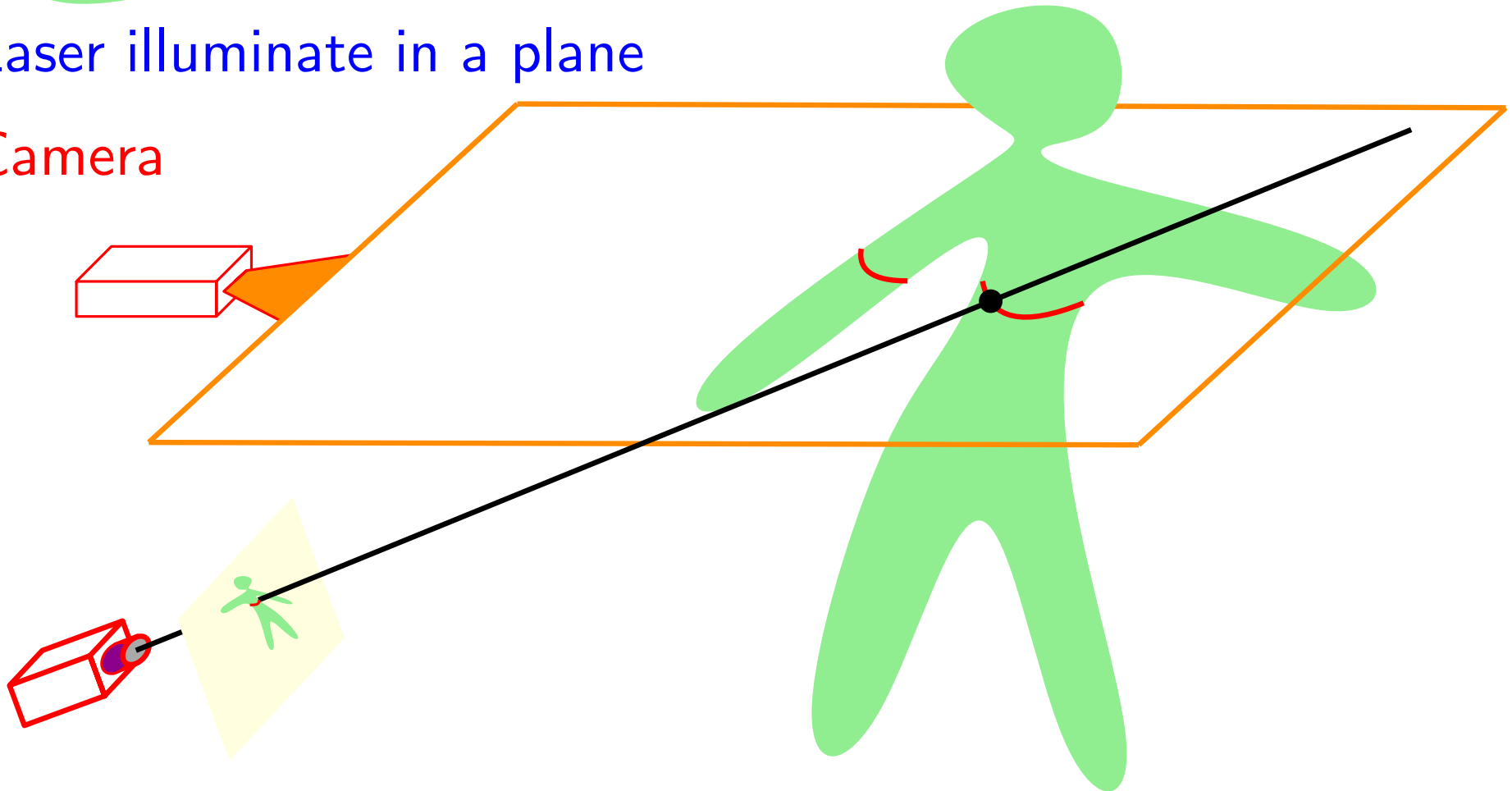
Sensor



Point set (no structure or unknown)

Laser illuminate in a plane

Camera

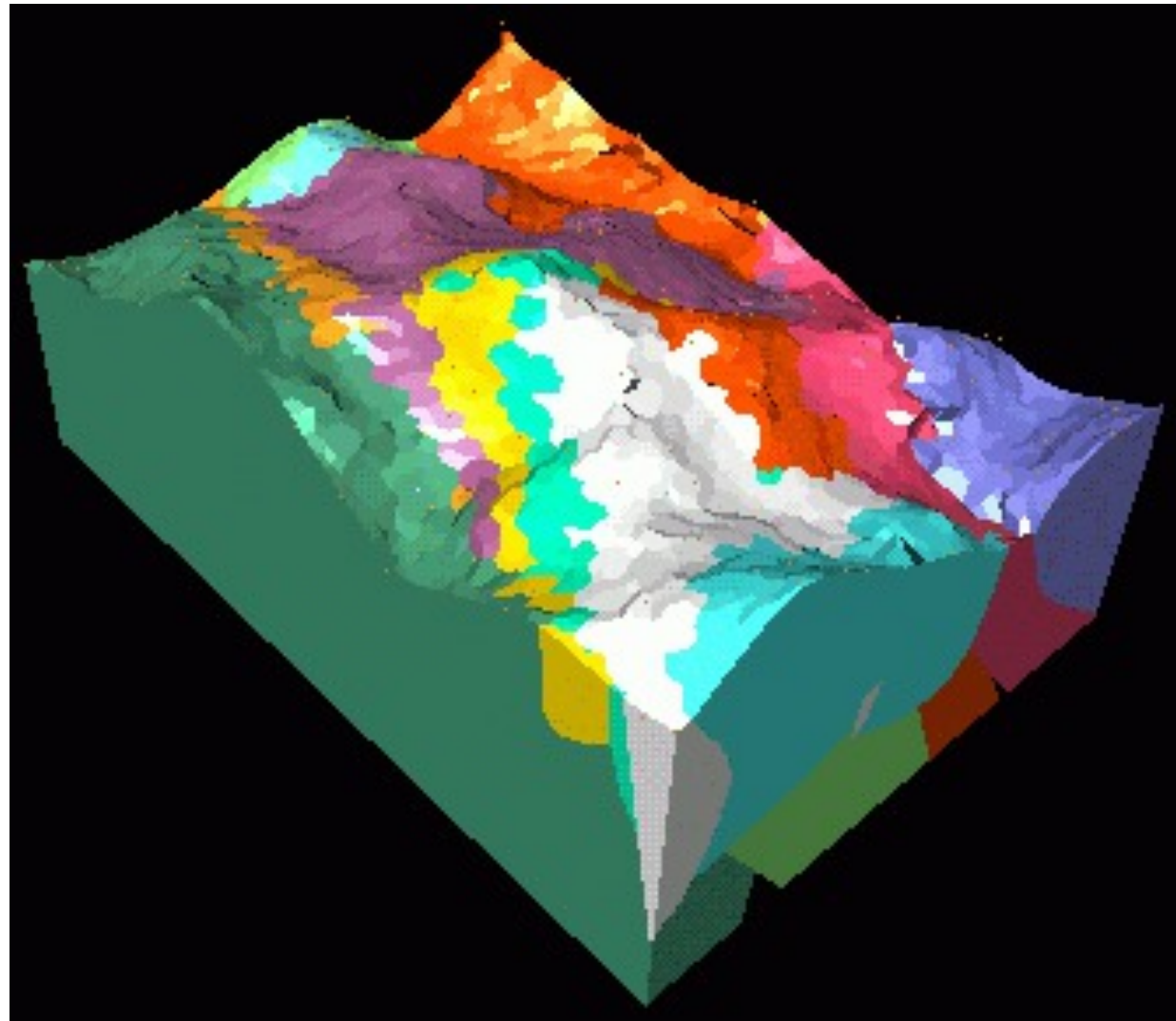


Get 3D position

Reconstruction

Context

Geology



Reconstruction

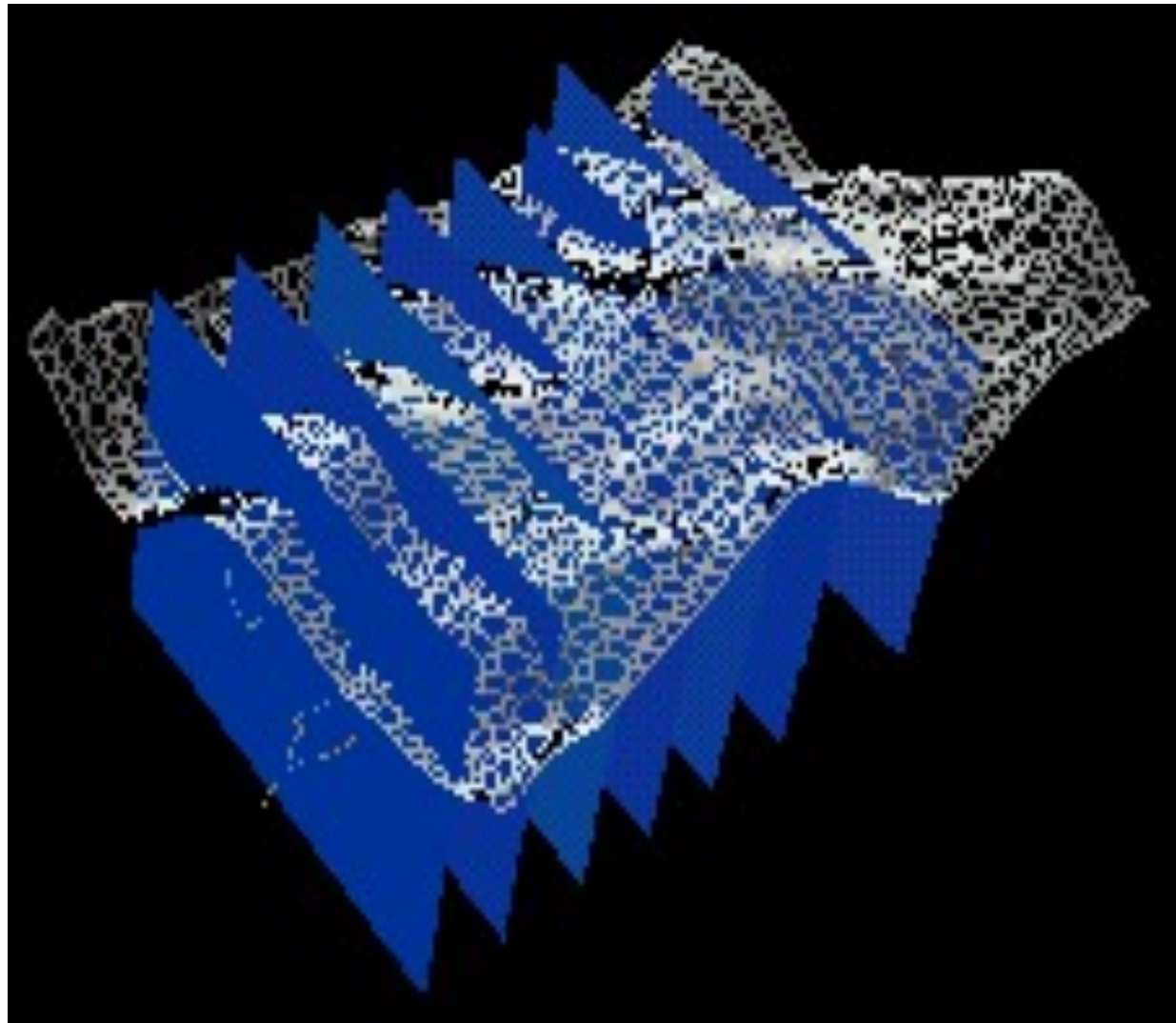
Context

Sensor



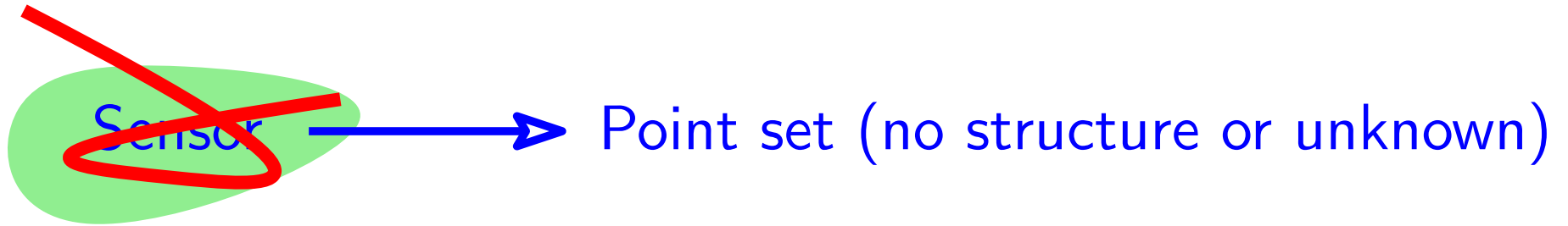
Point set (no structure or unknown)

Geology



Reconstruction

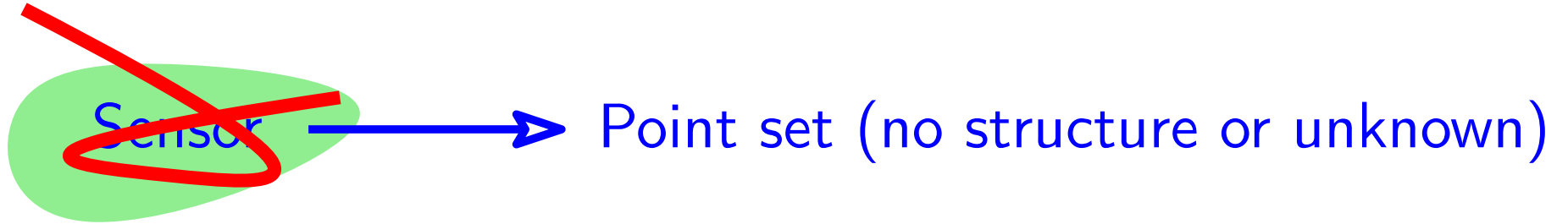
Context



Abstract 3D problem that we can solve in 2D section

Reconstruction

Context

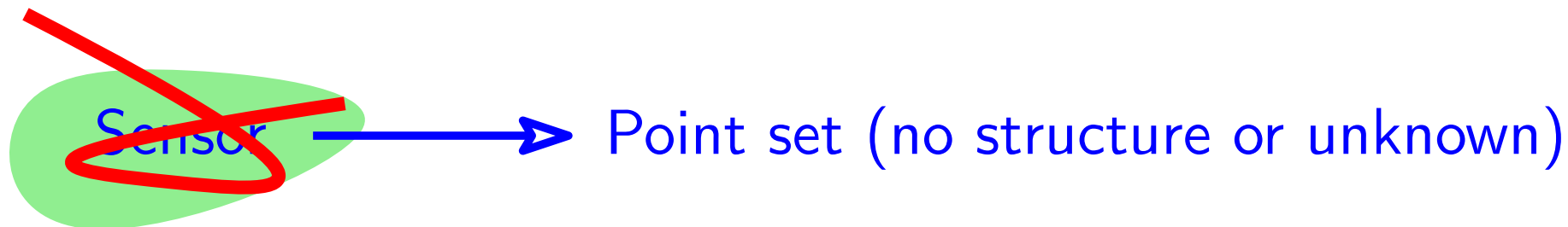


Abstract 3D problem that we can solve in 2D section

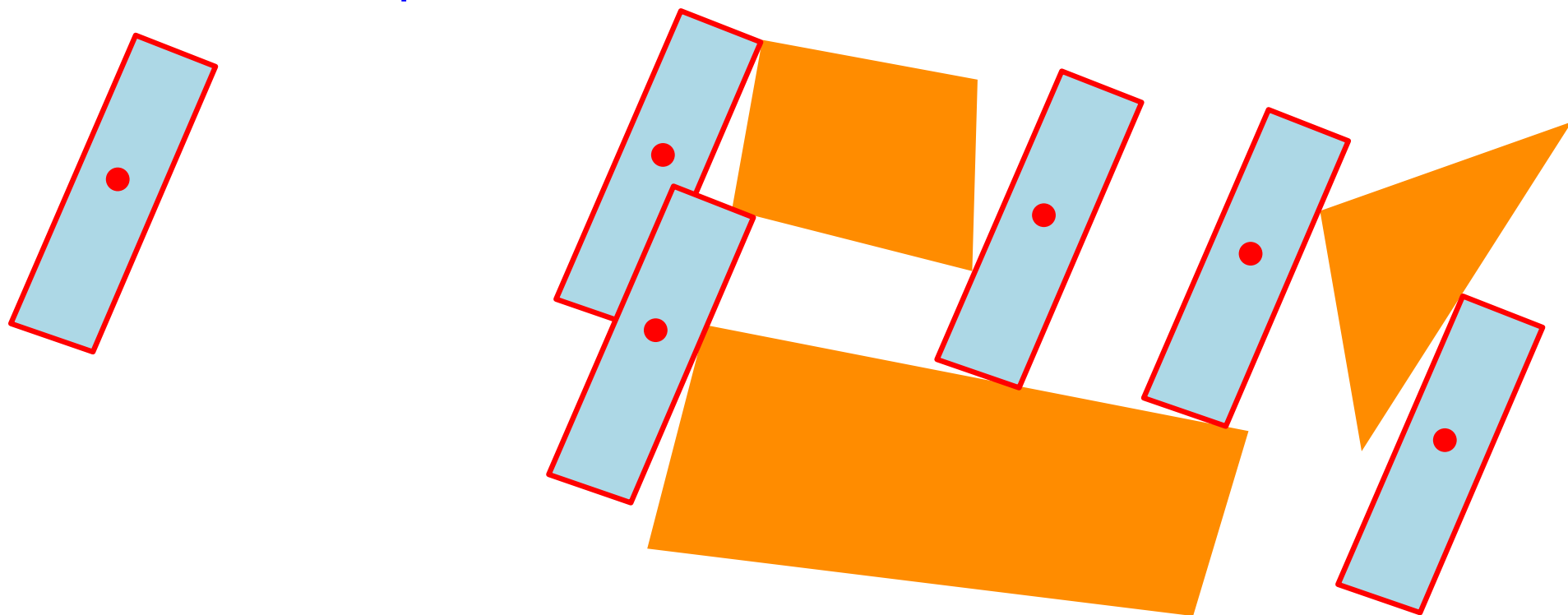


Reconstruction

Context

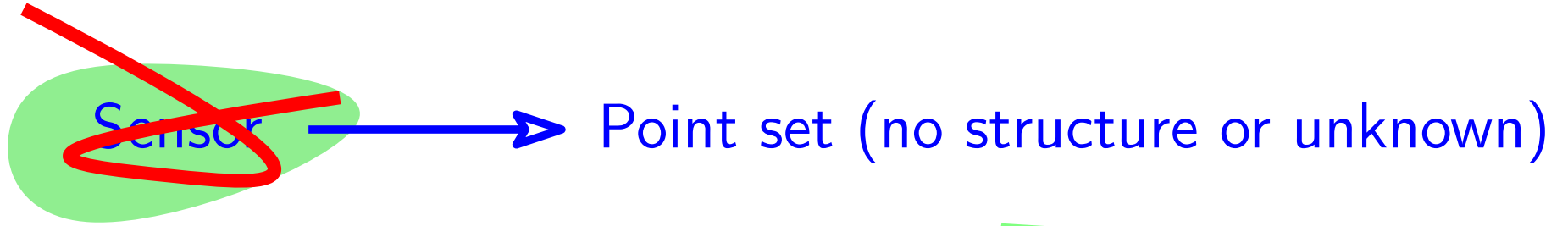


Abstract 3D problem that we can solve in 2D section

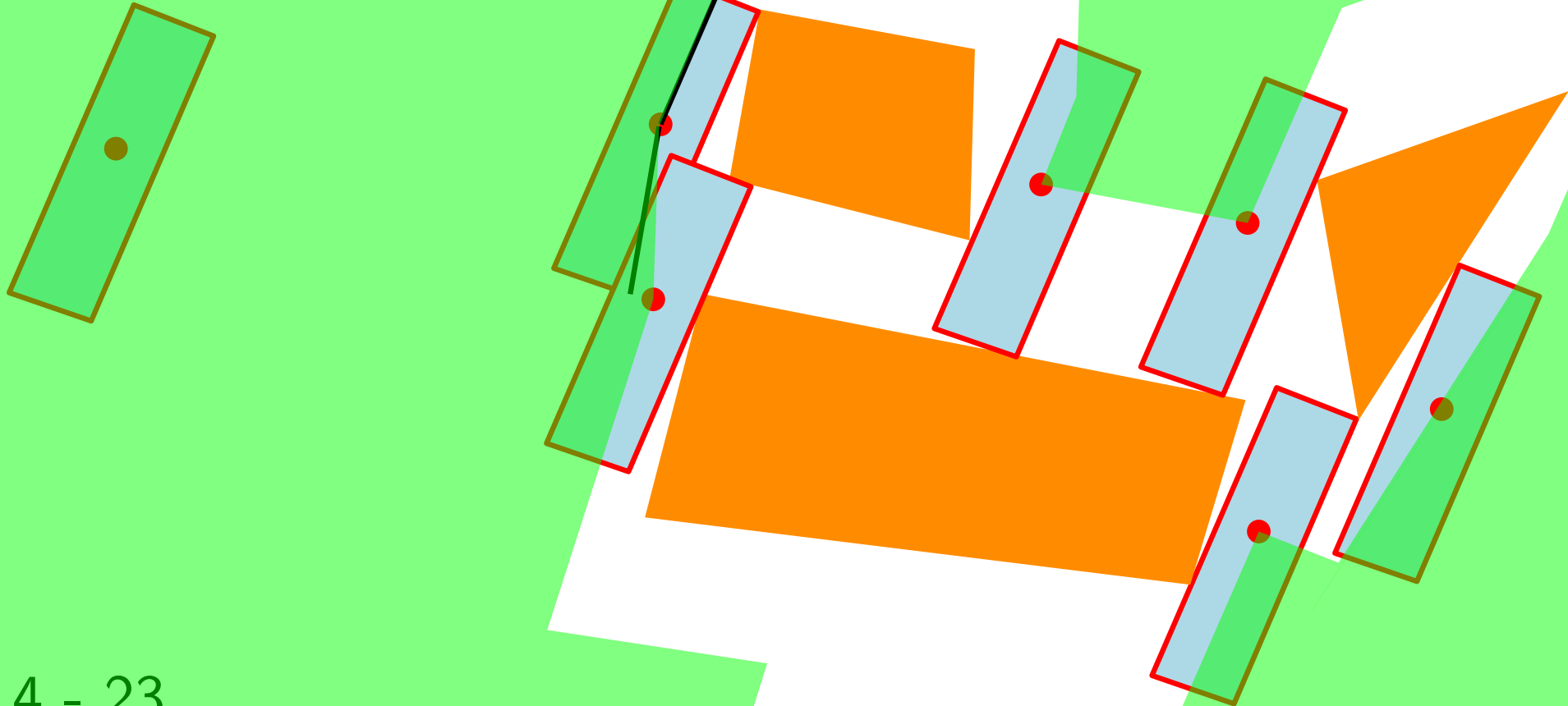


Reconstruction

Context

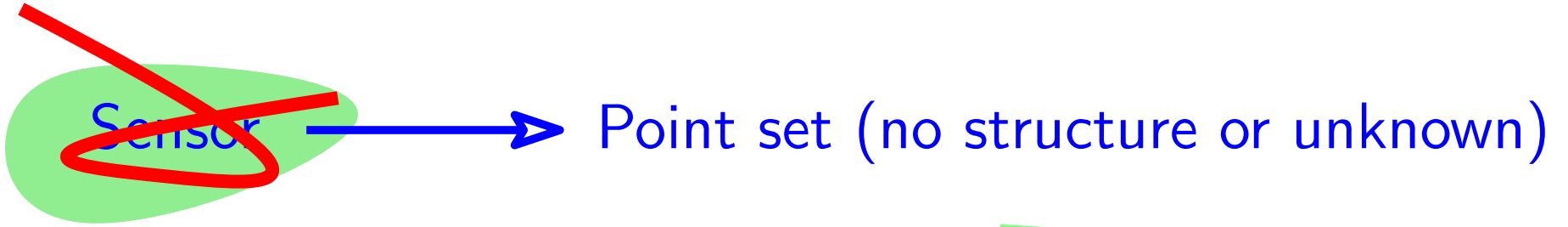


Abstract 3D problem that we can solve in 2D section

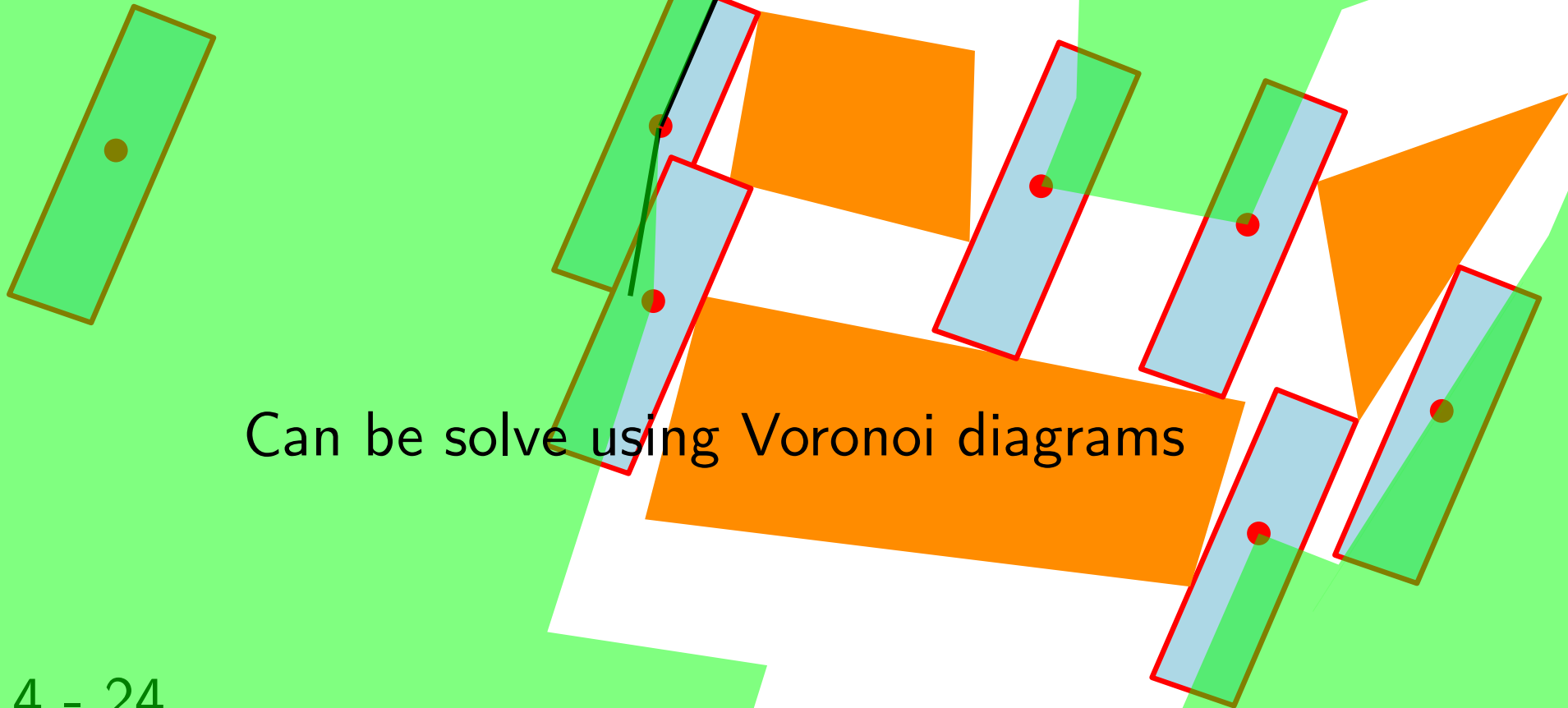


Reconstruction

Context

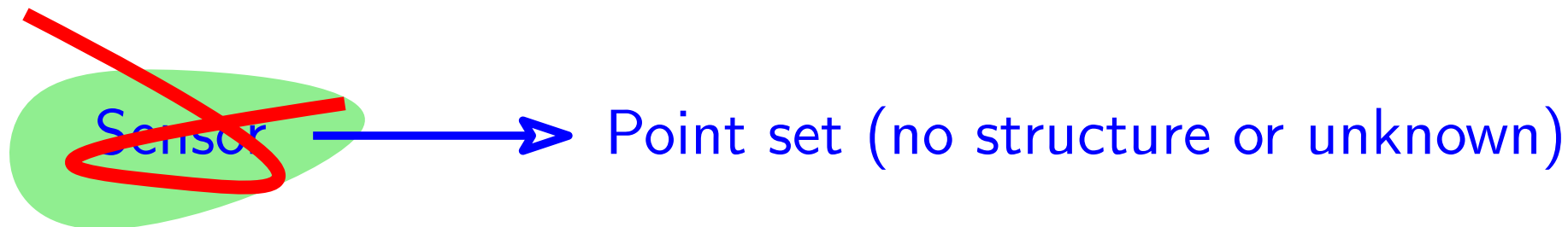


Abstract 3D problem that we can solve in 2D section

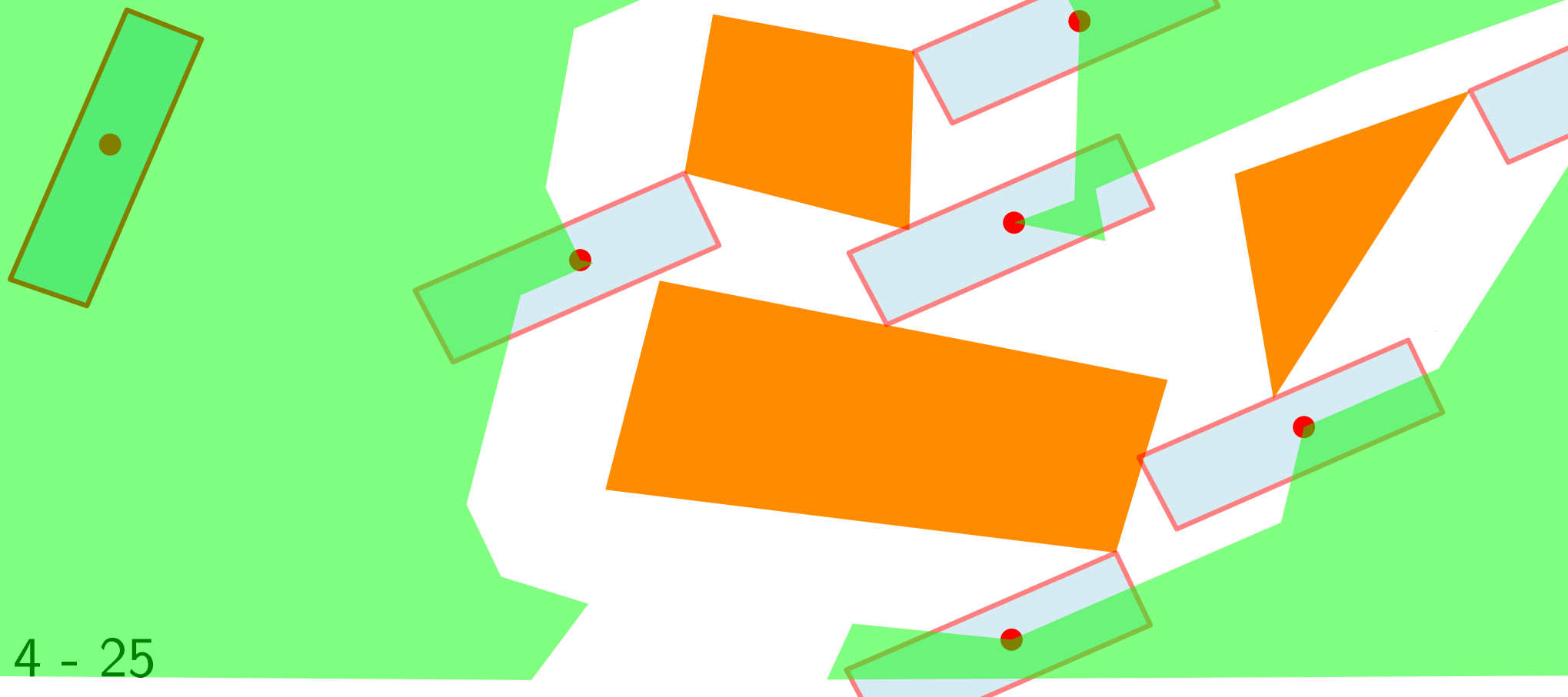


Reconstruction

Context

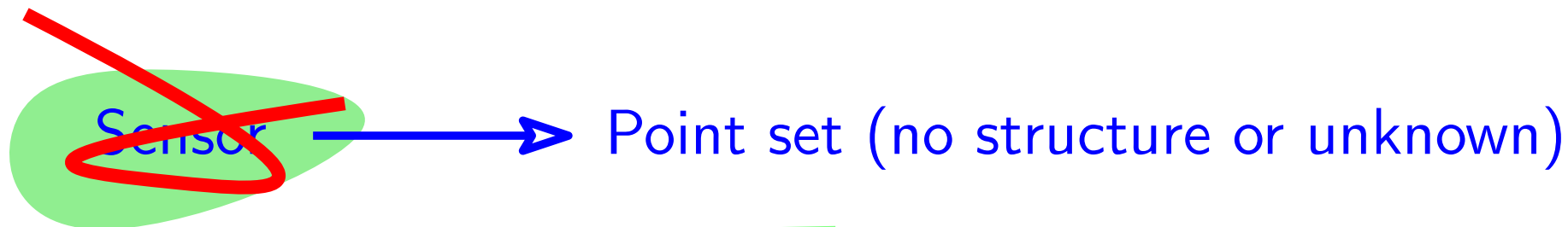


Abstract 3D problem that we can solve in 2D section

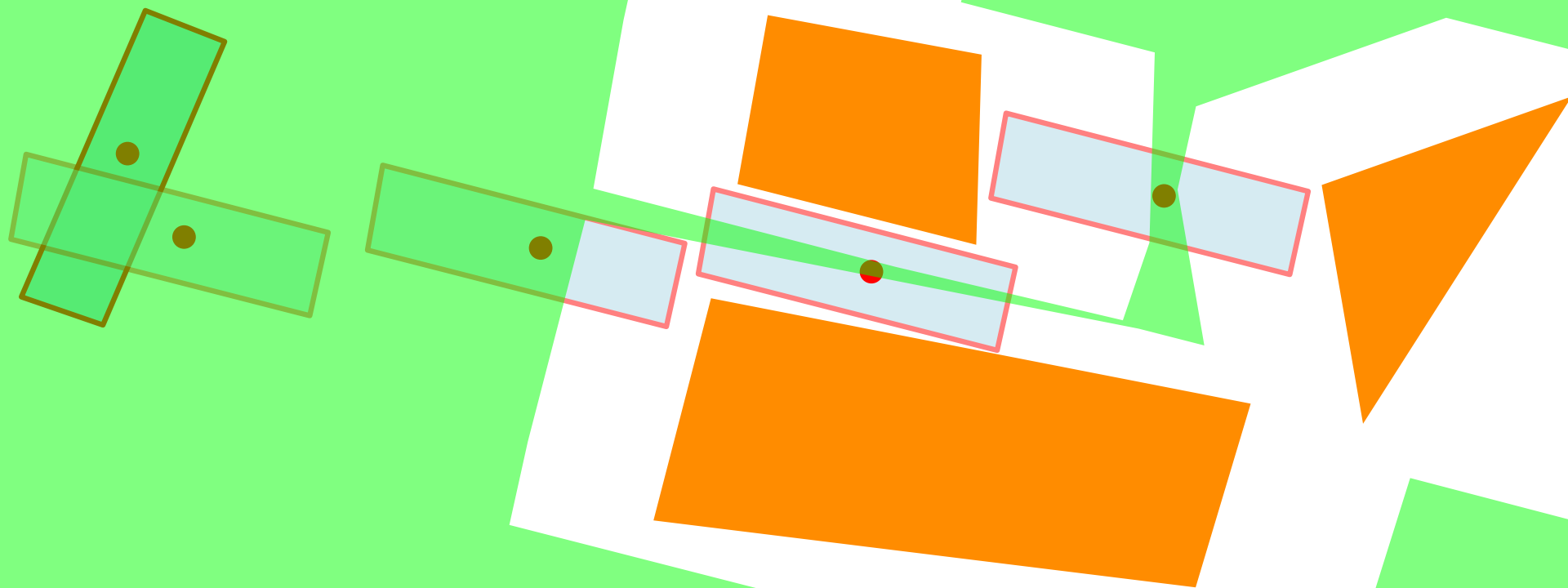


Reconstruction

Context

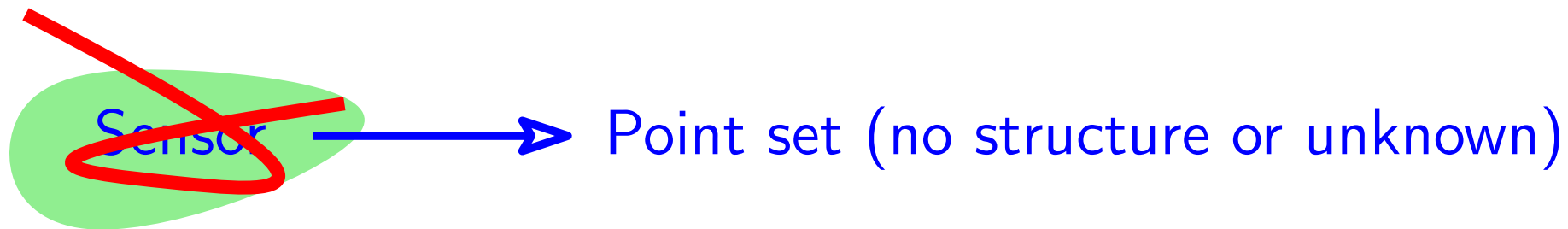


Abstract 3D problem that we can solve in 2D section

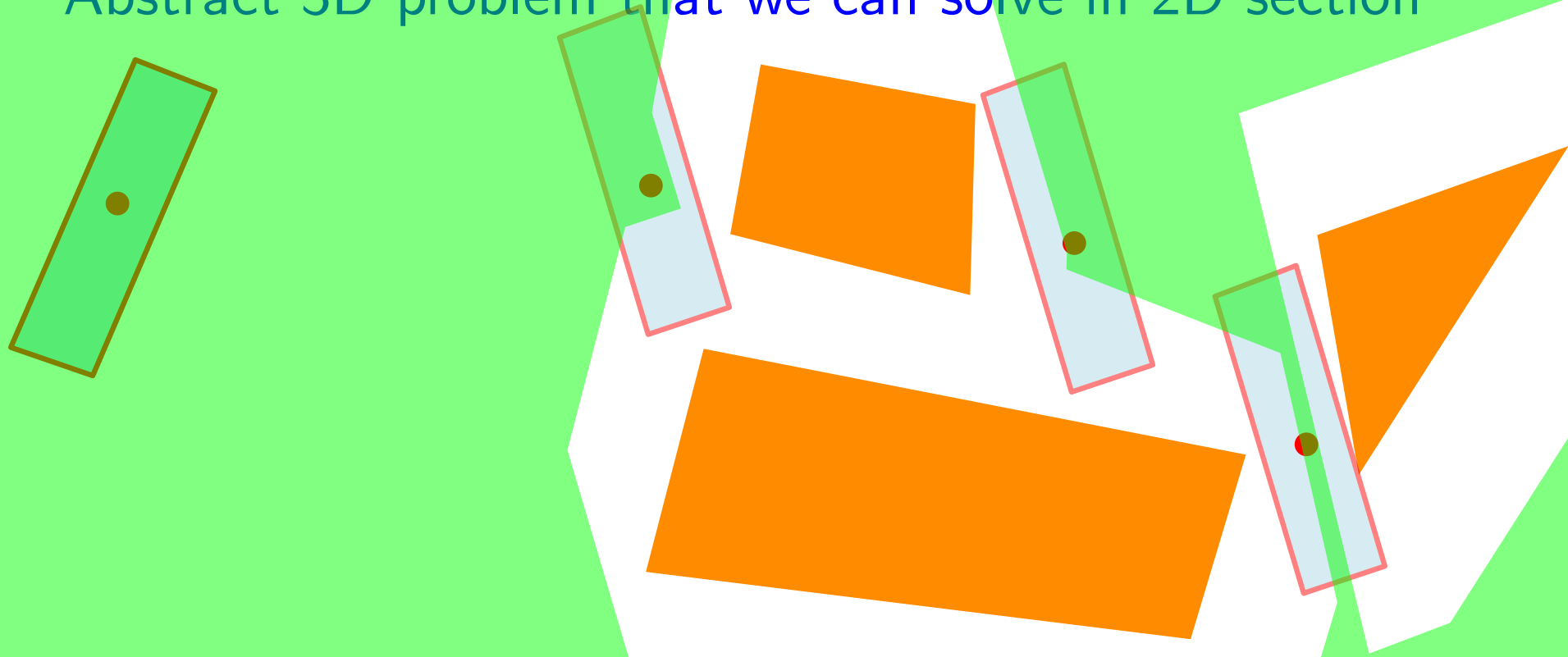


Reconstruction

Context

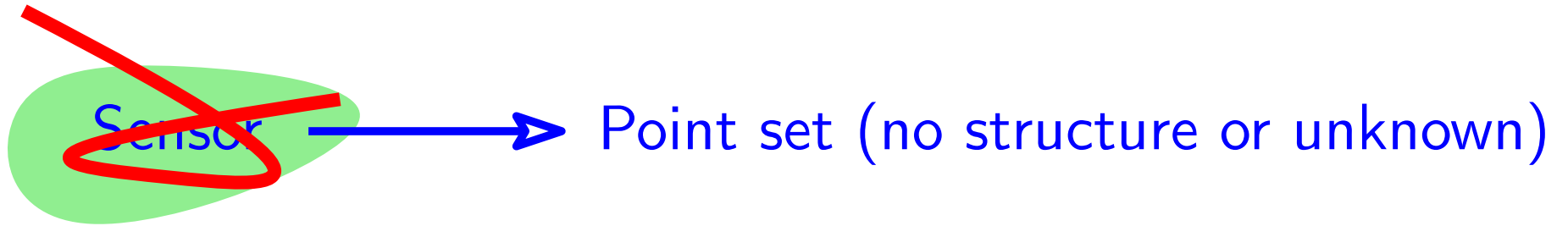


Abstract 3D problem that we can solve in 2D section

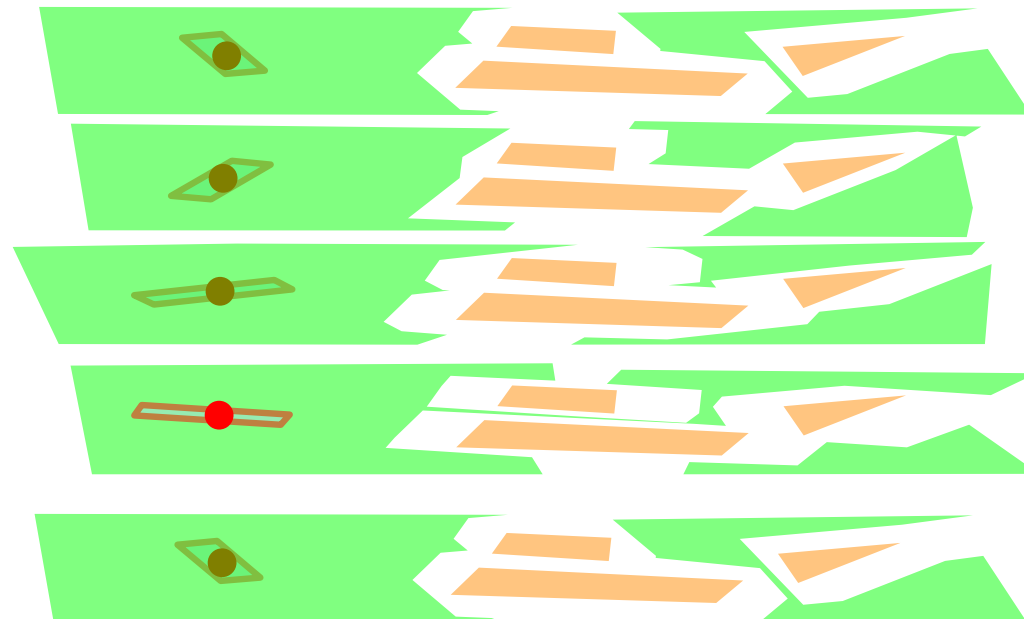


Reconstruction

Context

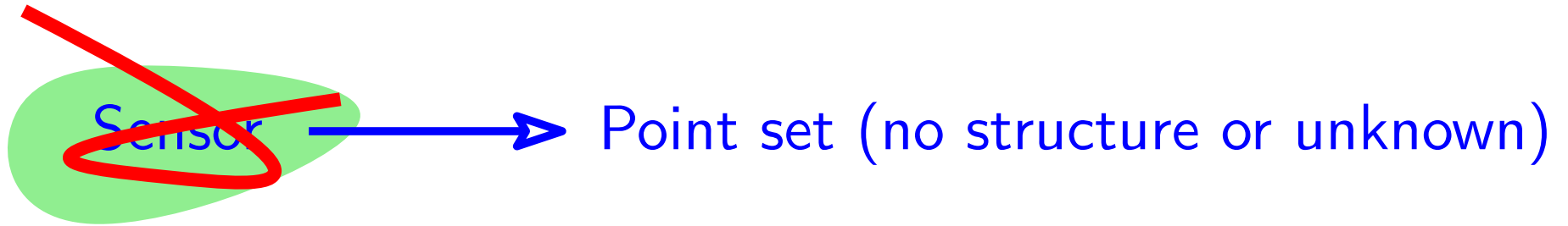


Abstract 3D problem that we can solve in 2D section

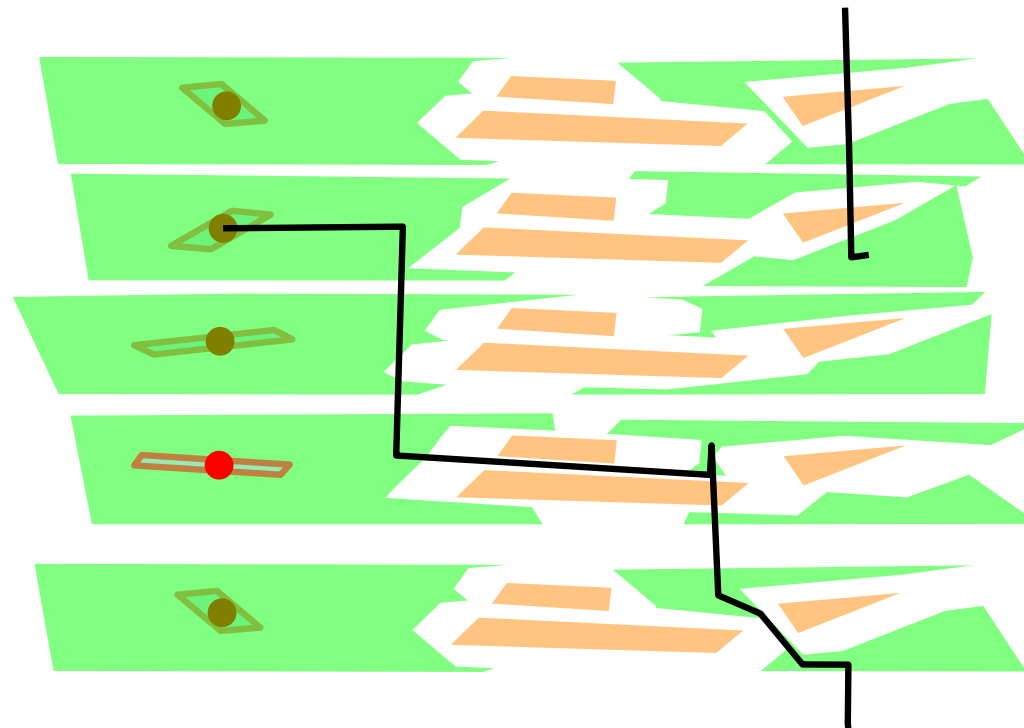


Reconstruction

Context

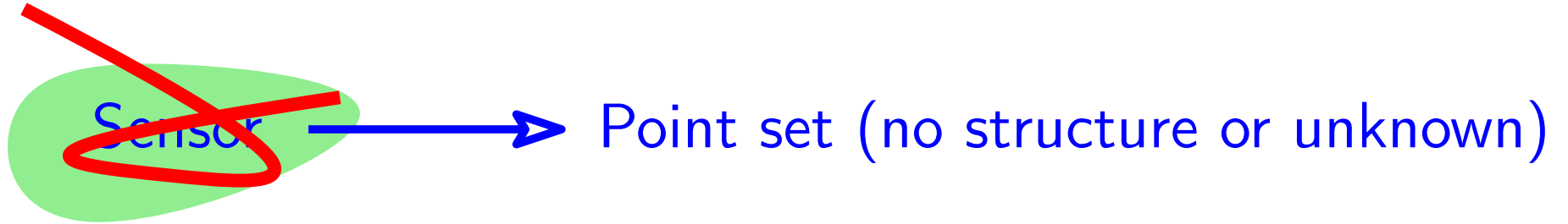


Abstract 3D problem that we can solve in 2D section

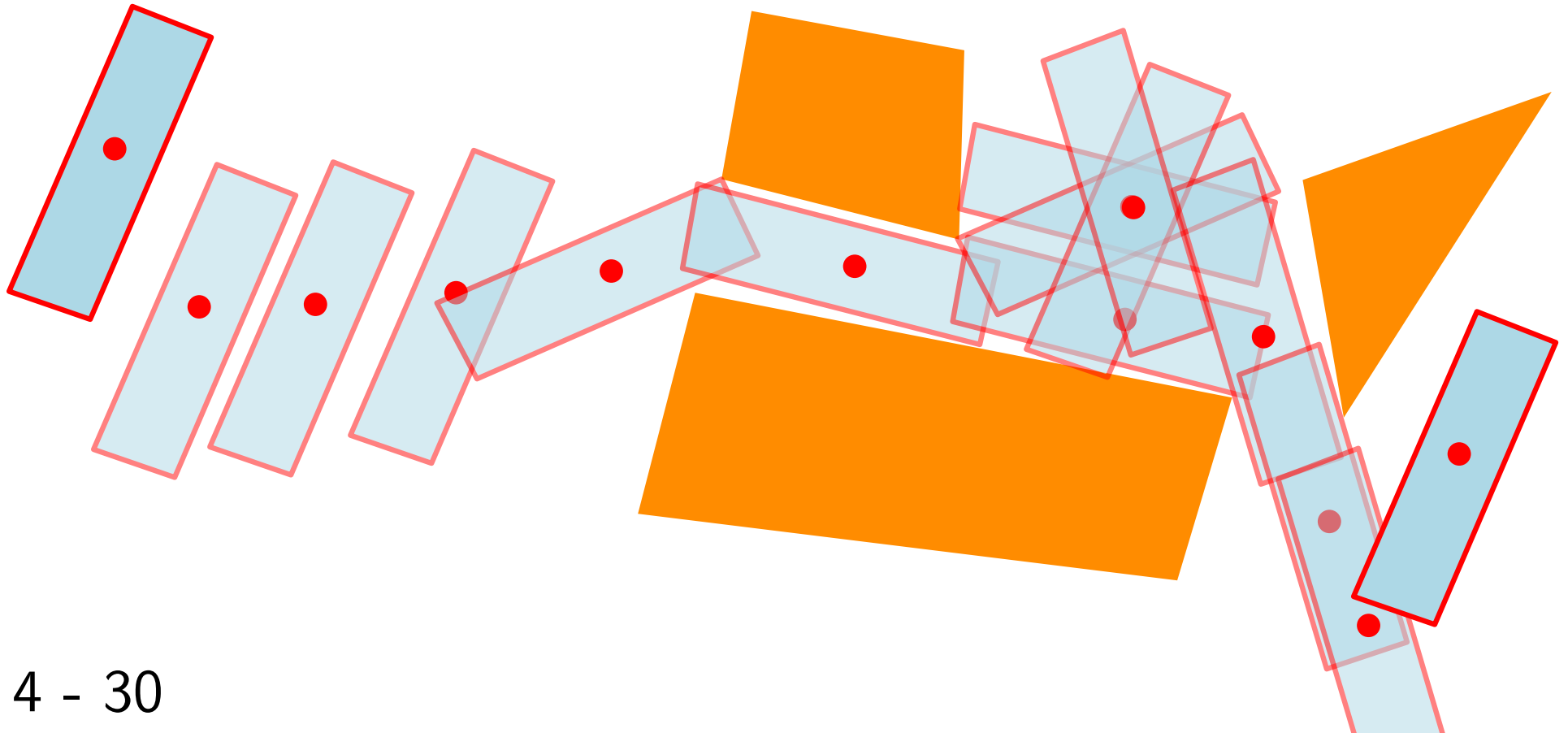


Reconstruction

Context



Abstract 3D problem that we can solve in 2D section



Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

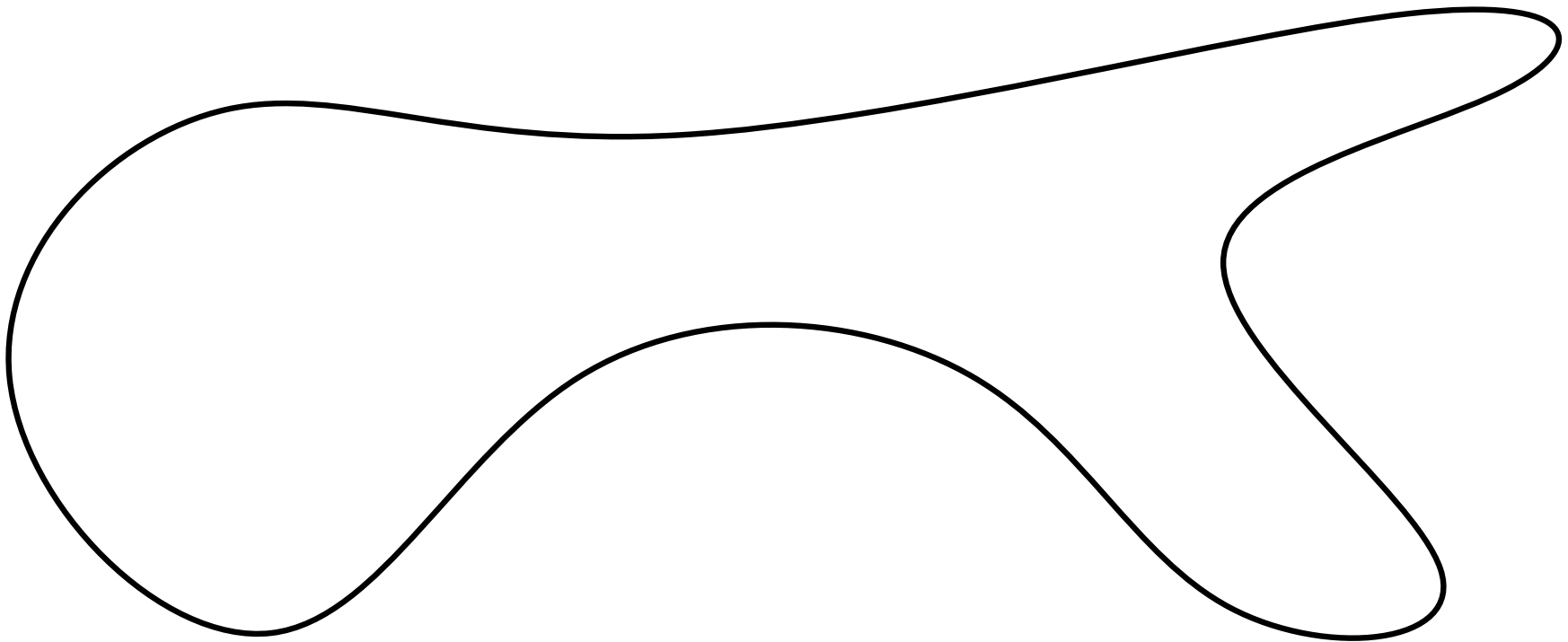
Locus of center of bitangent spheres

Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

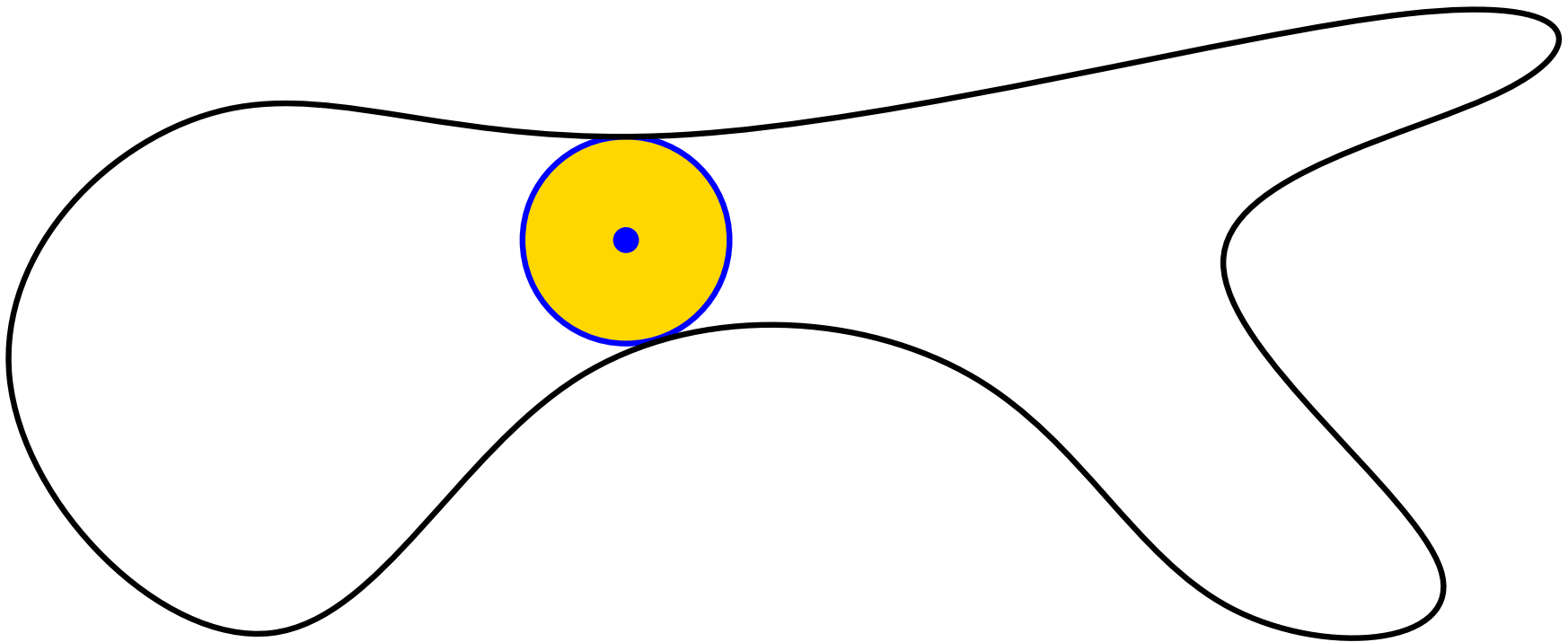


Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

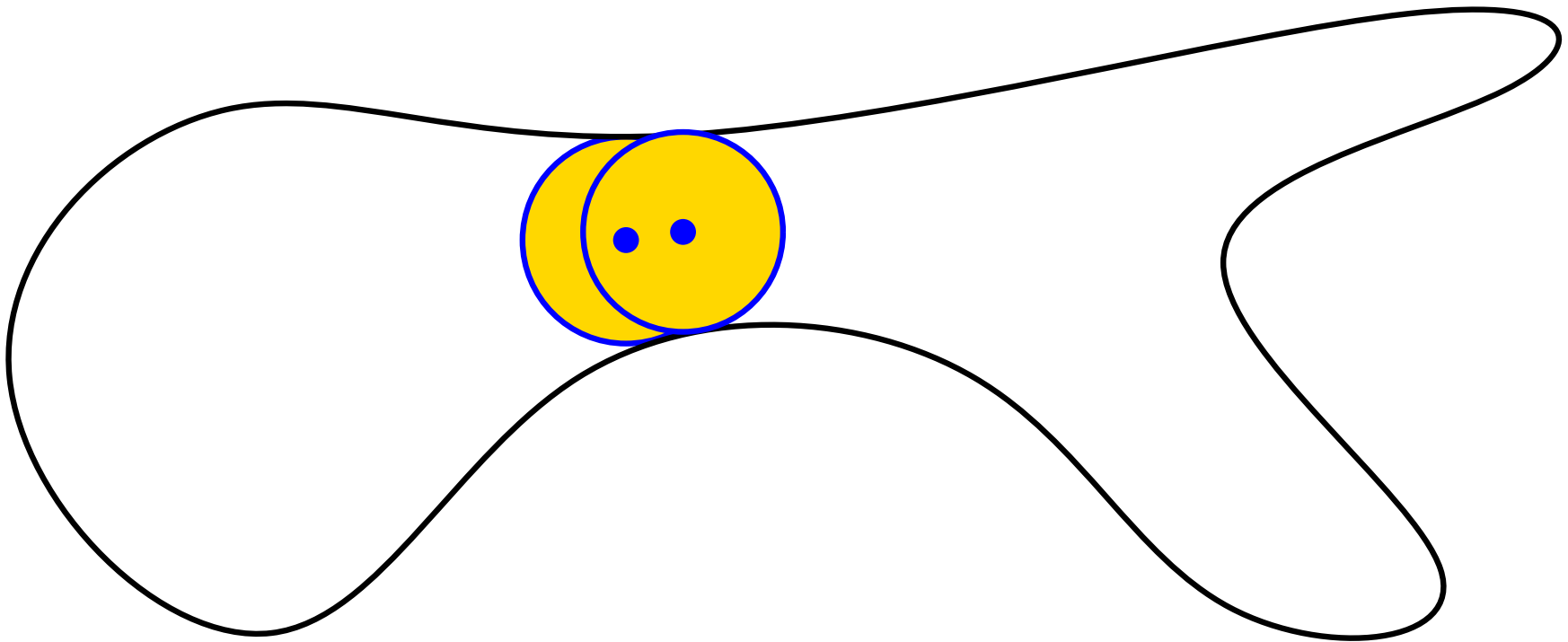


Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

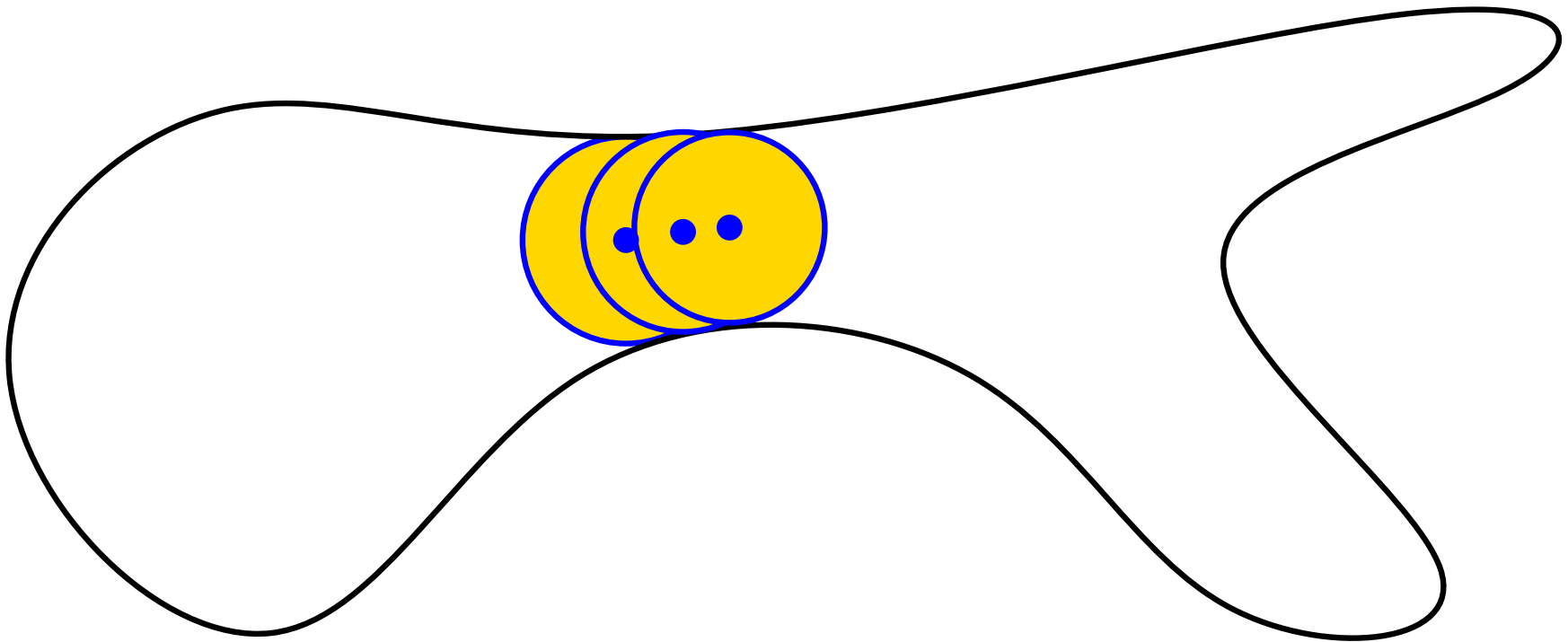


Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

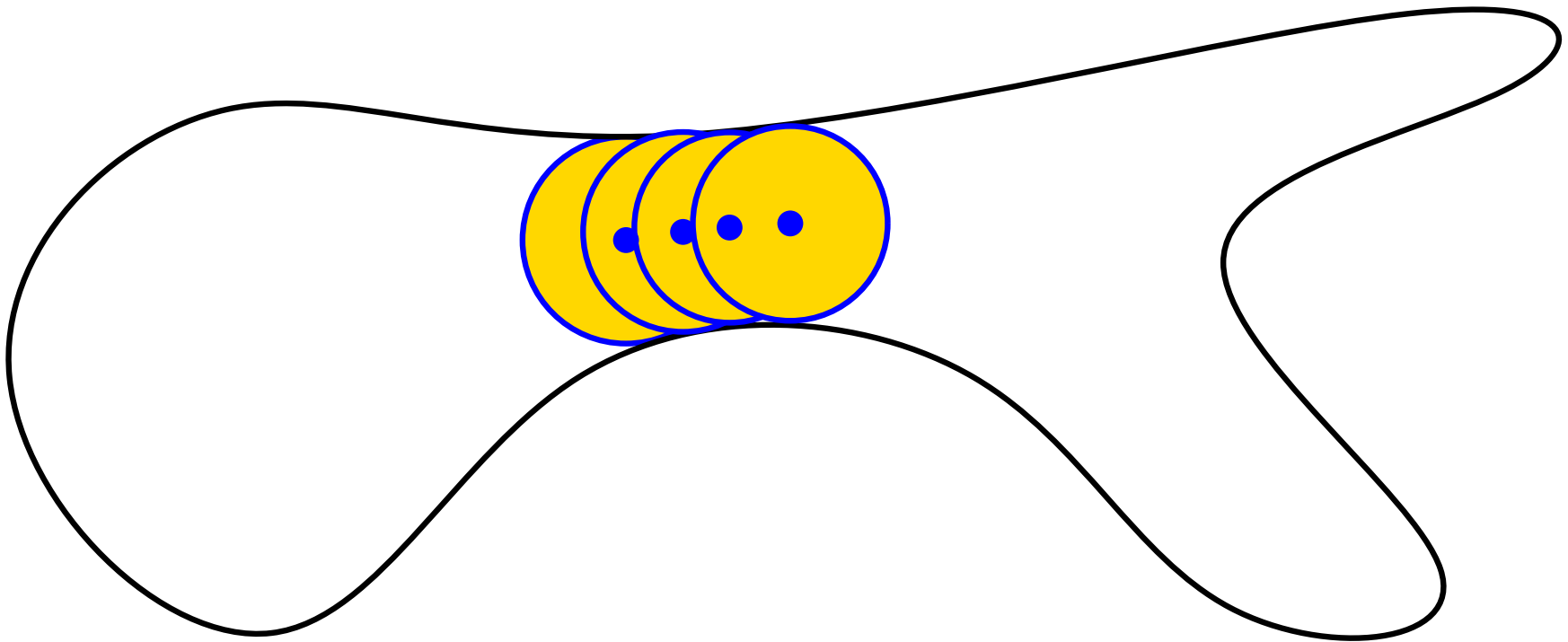


Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

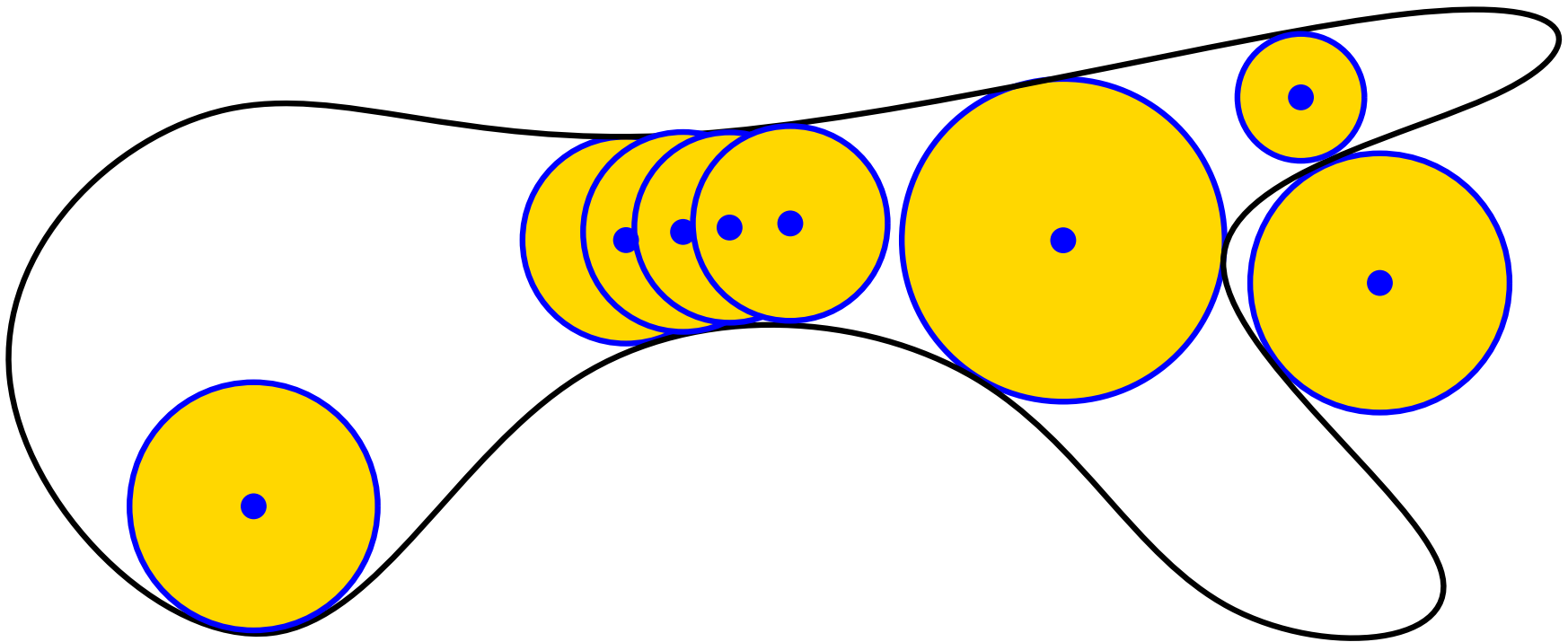


Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

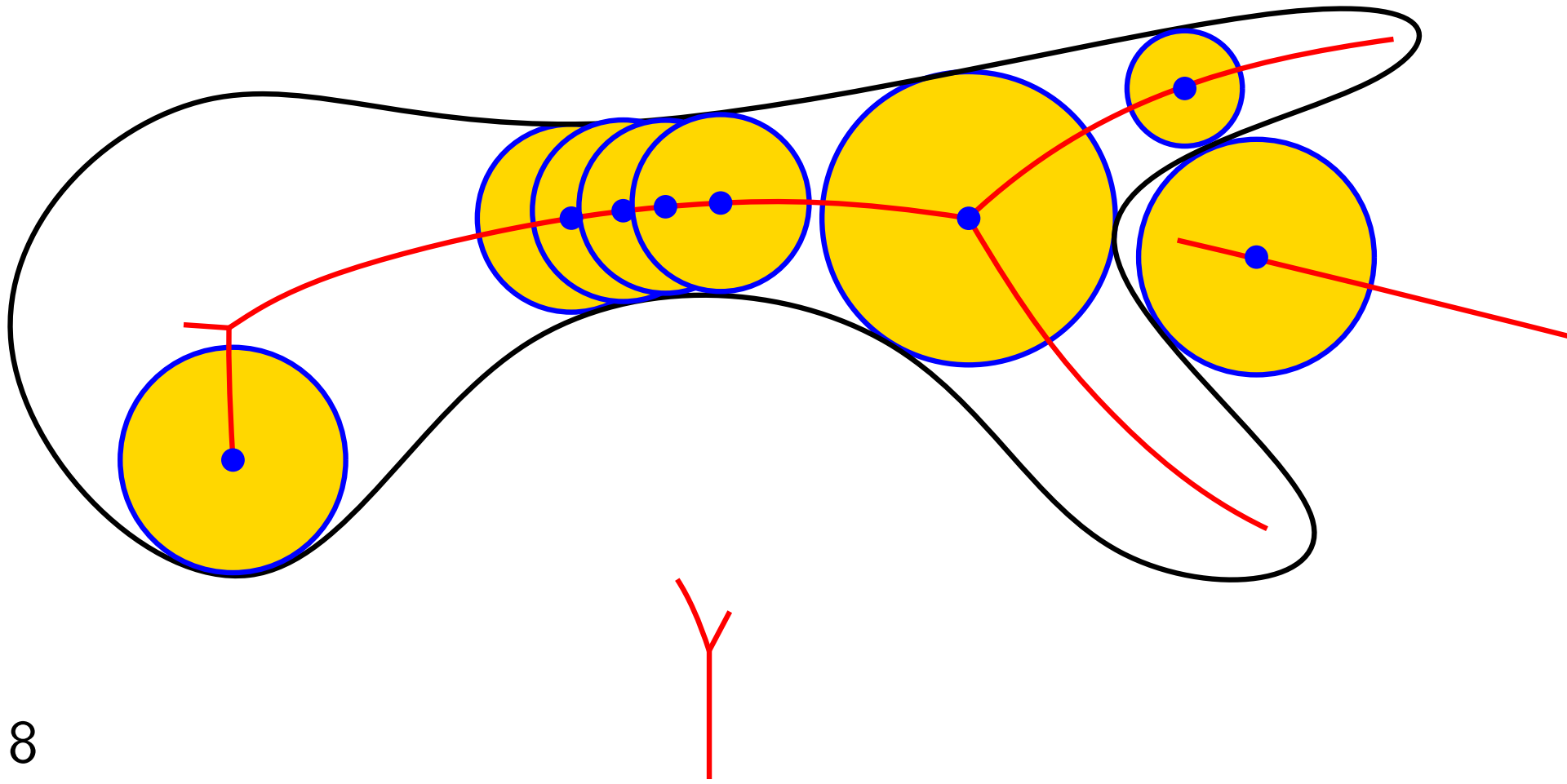


Reconstruction

Delaunay is a good start

Medial axis of a curve (surface in 3D)

Locus of center of bitangent spheres

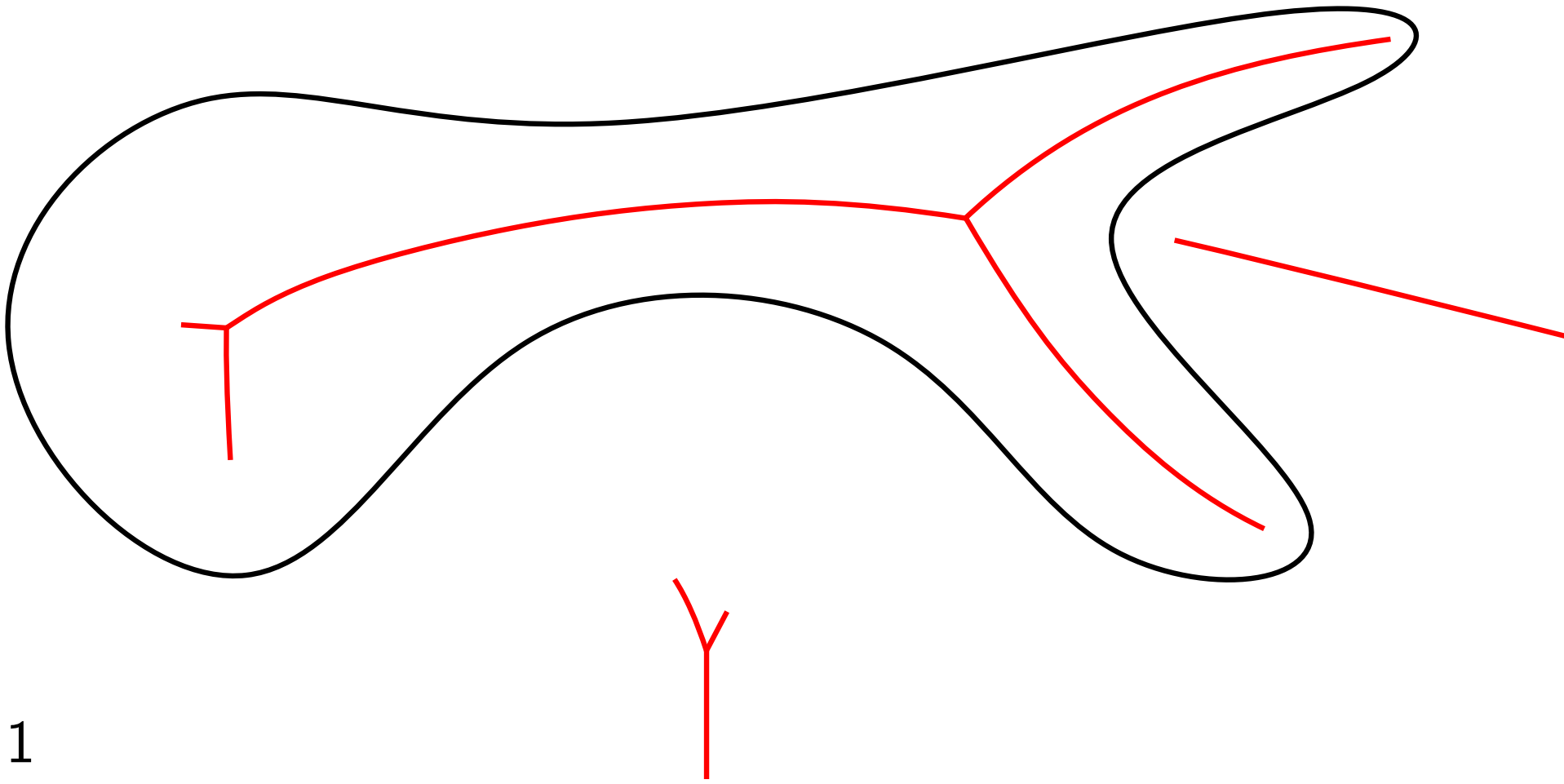


Reconstruction

Delaunay is a good start

ϵ -sample of a curve

Local feature size:

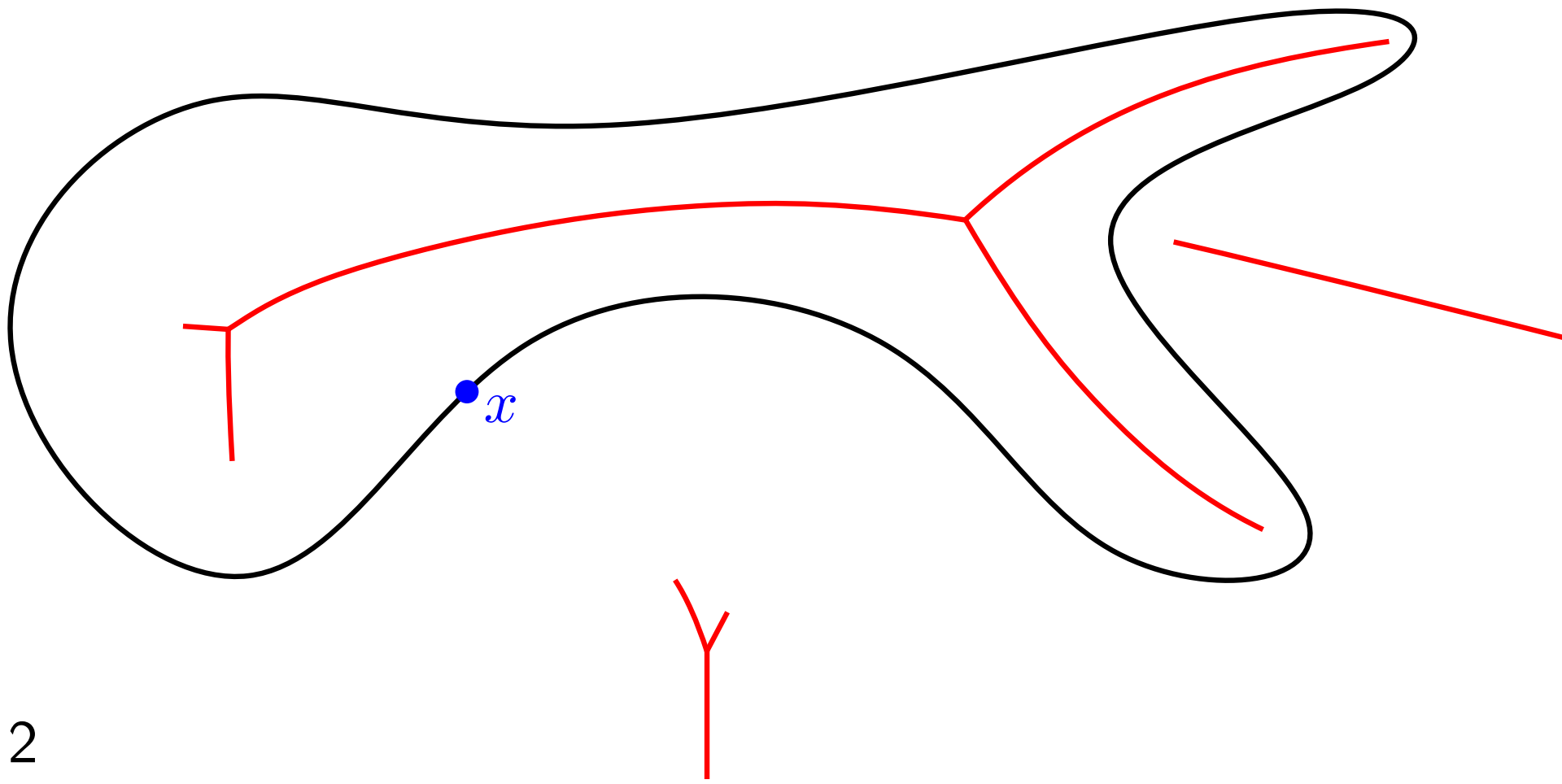


Reconstruction

Delaunay is a good start

ϵ -sample of a curve

Local feature size: $lfs(x) =$

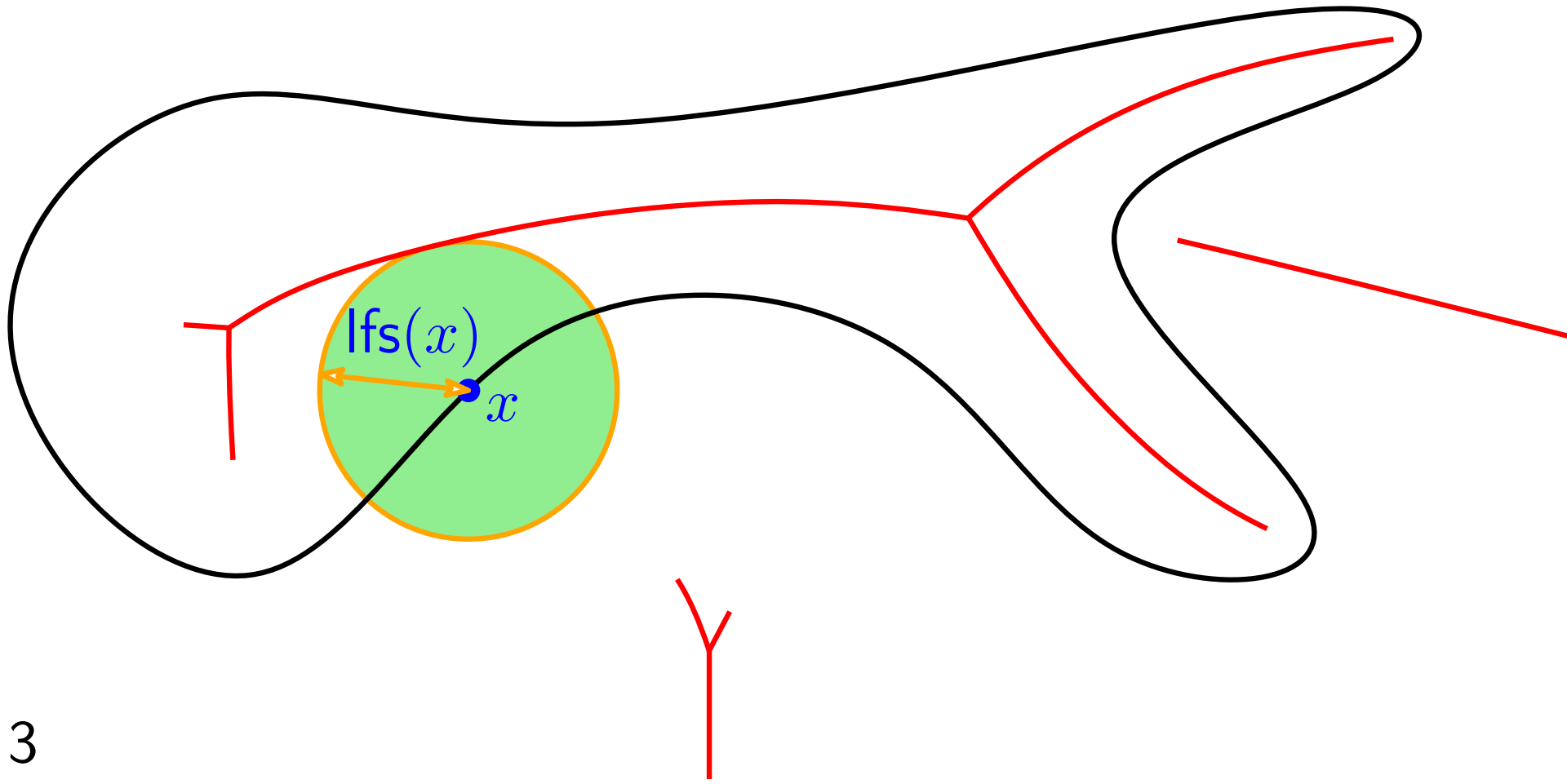


Reconstruction

Delaunay is a good start

ϵ -sample of a curve

Local feature size: $lfs(x) = \text{distance}(x, \text{medial axis})$

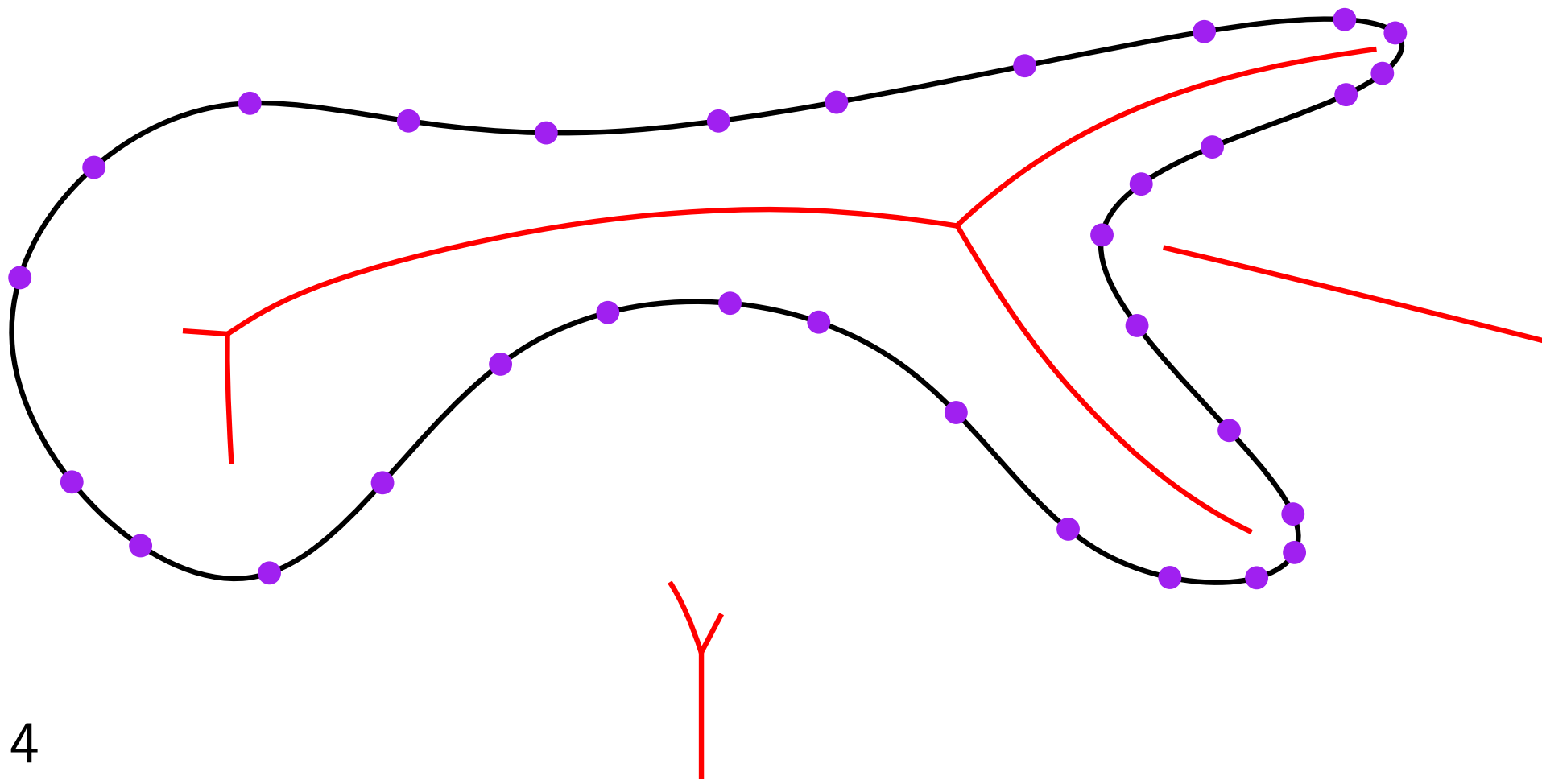


Reconstruction

Delaunay is a good start

Sample is an ϵ -sample of a curve

Local feature size:



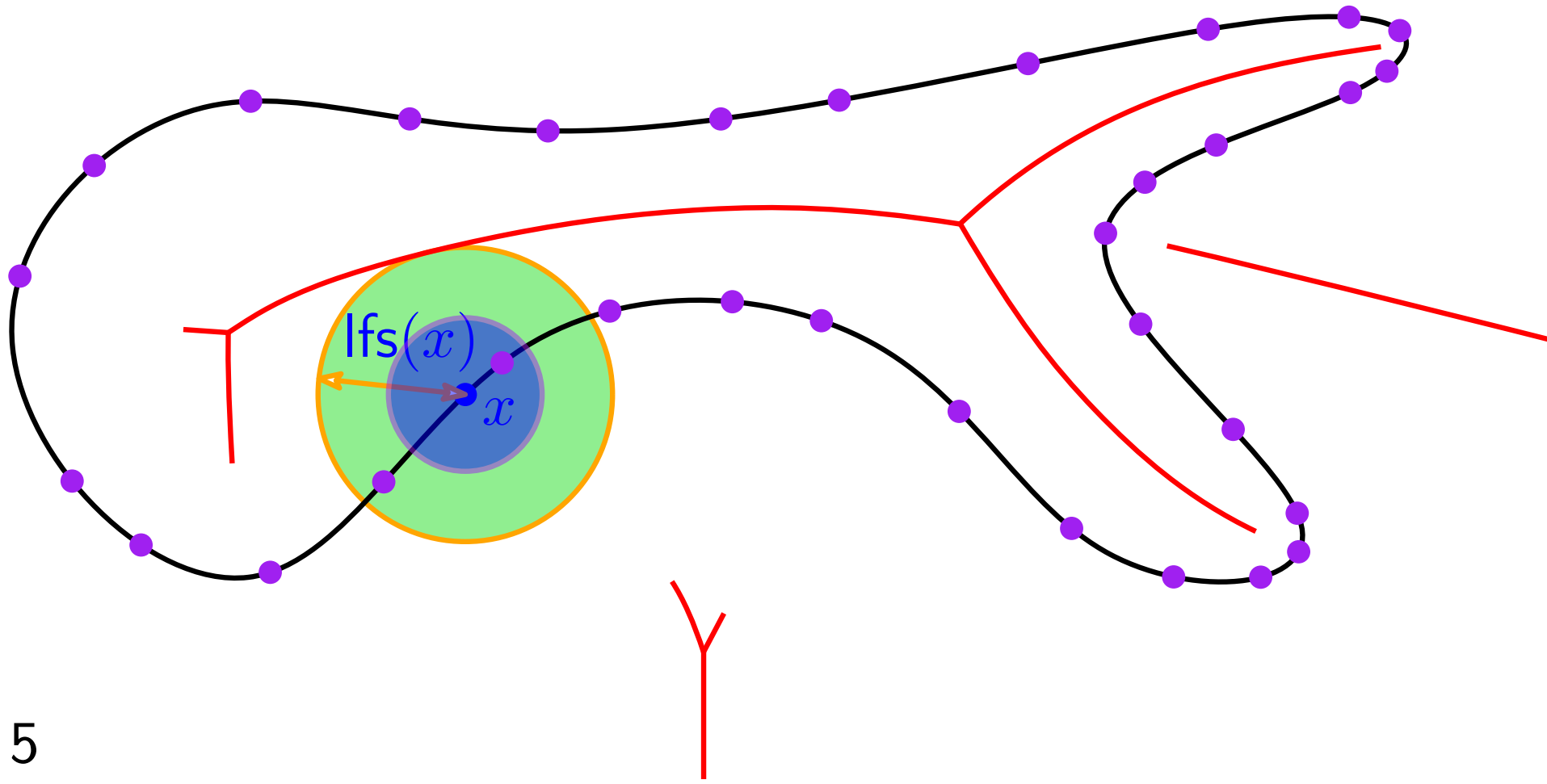
Reconstruction

Delaunay is a good start

Sample is an

ϵ -sample of a curve if $\forall x, \text{Disk}(x, \epsilon \cdot \text{lfs}(x)) \cap \text{Sample} \neq \emptyset$

Local feature size: $\text{lfs}(x) = \text{distance}(x, \text{medial axis})$

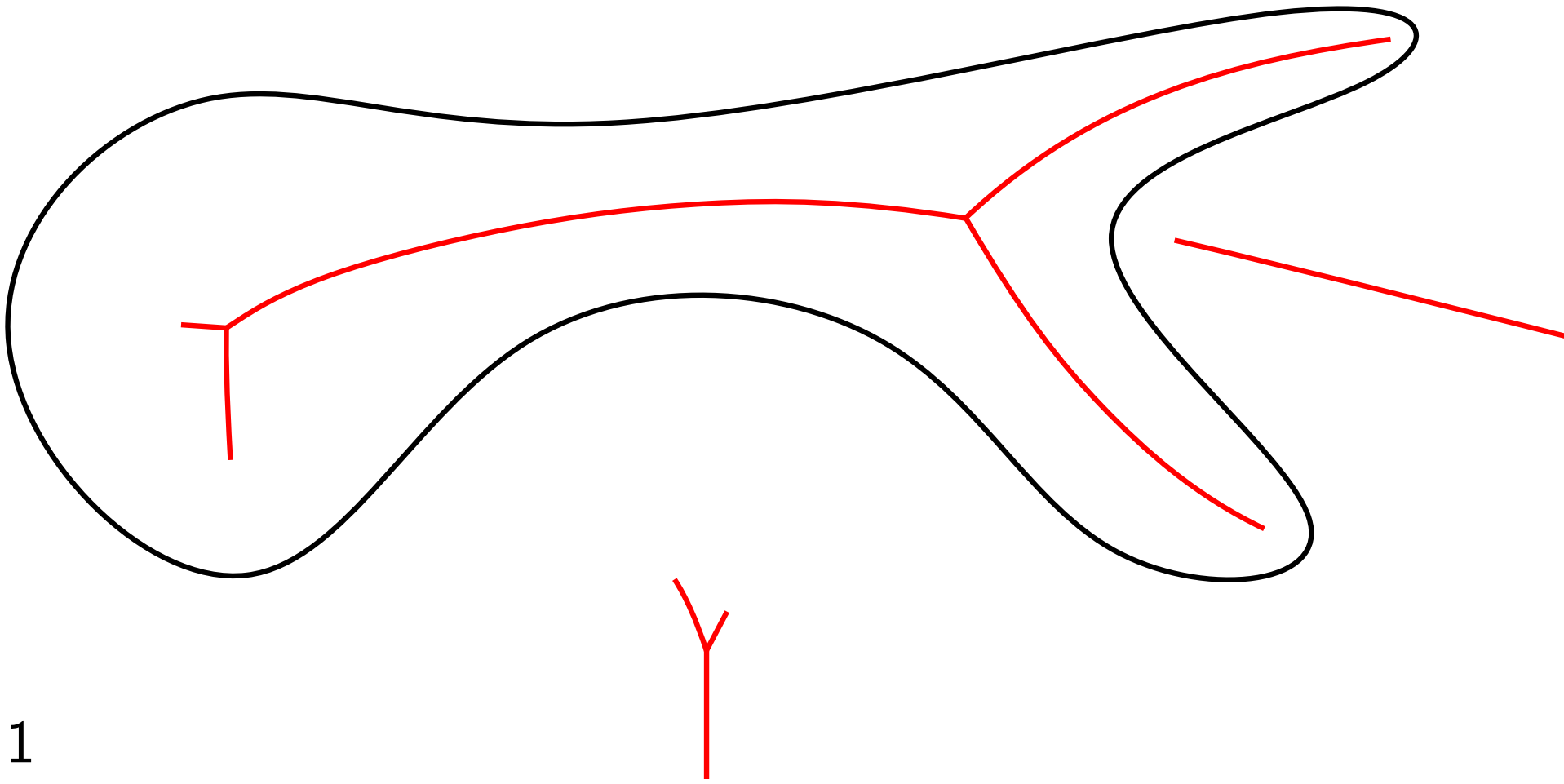


Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$

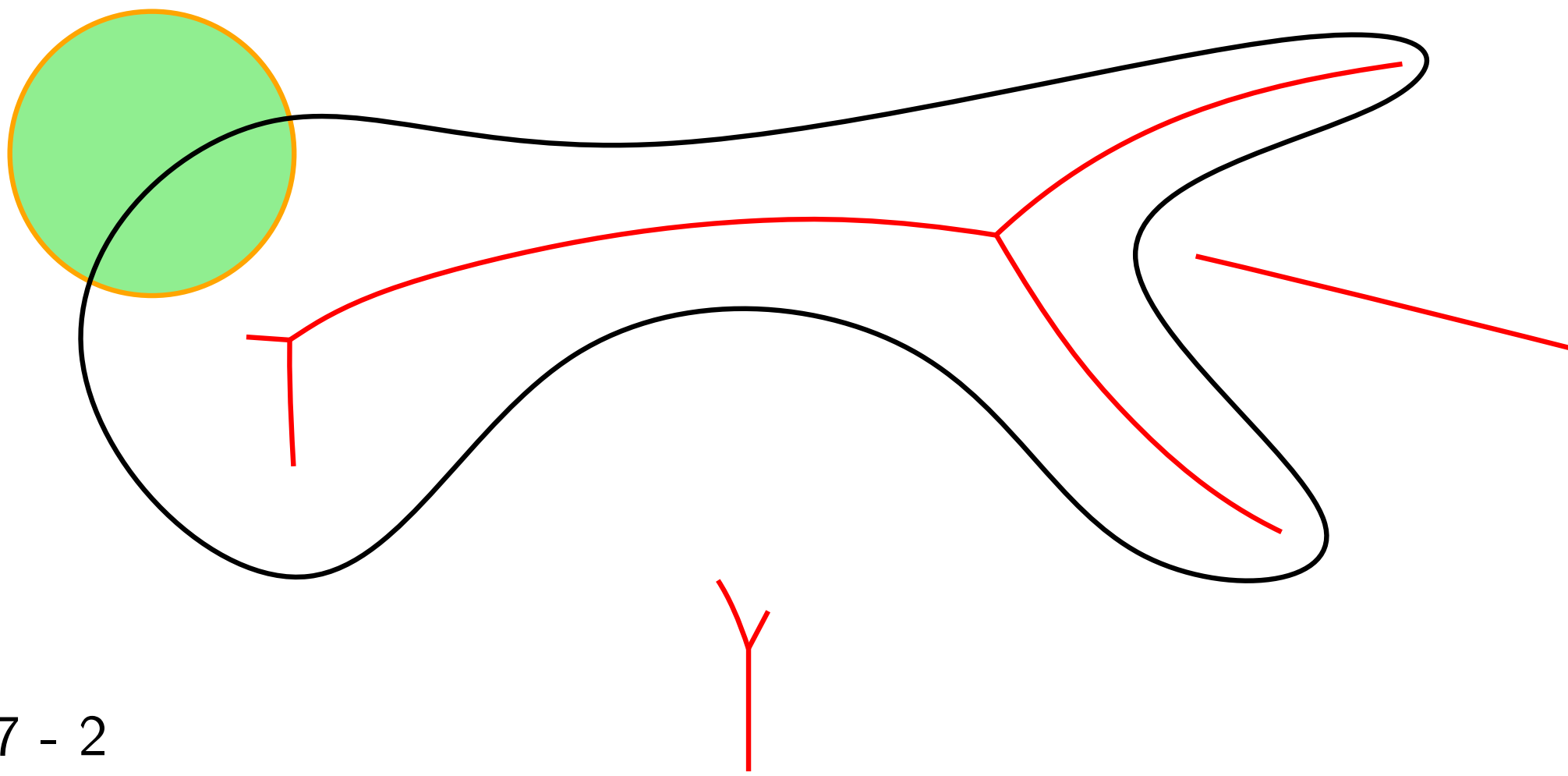


Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$

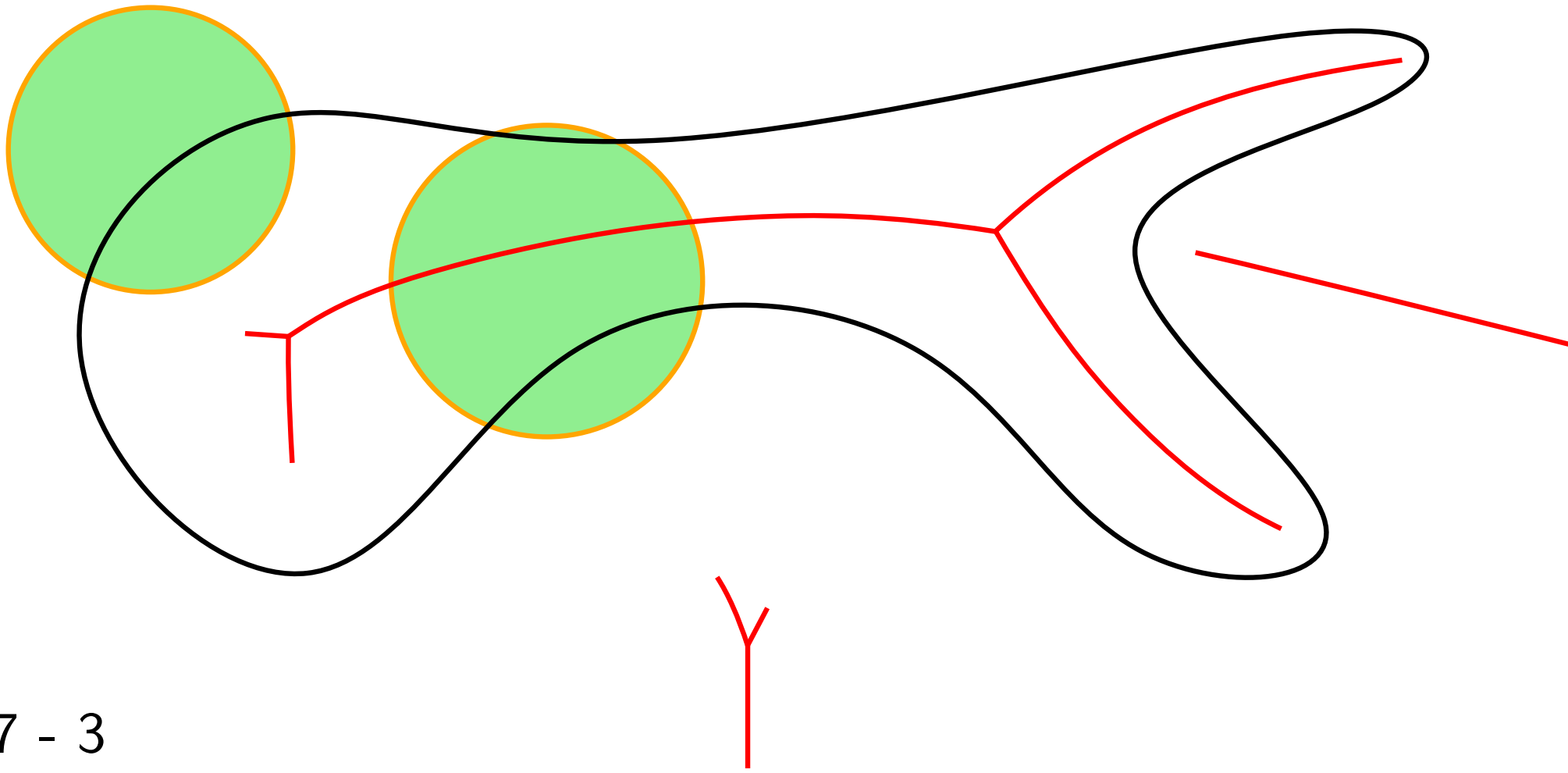


Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$

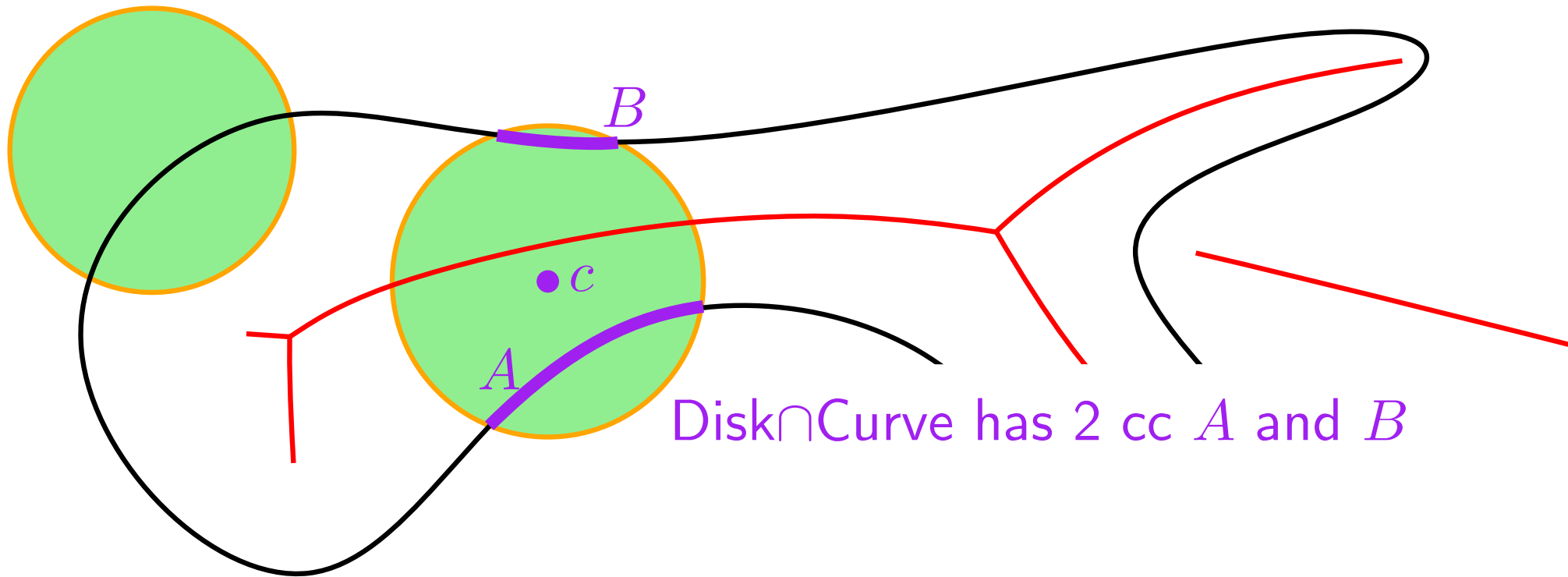


Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$

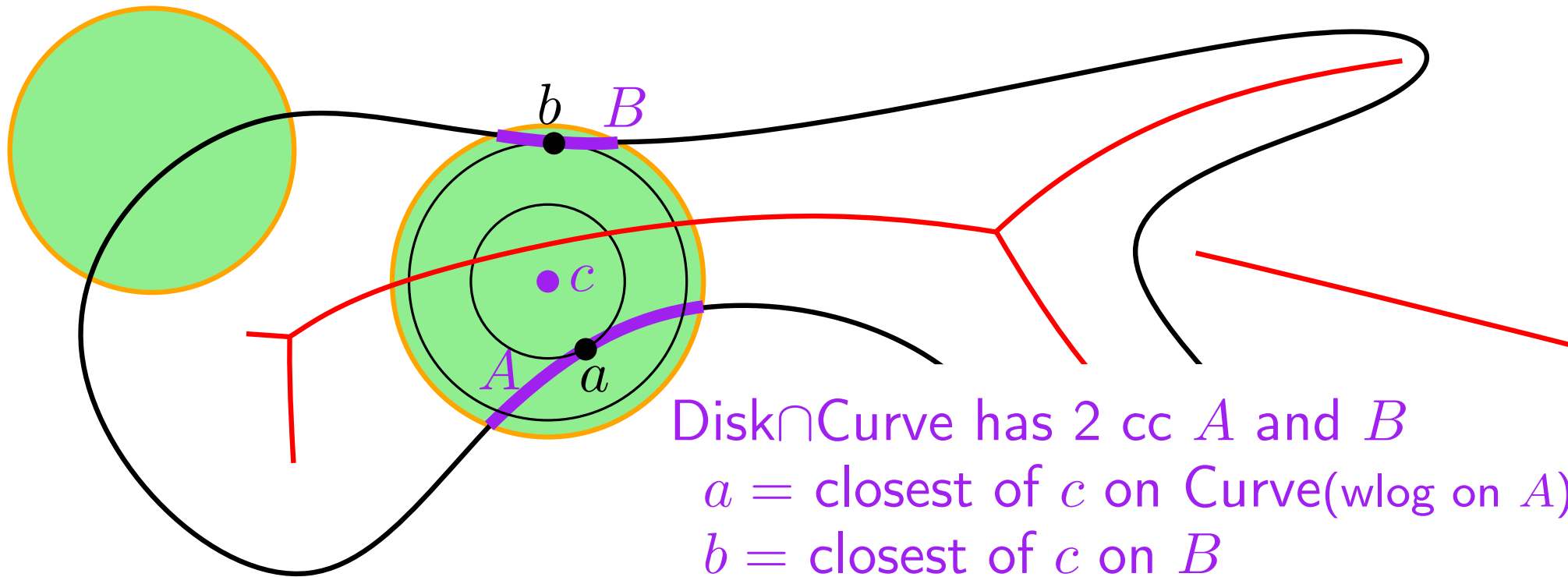


Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$

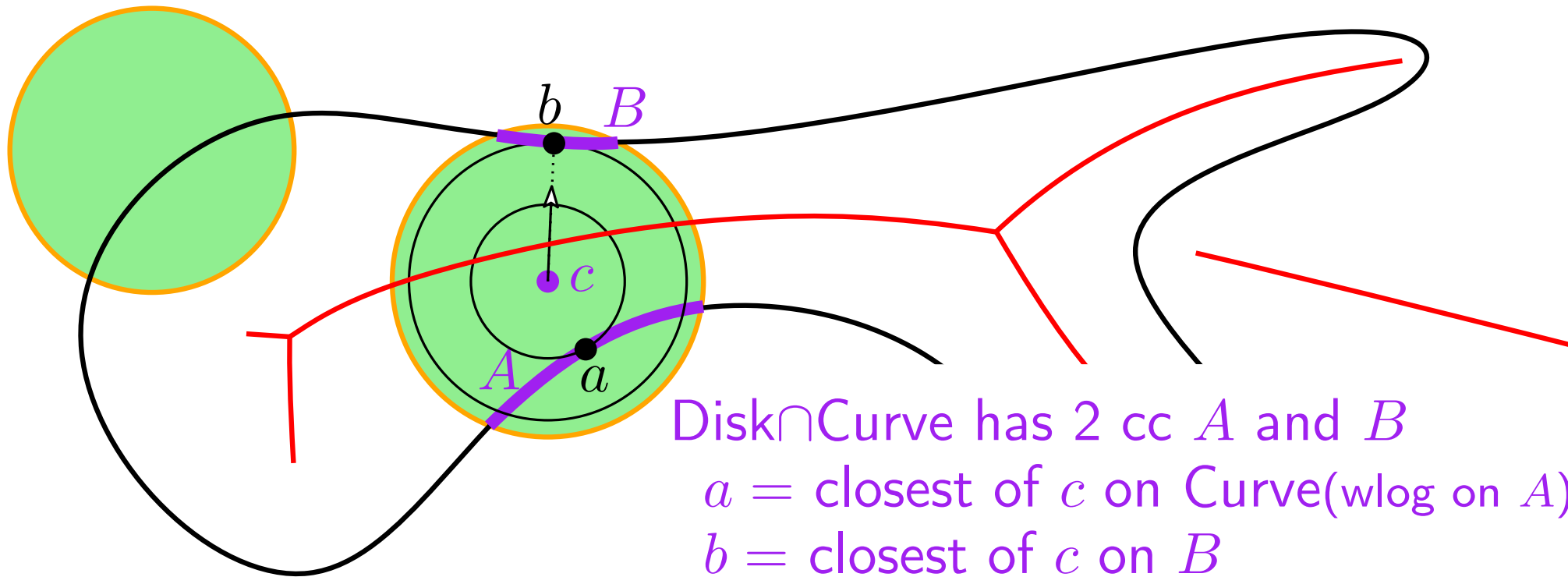


Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$



$\text{Disk} \cap \text{Curve}$ has 2 cc A and B
 $a =$ closest of c on Curve (wlog on A)
 $b =$ closest of c on B

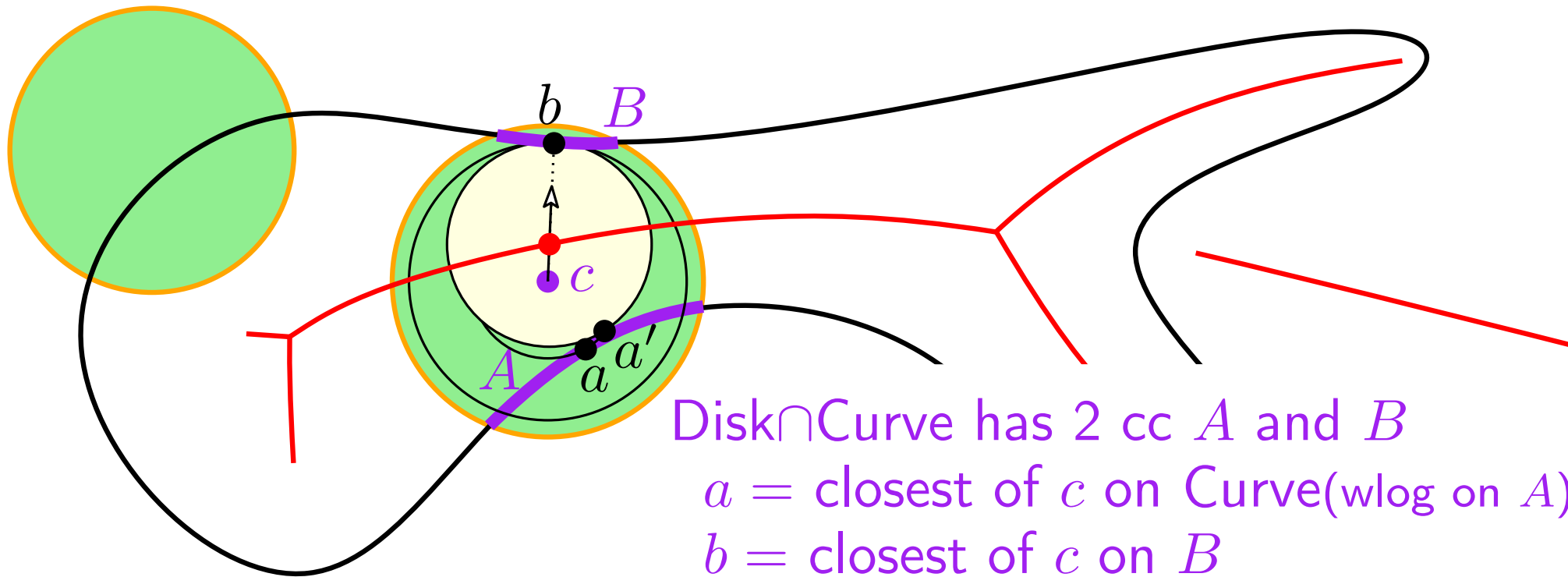
Moving from c to a dist to $B \nearrow$

Reconstruction

Delaunay is a good start

Lemma:

\forall Disk, $\text{Disk} \cap \text{Curve}$ has a single connected component
or $\text{Disk} \cap \text{Medial axis} \neq \emptyset$



$\text{Disk} \cap \text{Curve}$ has 2 cc A and B
 $a =$ closest of c on Curve (wlog on A)
 $b =$ closest of c on B

Moving from c to a dist to B \nearrow
reach center of bitangent disk

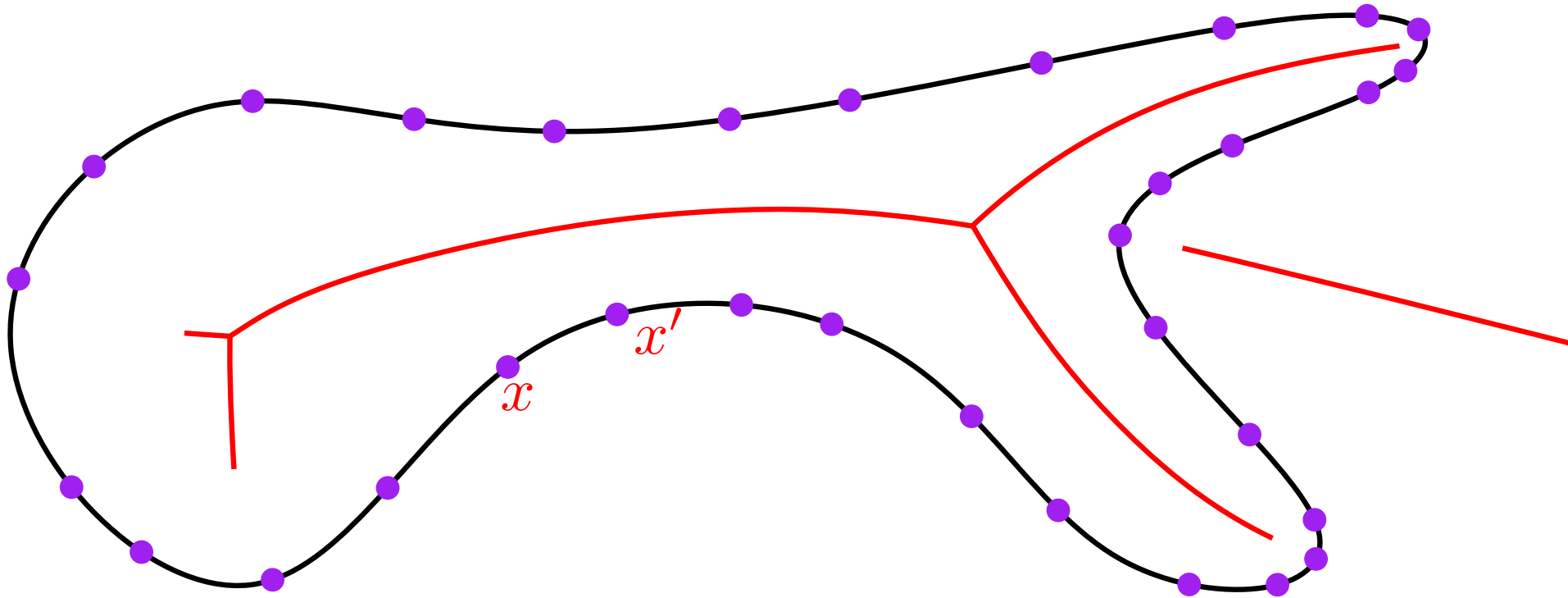
Reconstruction

Delaunay is a good start

Theorem

If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



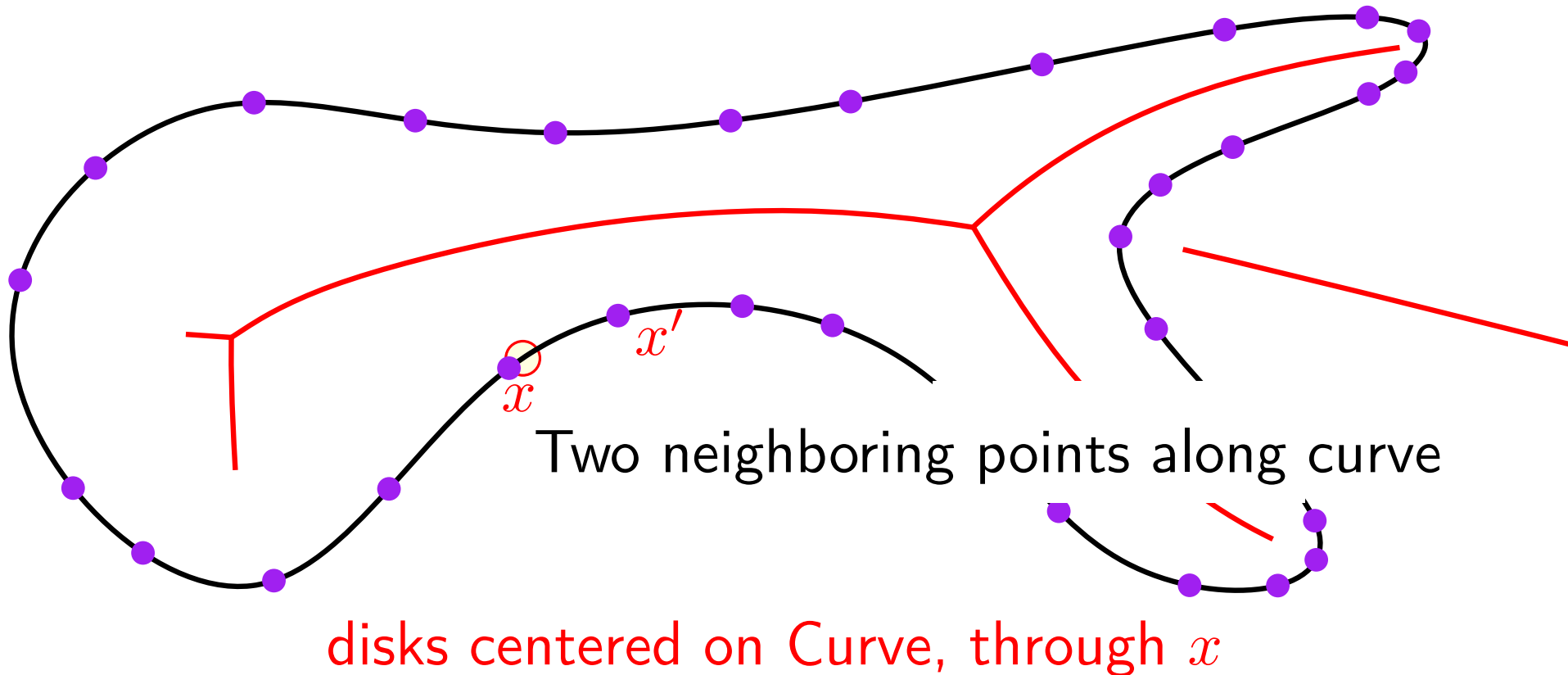
Reconstruction

Delaunay is a good start

Theorem

If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



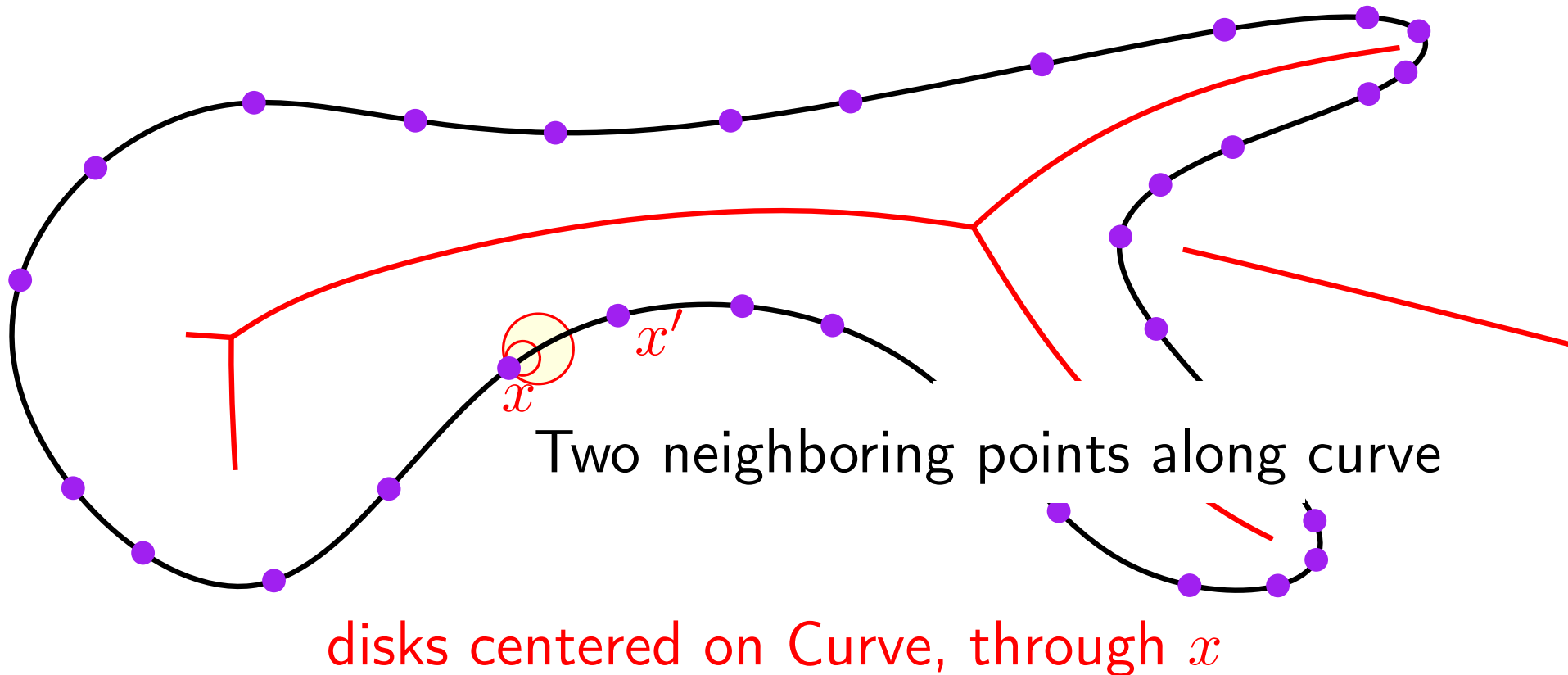
Reconstruction

Delaunay is a good start

Theorem

If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



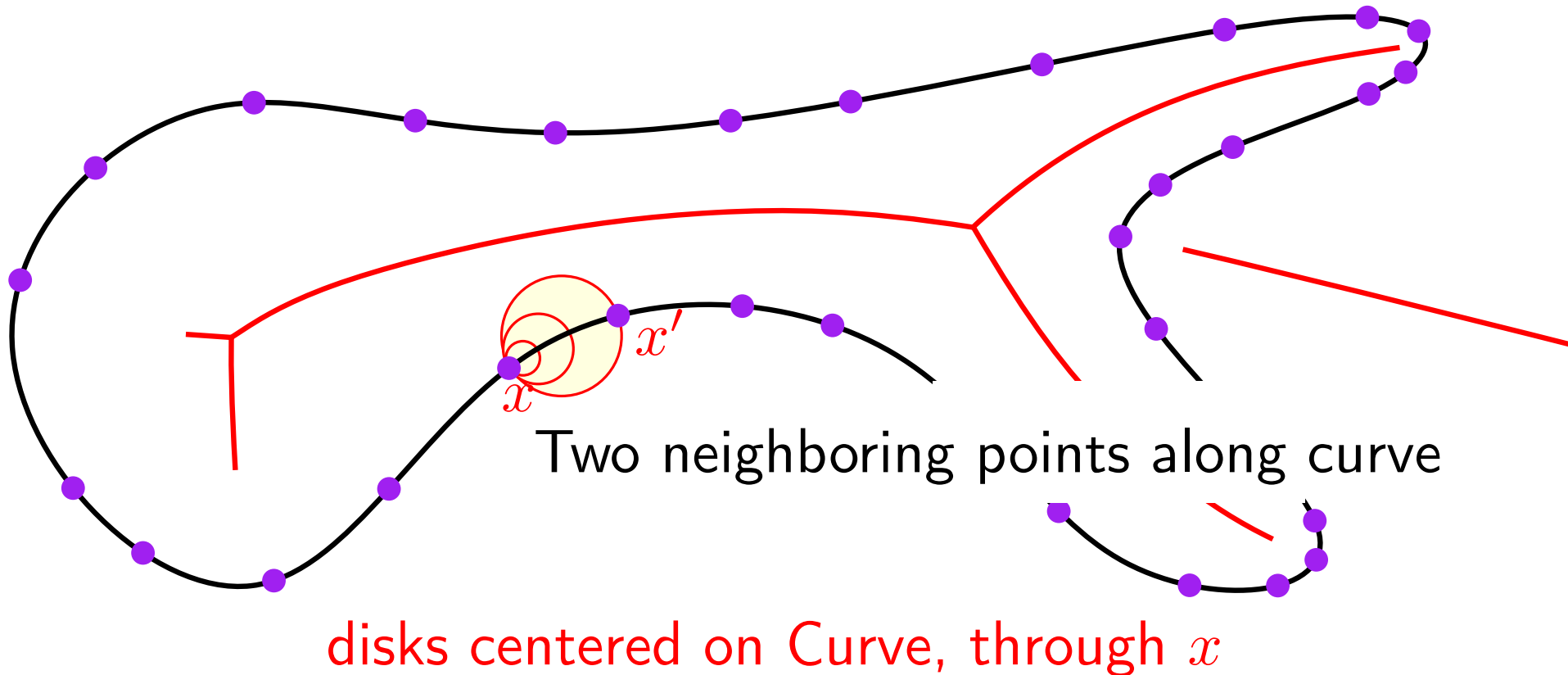
Reconstruction

Delaunay is a good start

Theorem

If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



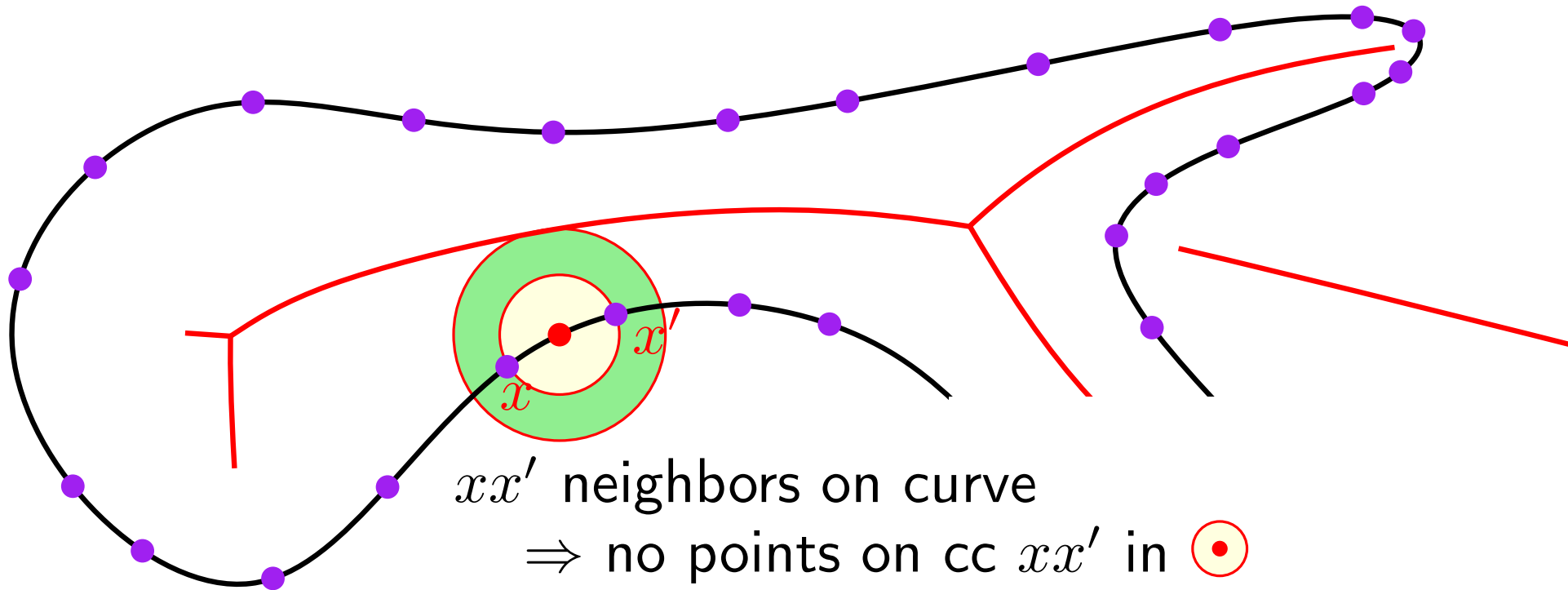
Reconstruction

Delaunay is a good start

Theorem

If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



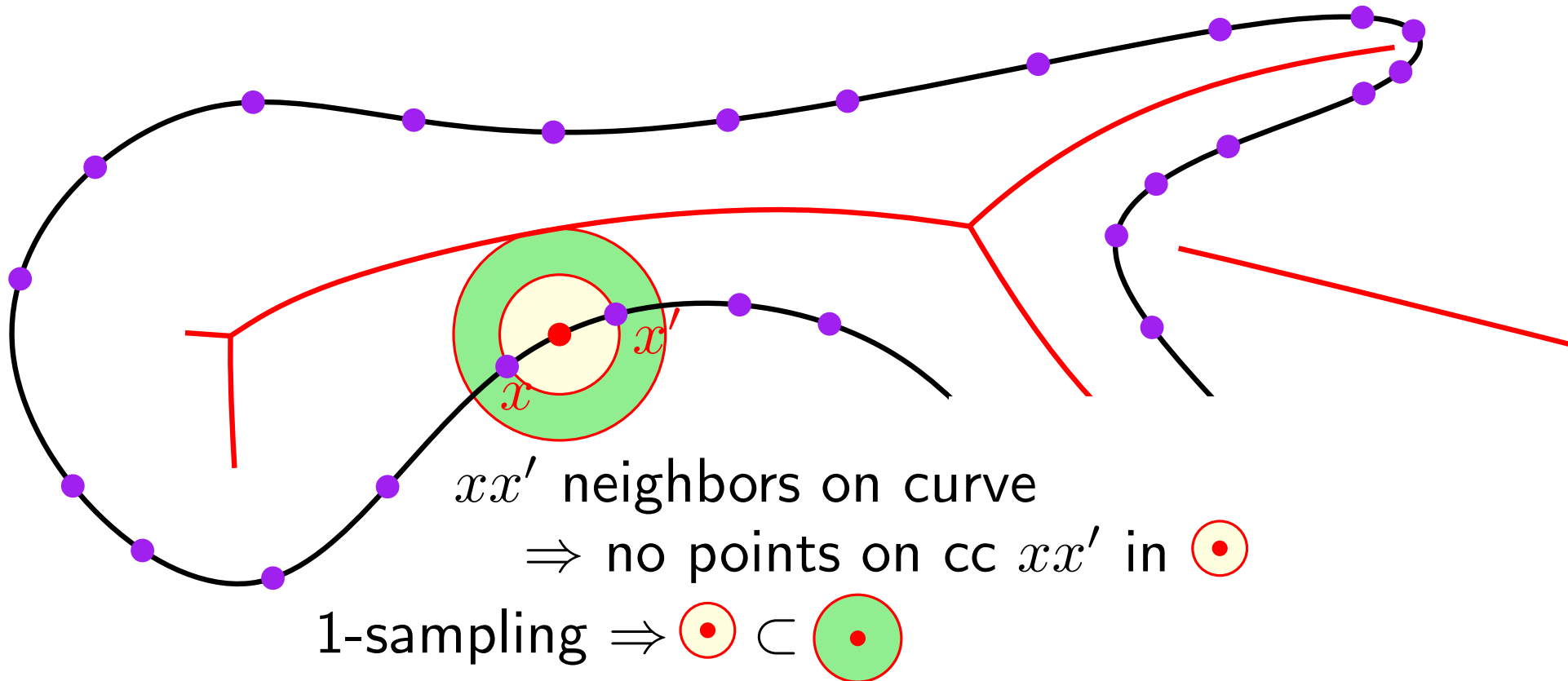
Reconstruction

Delaunay is a good start

Theorem

If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



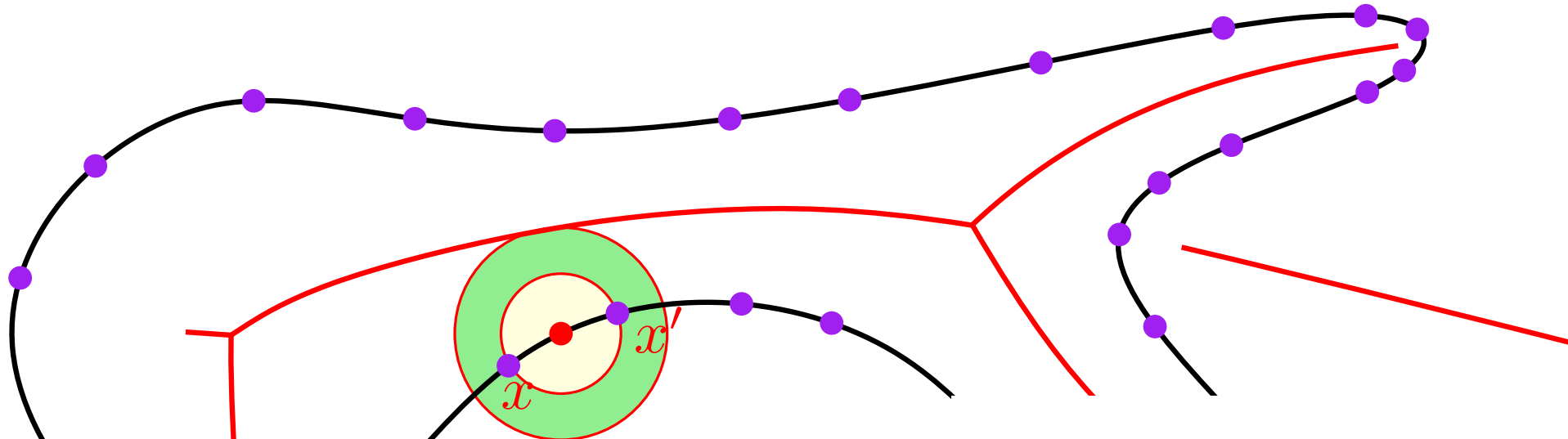
Reconstruction

Delaunay is a good start

Theorem


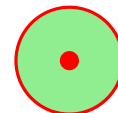
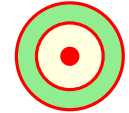
If Sample is a ϵ -sample, $\epsilon < 1$

neighboring points along Curve are Delaunay neighbors



xx' neighbors on curve

\Rightarrow no points on cc xx' in 

1-sampling \Rightarrow  \subset  \Rightarrow no other cc \cap 

Lemma

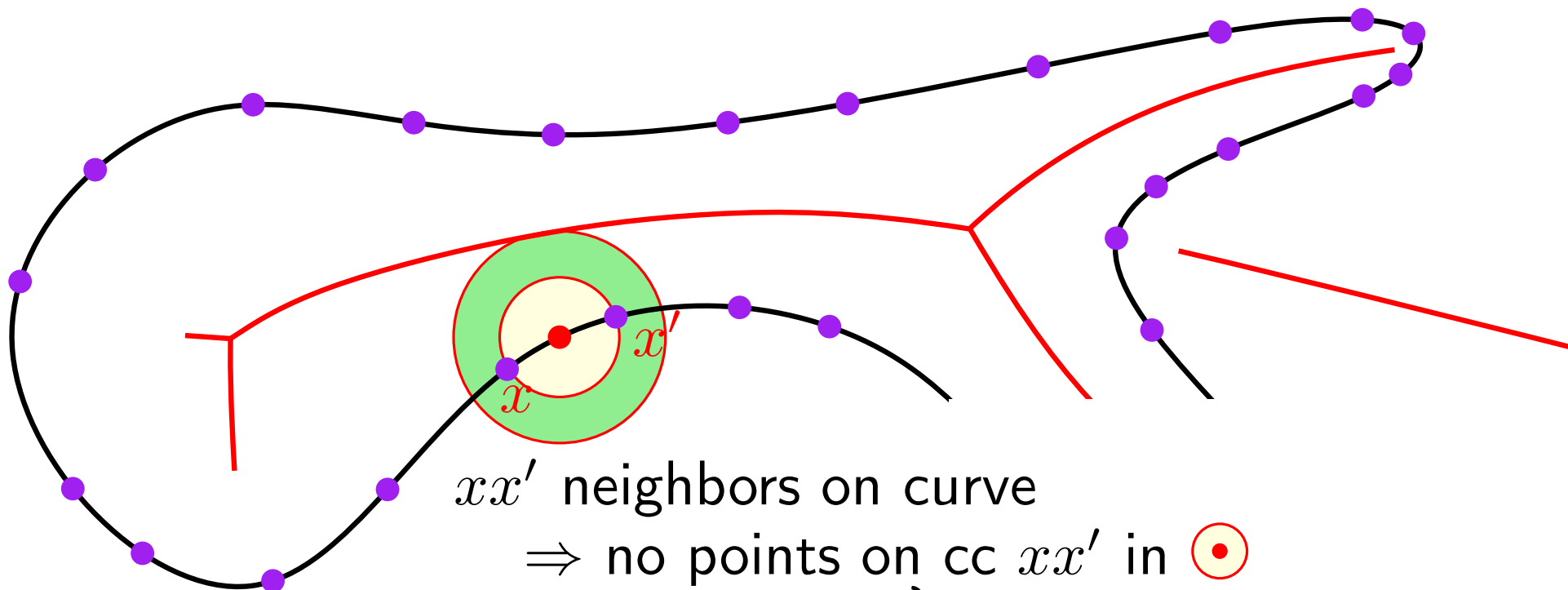
Reconstruction

Delaunay is a good start

Theorem


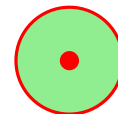
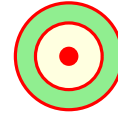
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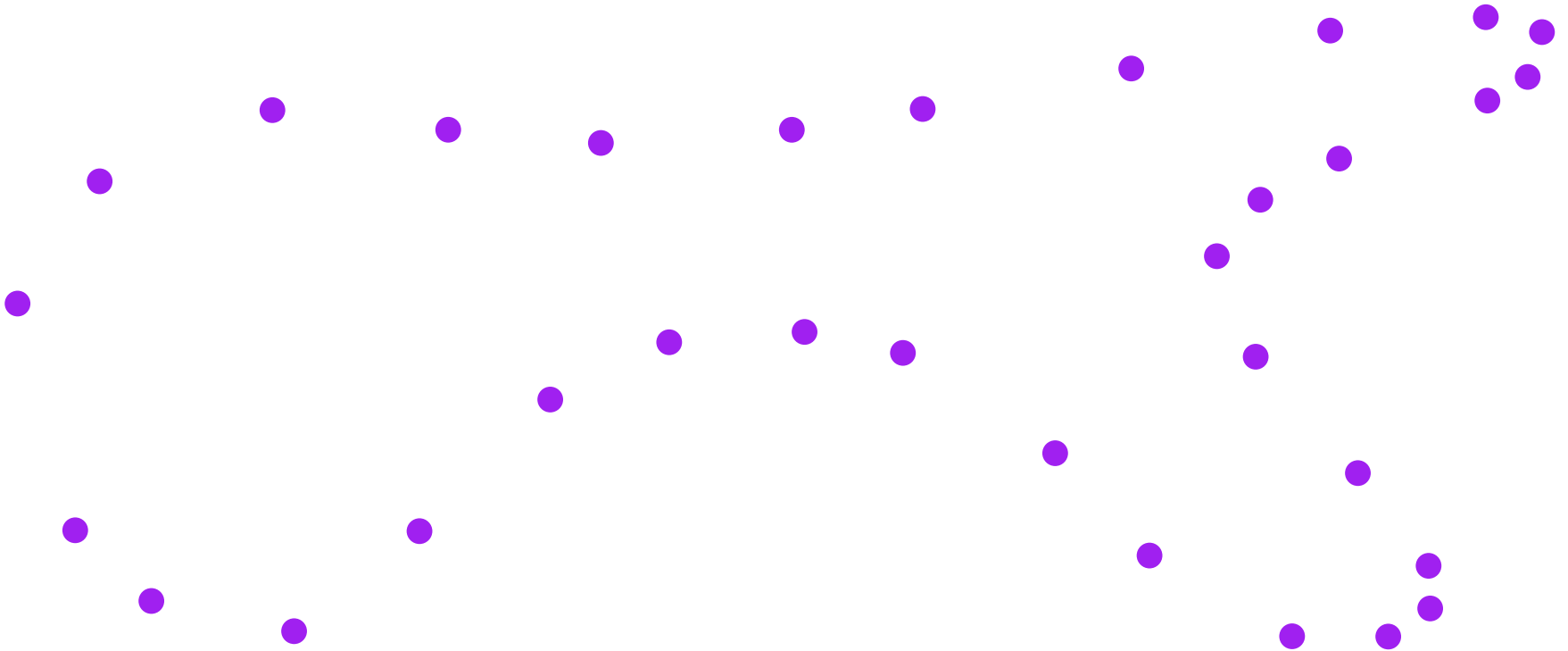
1-sampling \Rightarrow  \subset  \Rightarrow no other cc \cap 

Lemma \Rightarrow  empty

Reconstruction

Delaunay is a good start

Given a sampling

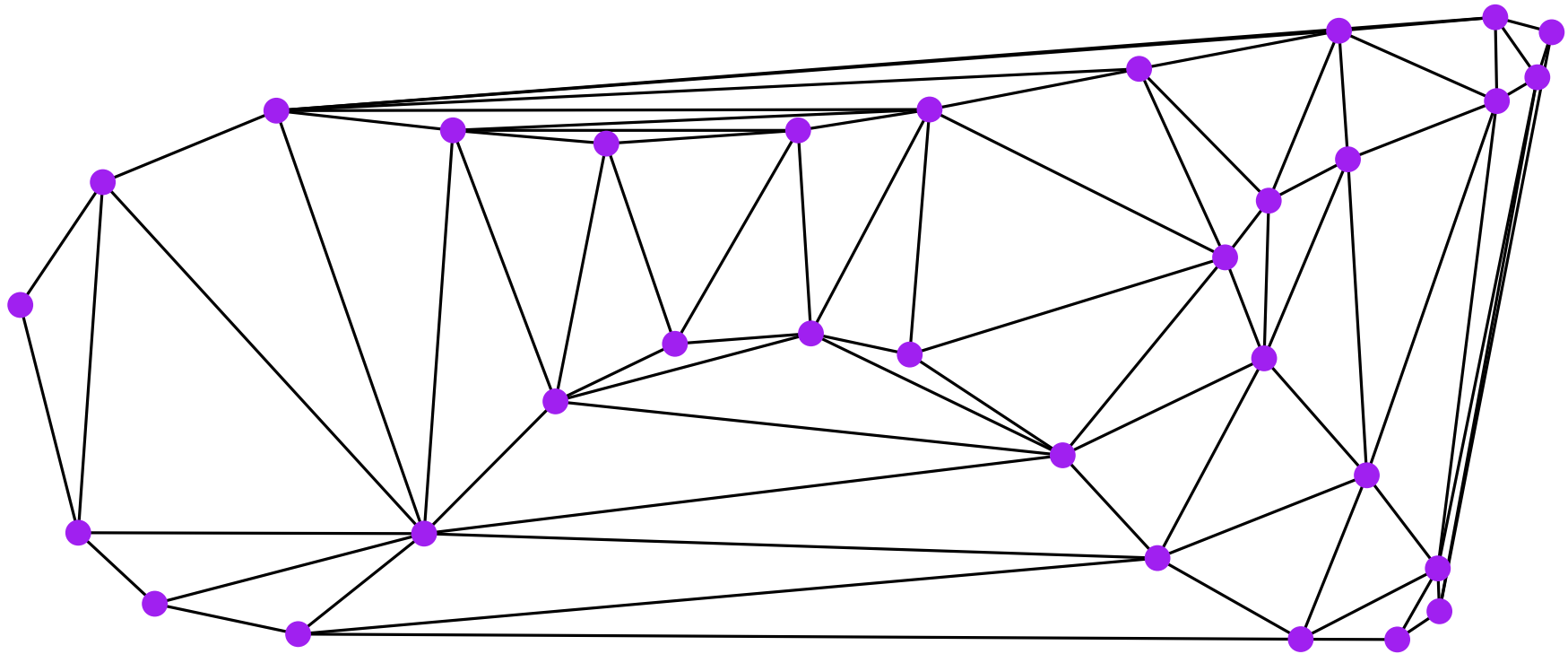


Reconstruction

Delaunay is a good start

Given a sampling

Compute Delaunay



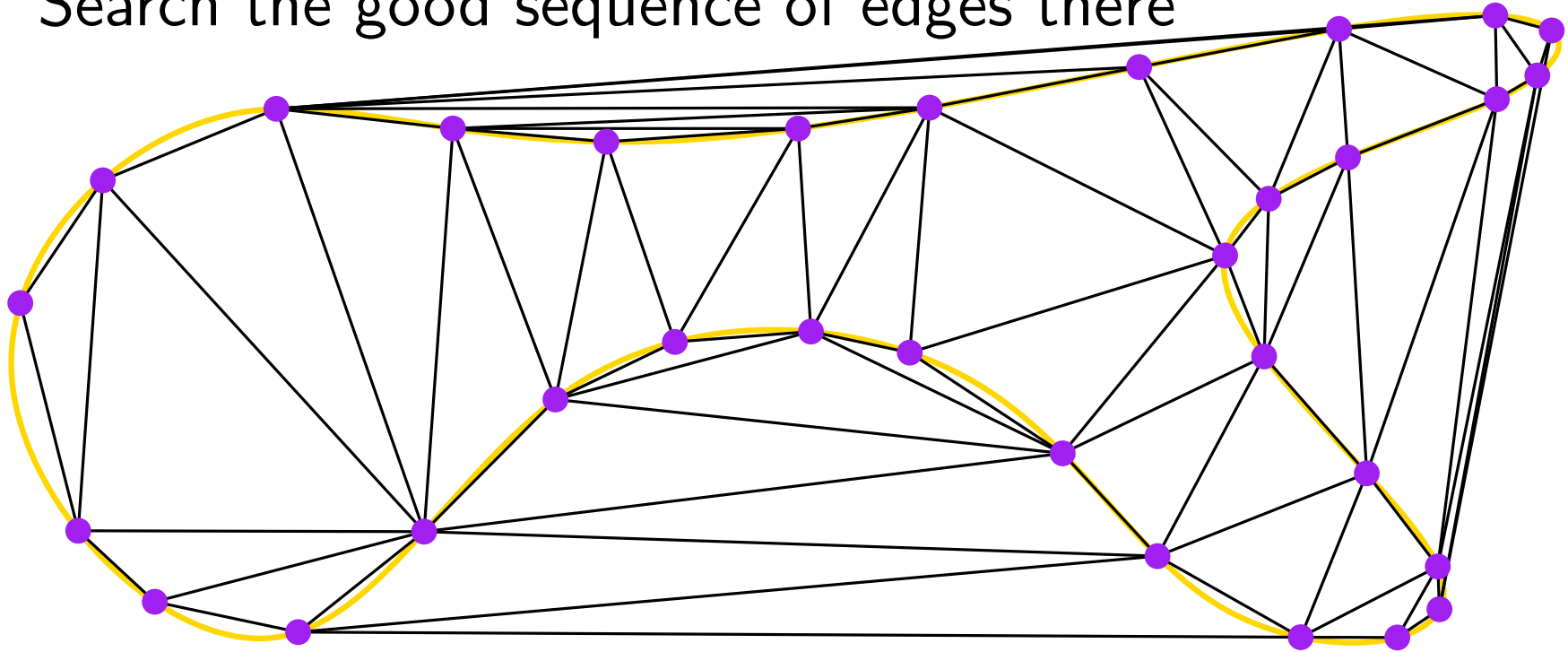
Reconstruction

Delaunay is a good start

Given a sampling

Compute Delaunay

Search the good sequence of edges there



Reconstruction

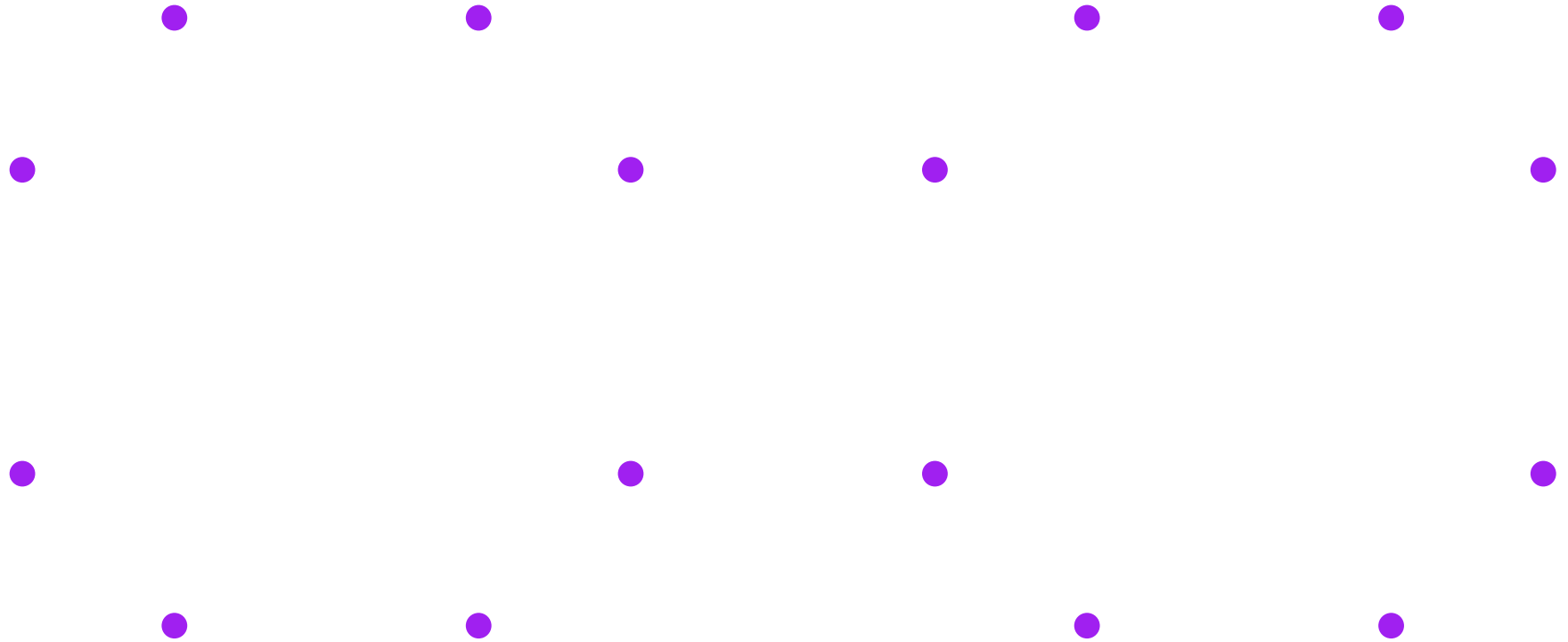
Delaunay is a good start

1-sample is not enough

Reconstruction

Delaunay is a good start

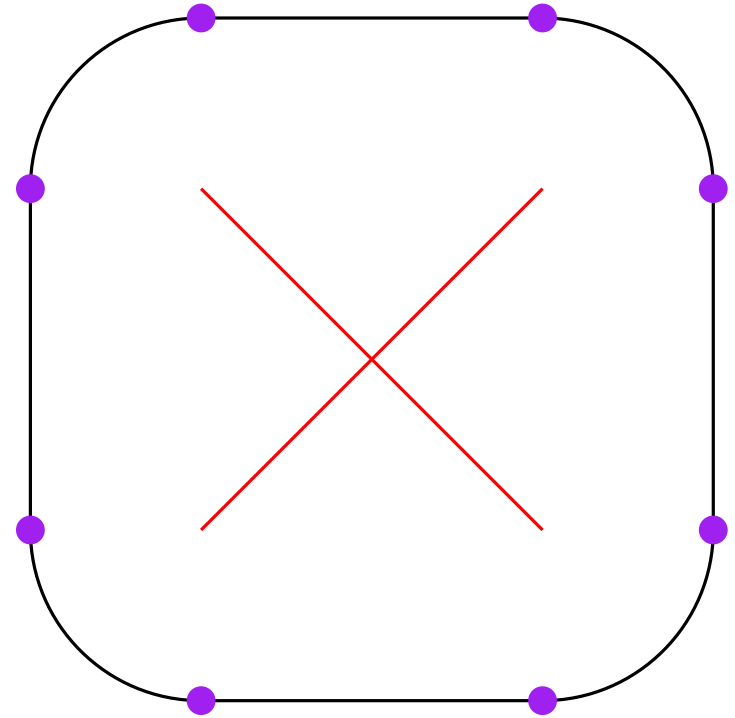
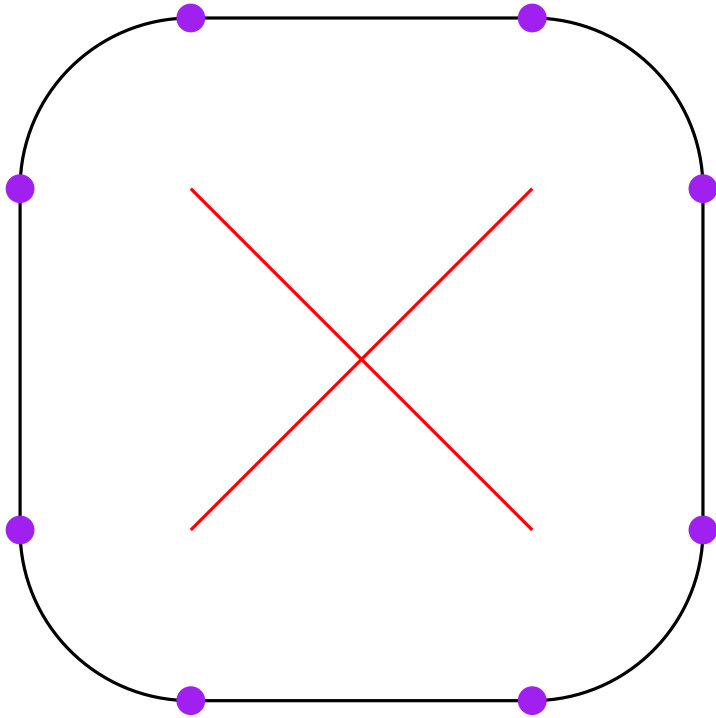
1-sample is not enough



Reconstruction

Delaunay is a good start

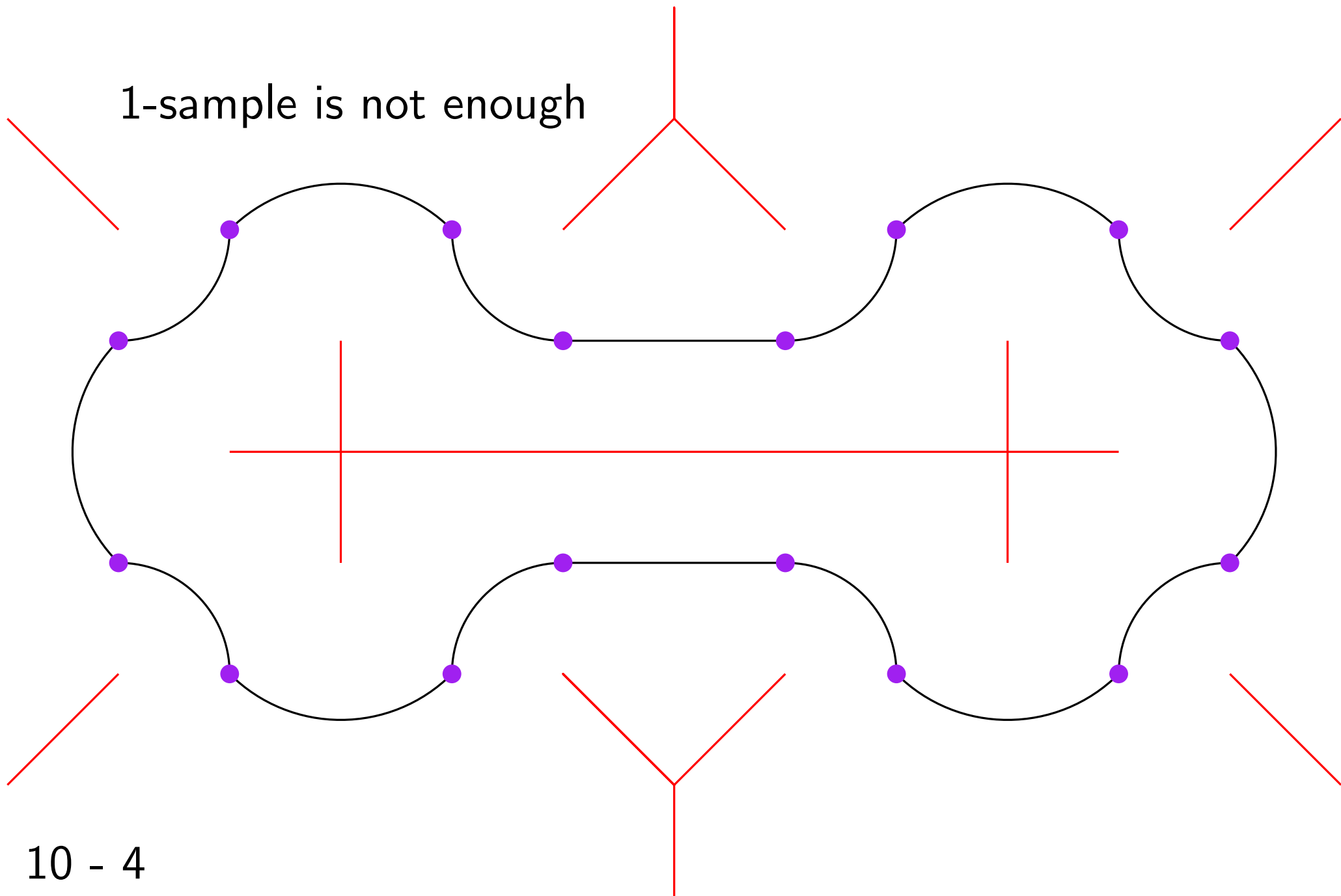
1-sample is not enough



Reconstruction

Delaunay is a good start

1-sample is not enough

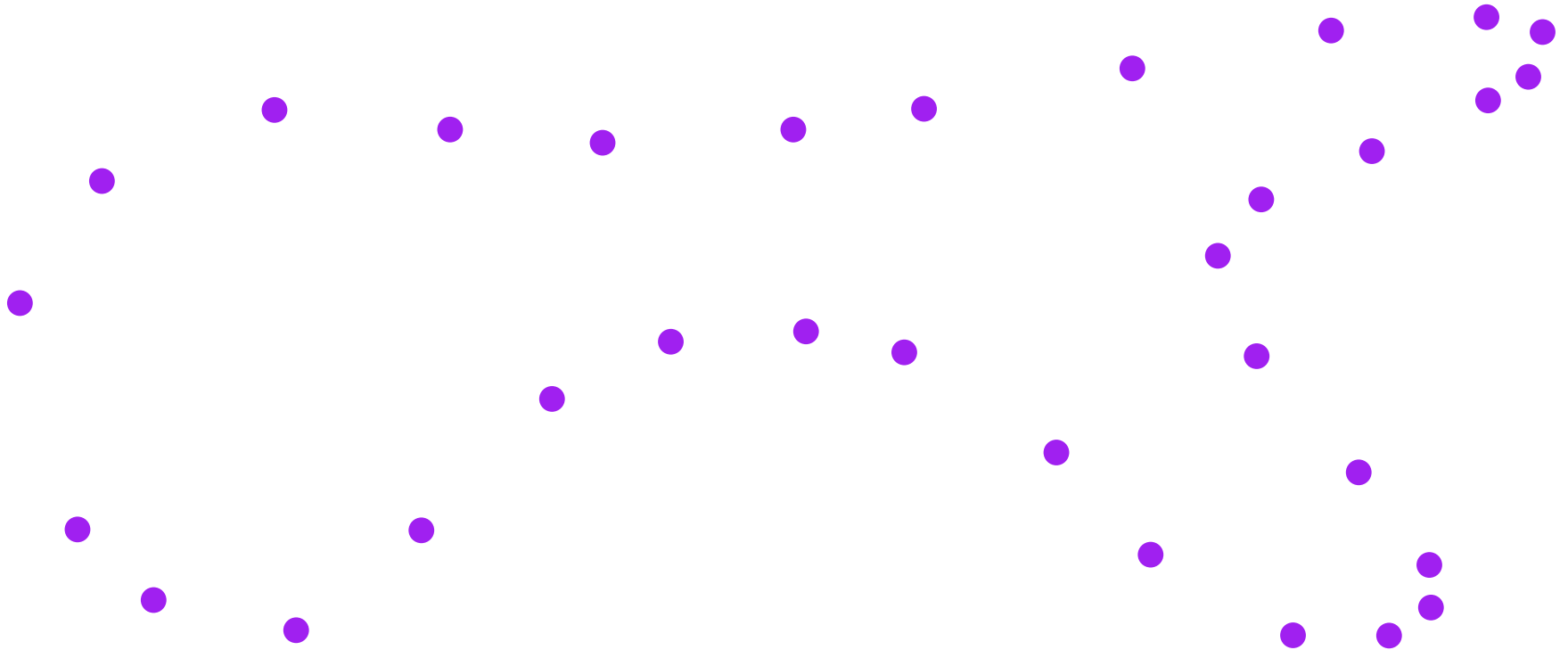


10 - 4

Reconstruction

Crust 2D

Algorithm

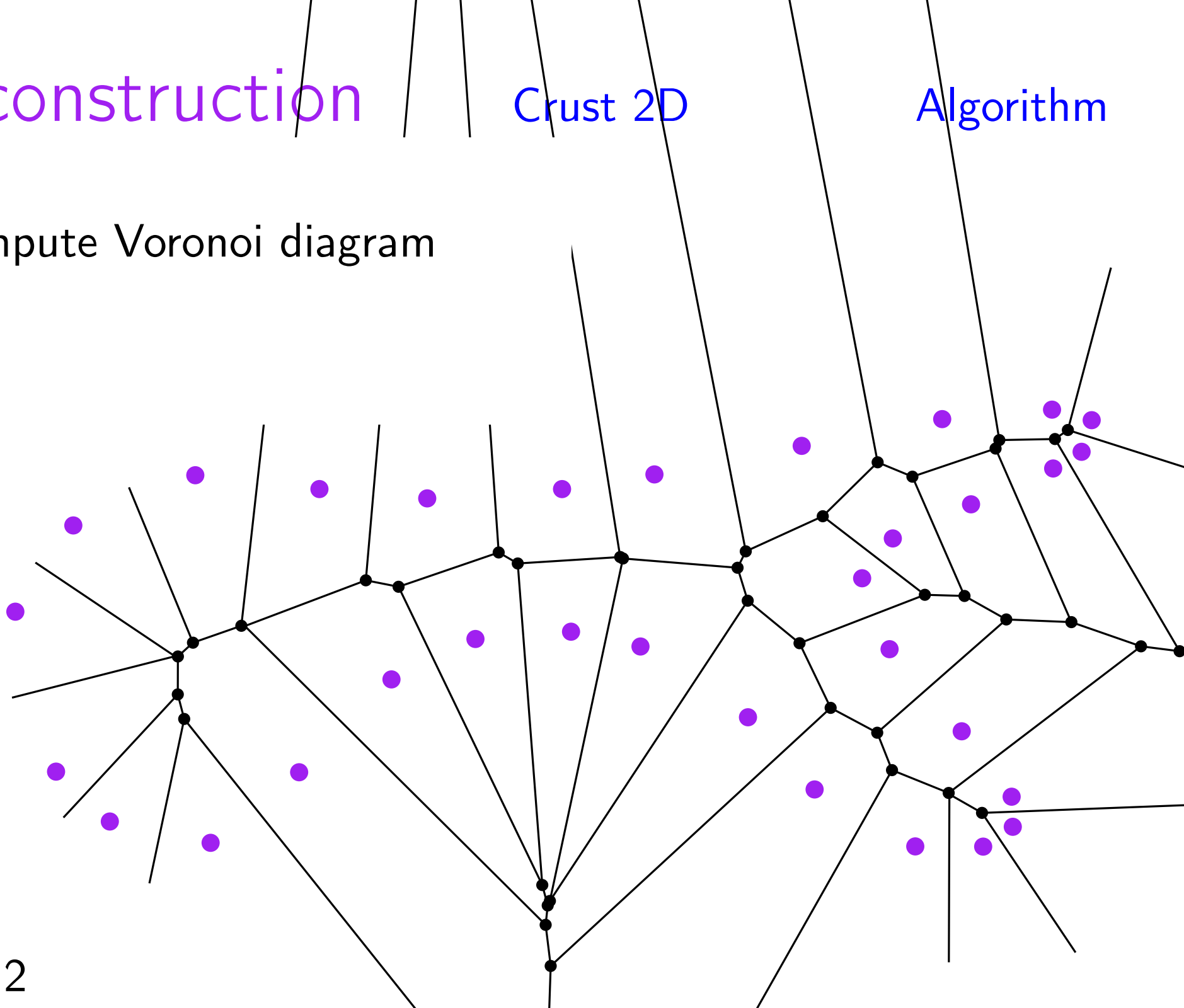


Reconstruction

Crust 2D

Algorithm

Compute Voronoi diagram

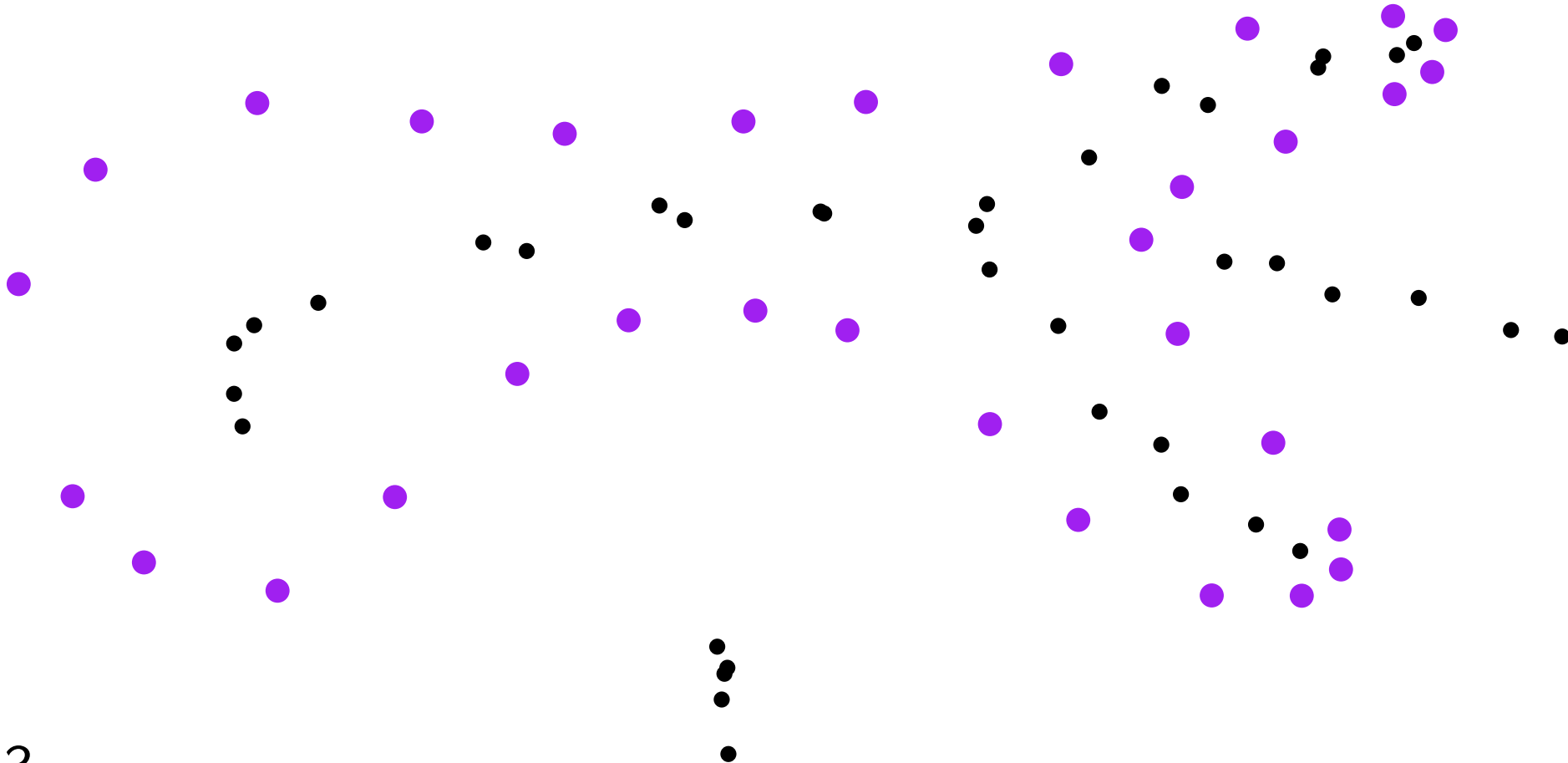


Reconstruction

Keep Voronoi vertices

Crust 2D

Algorithm



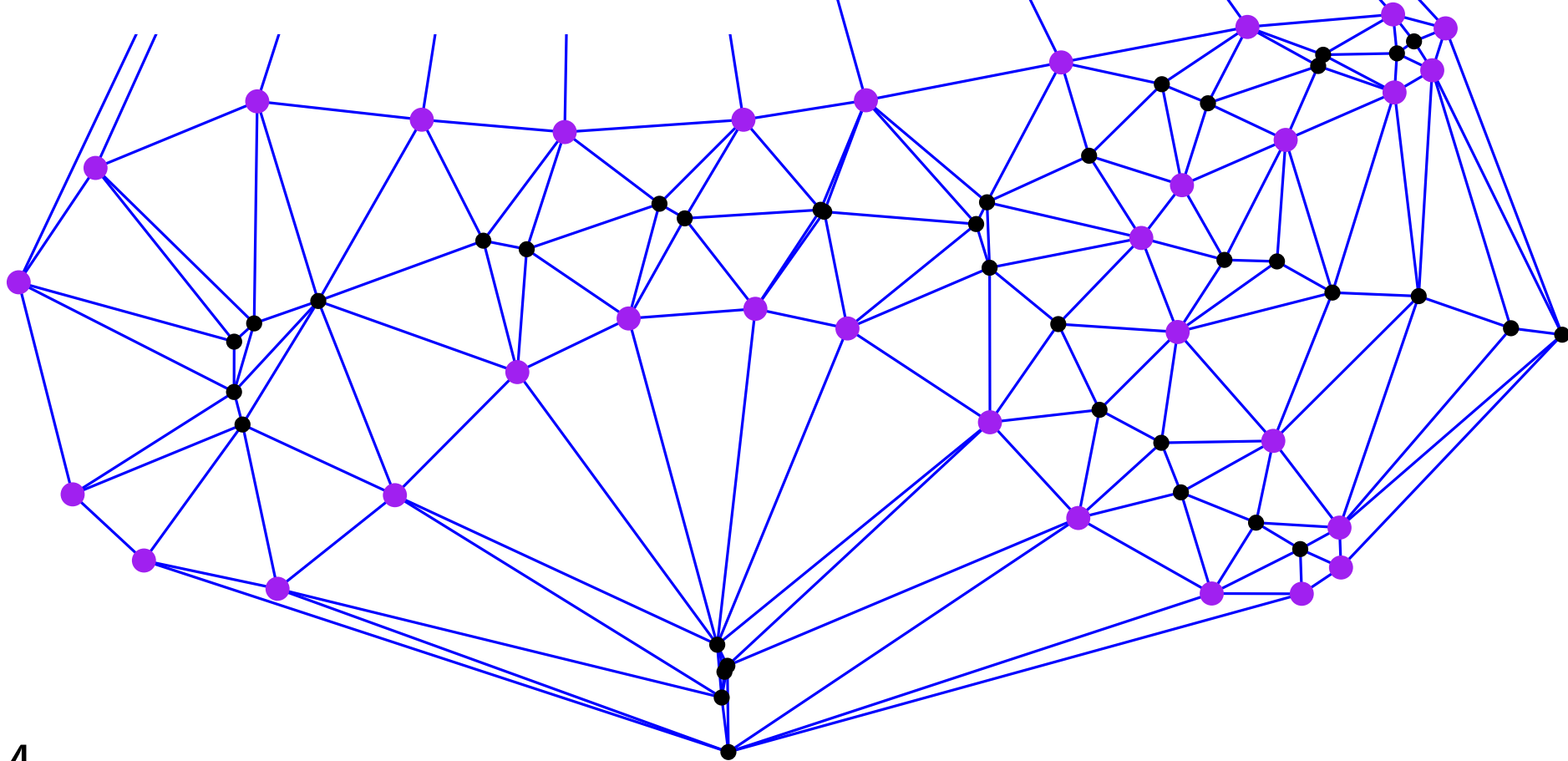
Reconstruction

Crust 2D

Algorithm

Keep Voronoi vertices

Compute Delaunay triangulation



Reconstruction

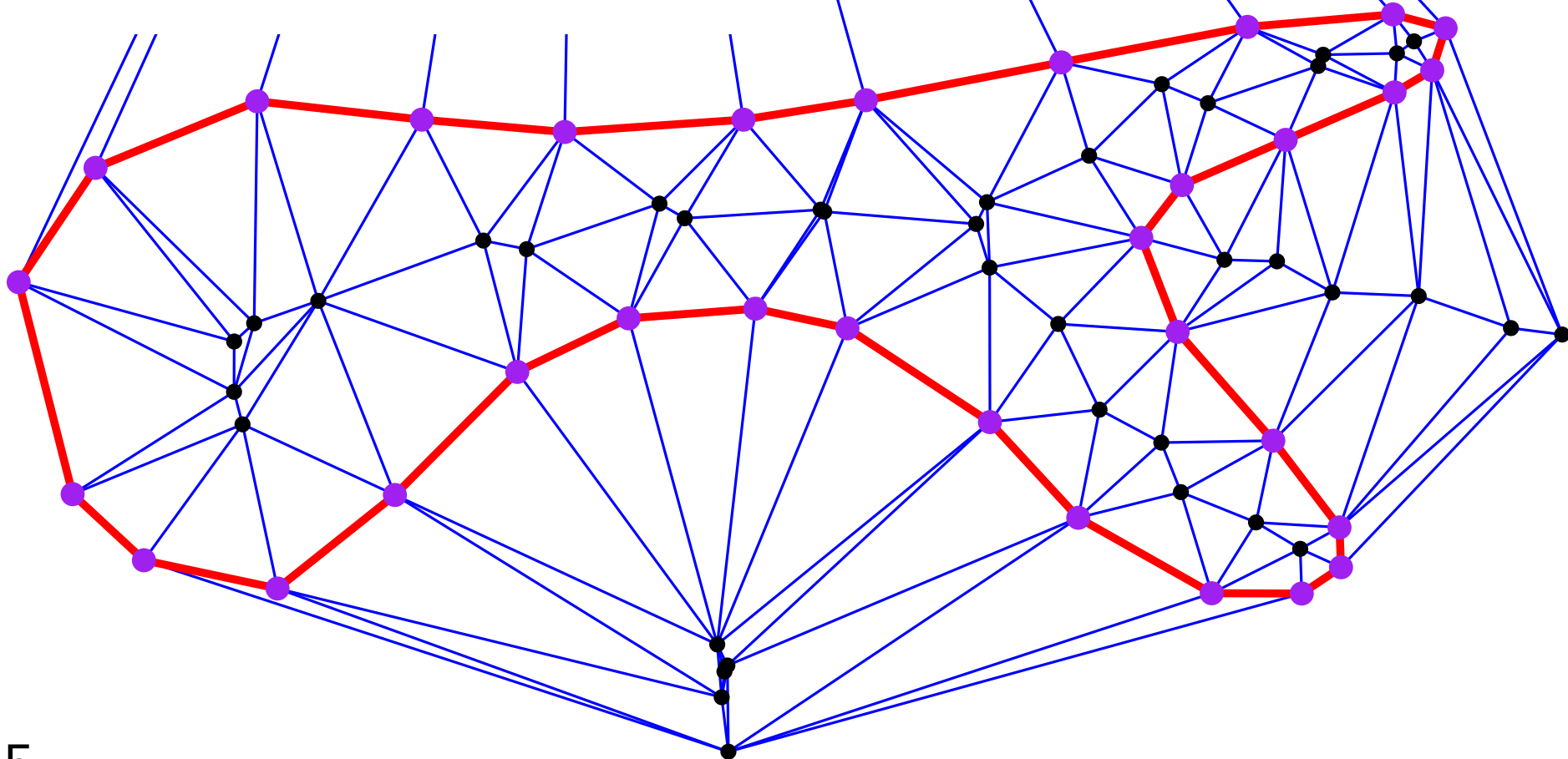
Crust 2D

Algorithm

Keep Voronoi vertices

Compute Delaunay triangulation

Keep edges between original points

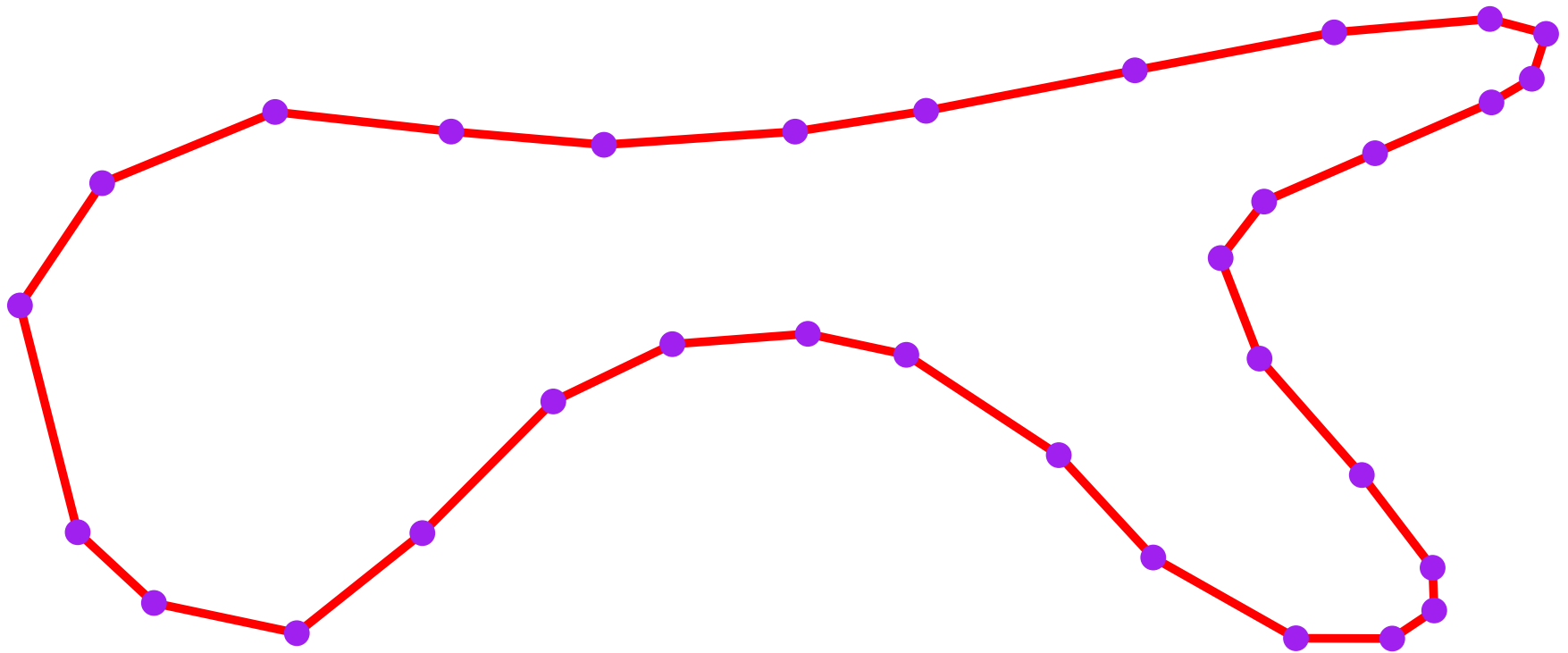


Reconstruction

Crust 2D

Algorithm

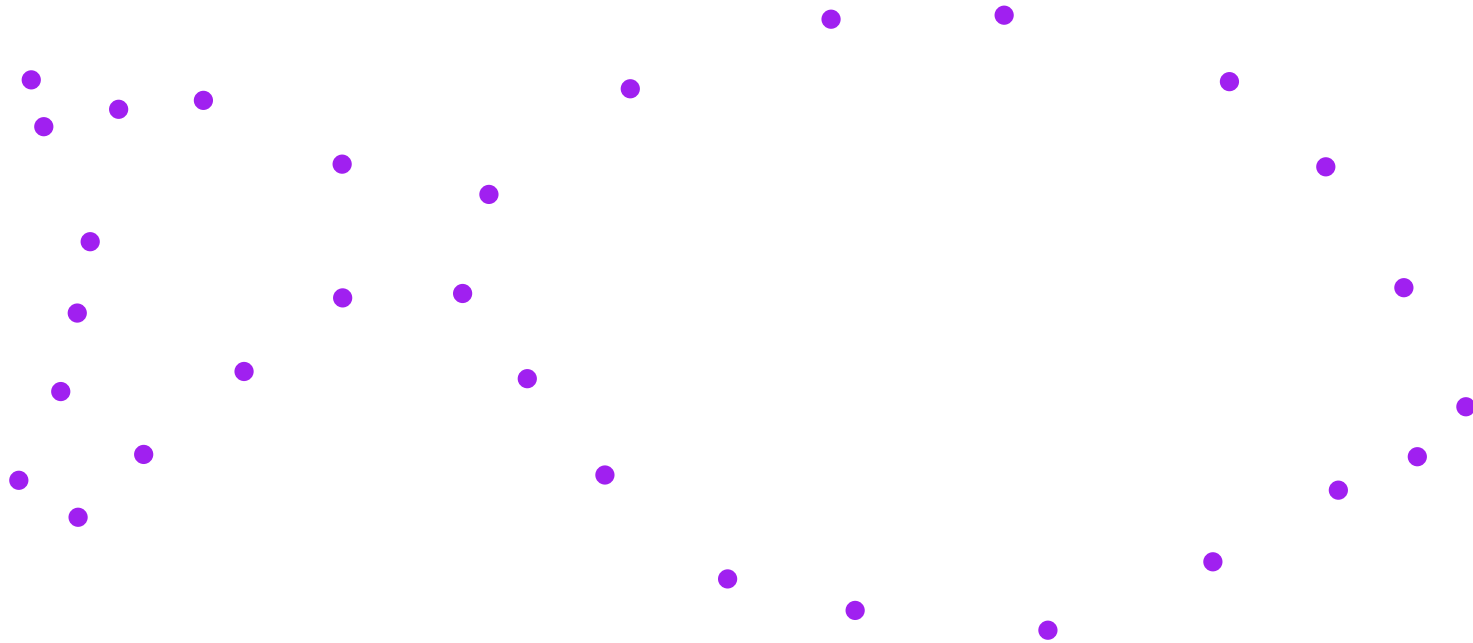
Keep edges between original points



Reconstruction

Crust 2D

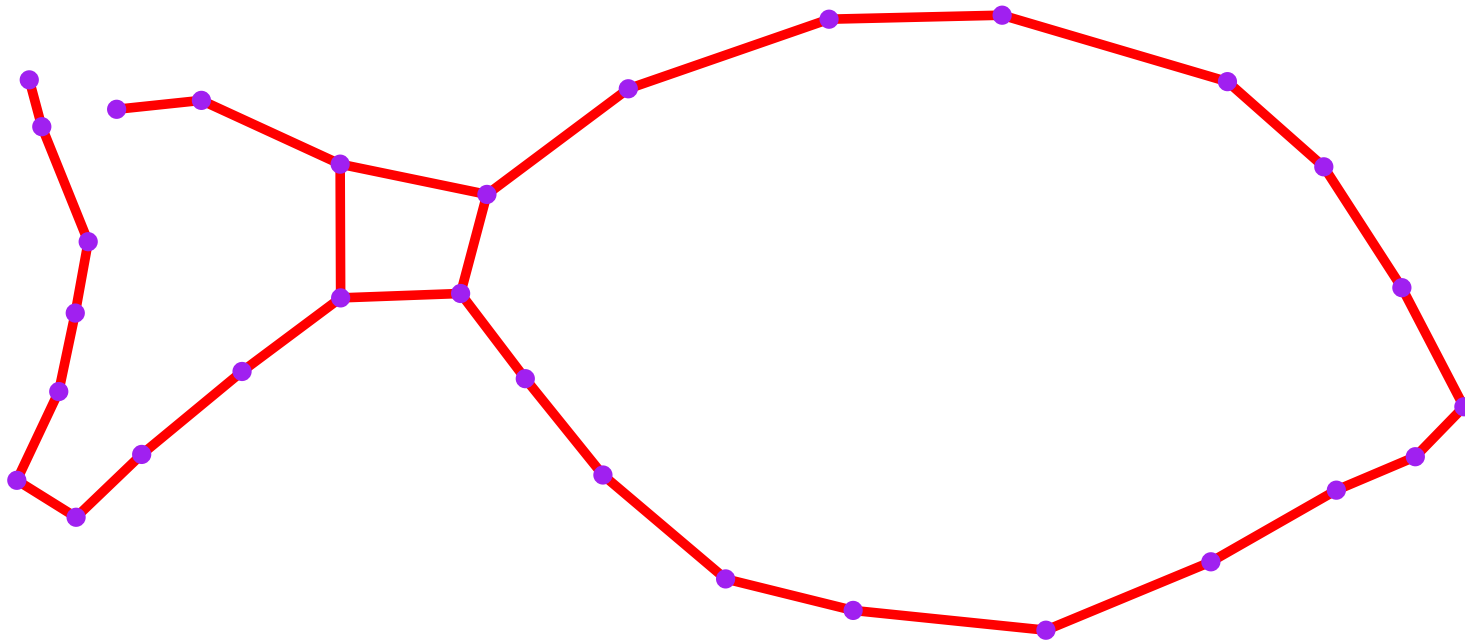
Algorithm



Reconstruction

Crust 2D

Algorithm



Reconstruction

Crust 2D

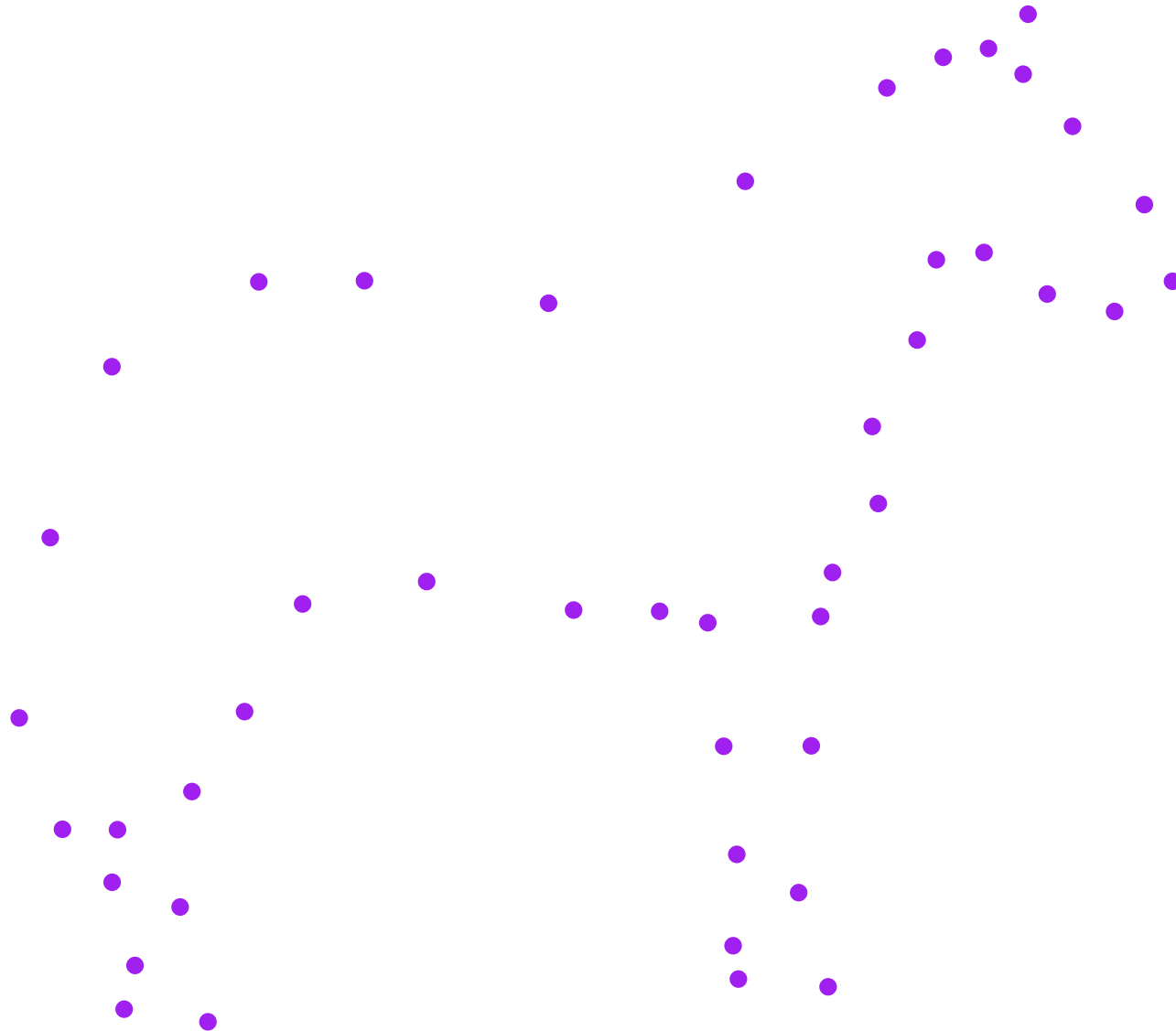
Algorithm



Reconstruction

Crust 2D

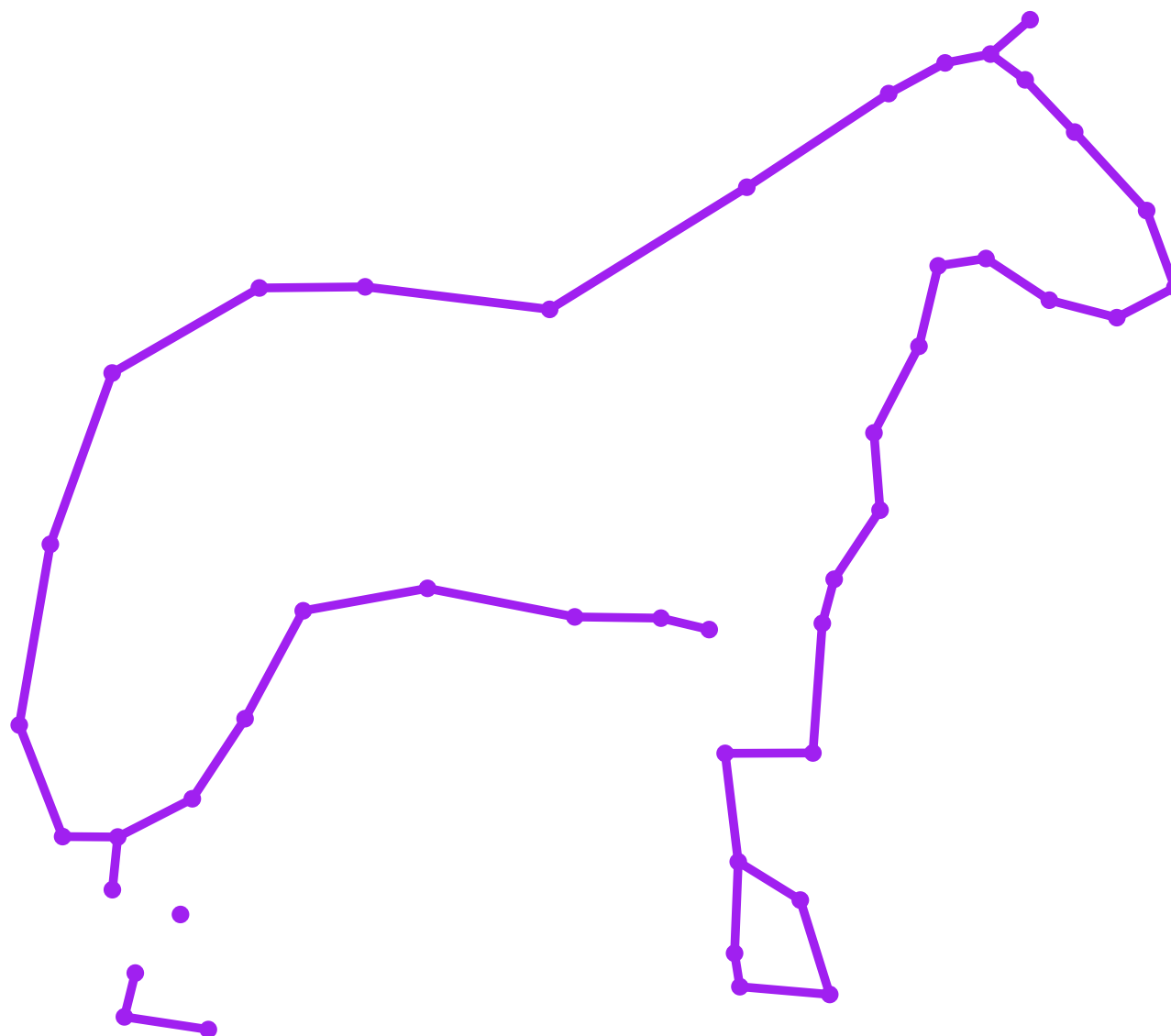
Algorithm



Reconstruction

Crust 2D

Algorithm



Reconstruction

Crust 2D

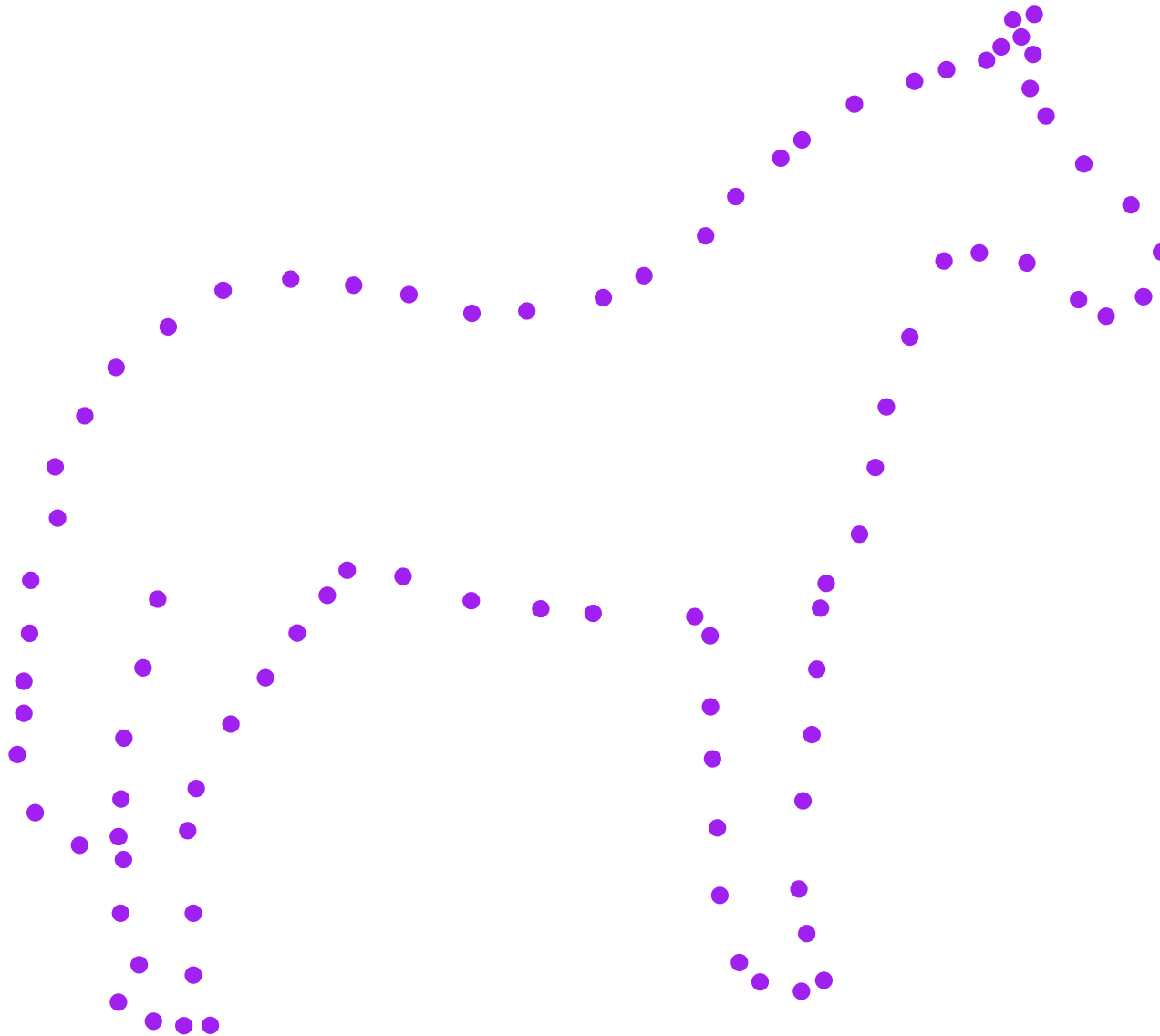
Algorithm



Reconstruction

Crust 2D

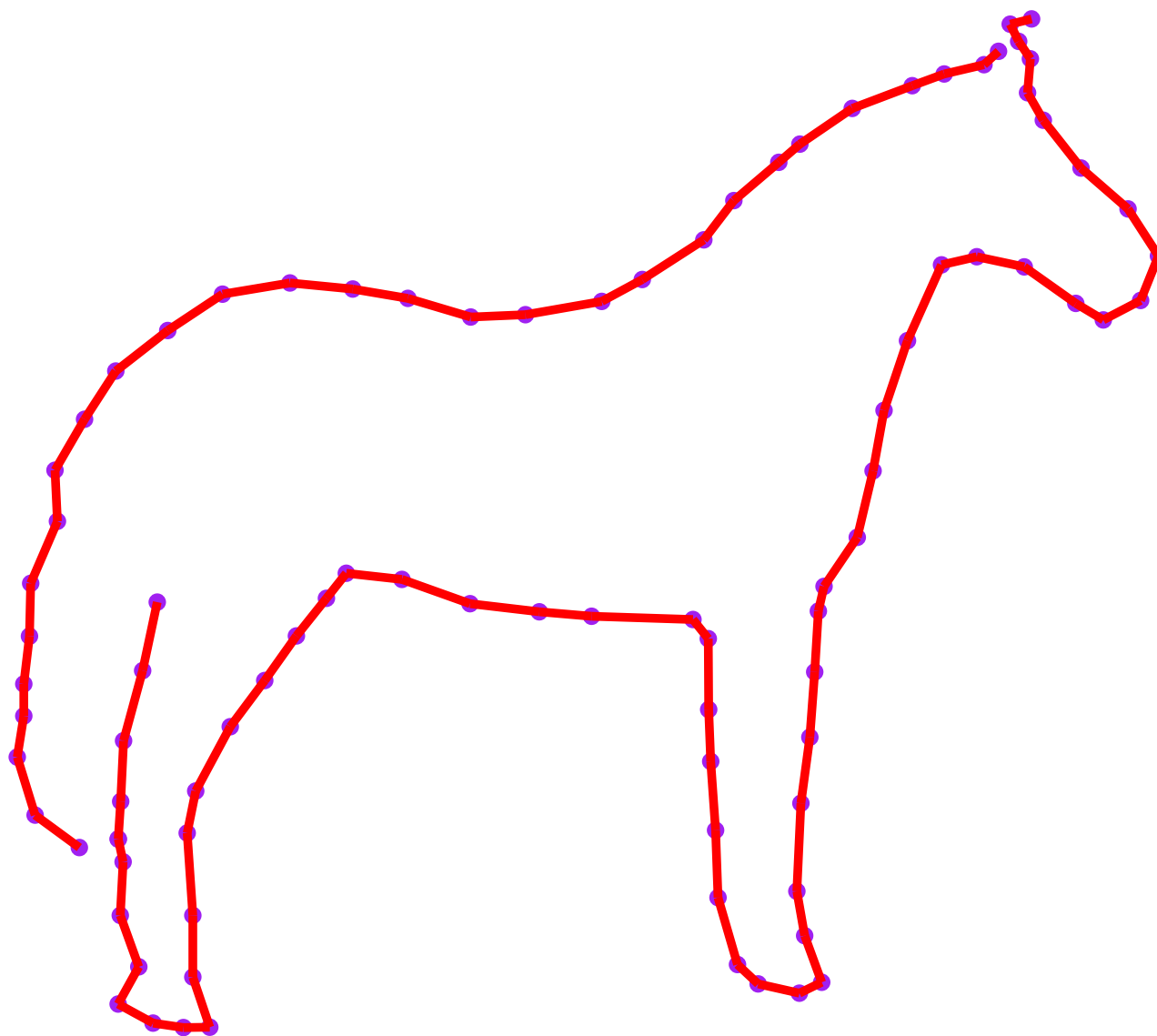
Algorithm



Reconstruction

Crust 2D

Algorithm



Reconstruction

Crust 2D $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

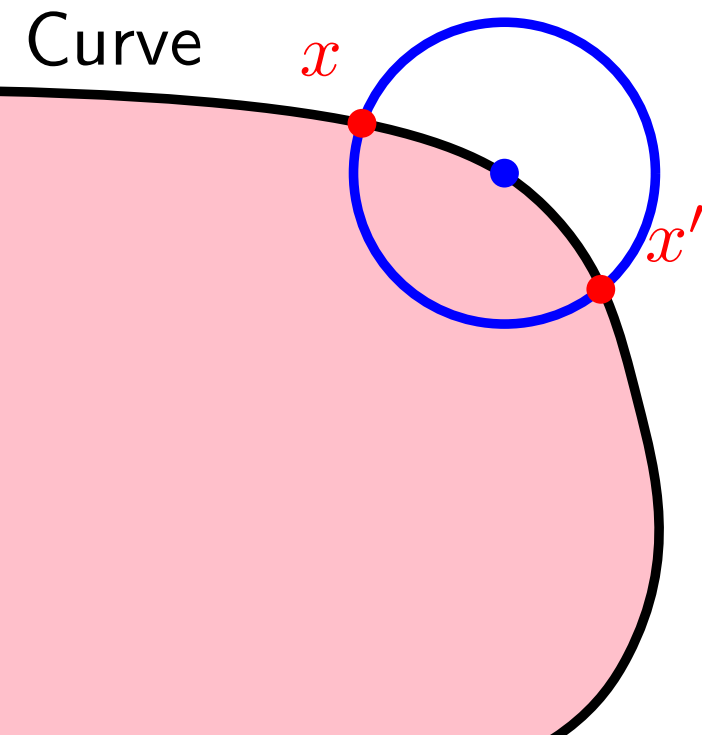
Theorem: $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

Reconstruction

Crust 2D 0.4 sample \Rightarrow wanted result \subset crust

Theorem: **0.4 sample \Rightarrow wanted result \subset crust**

x, x' two neighboring points on Curve
Circle thru x and x' centered on Curve



Reconstruction

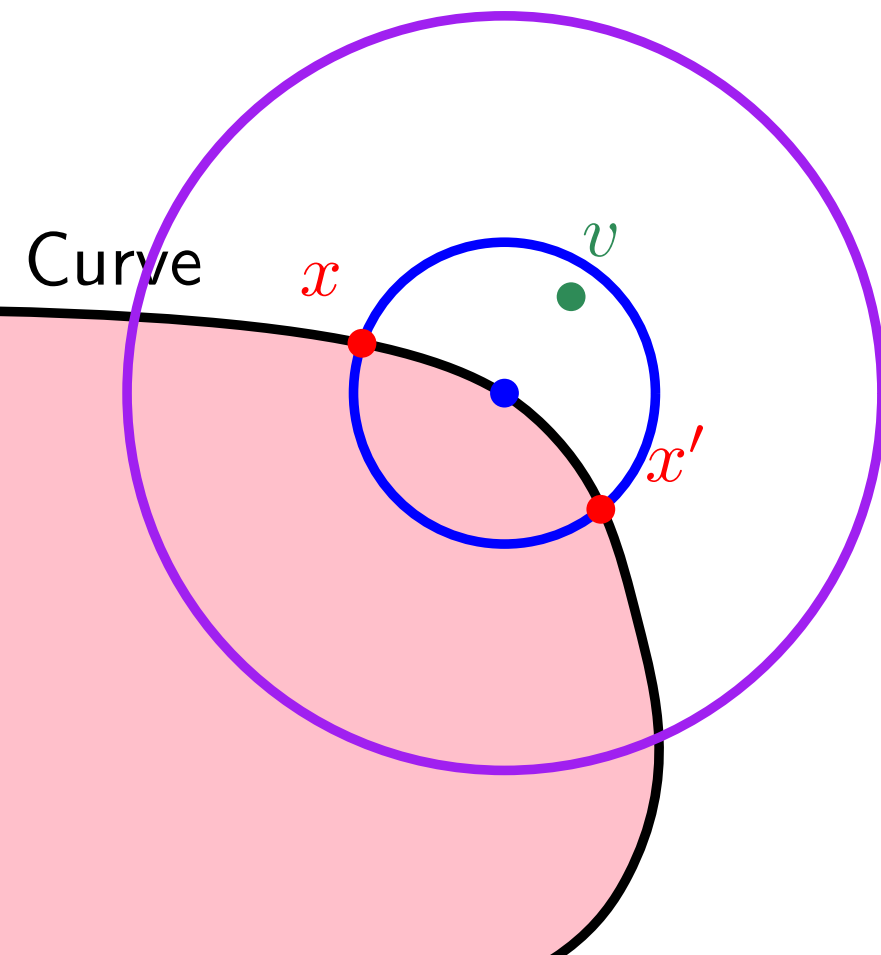
Crust 2D $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

Theorem: $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

By contradiction assume $v \in$ 



Reconstruction

Crust 2D $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

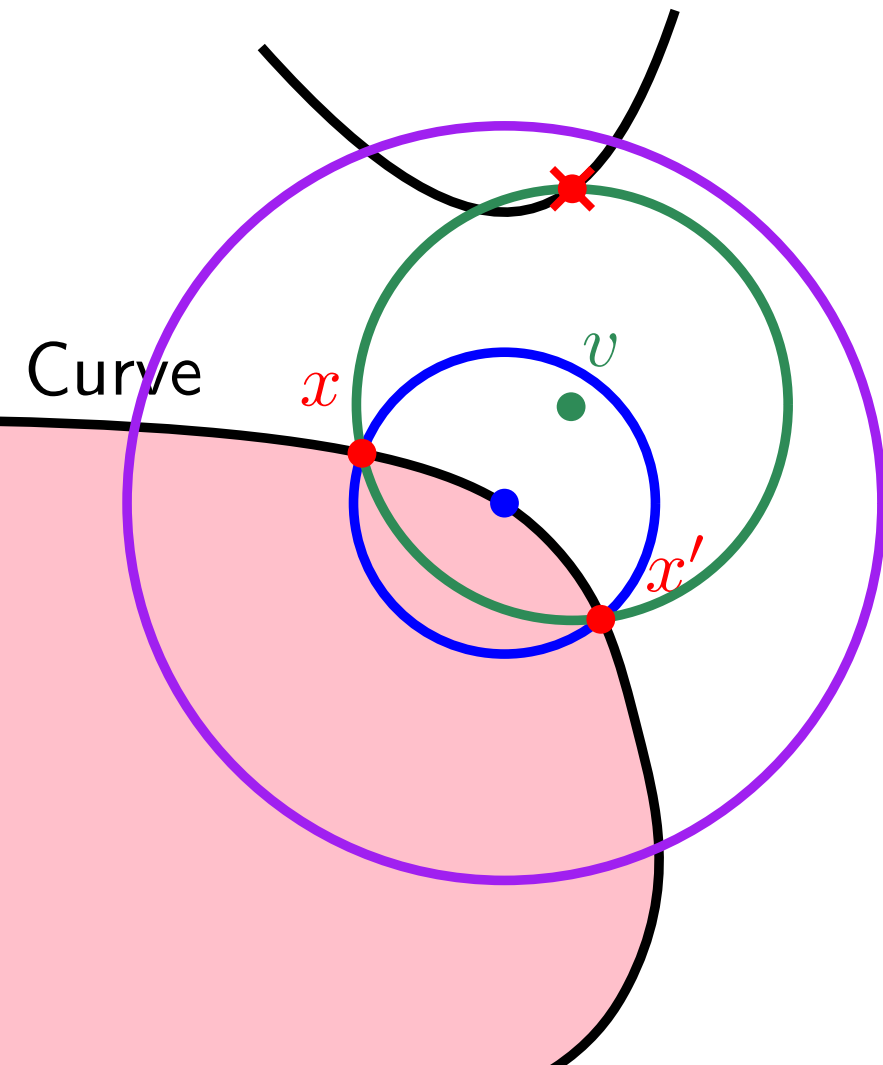
Theorem: $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

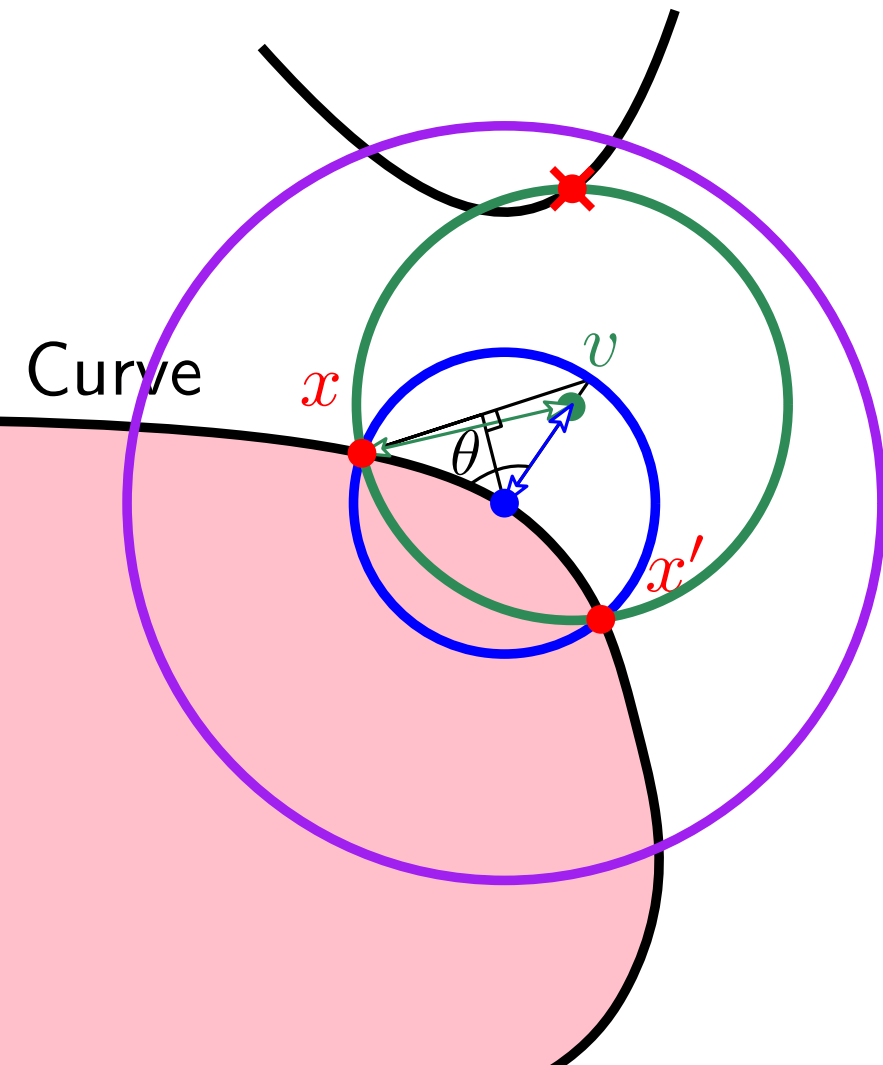
x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

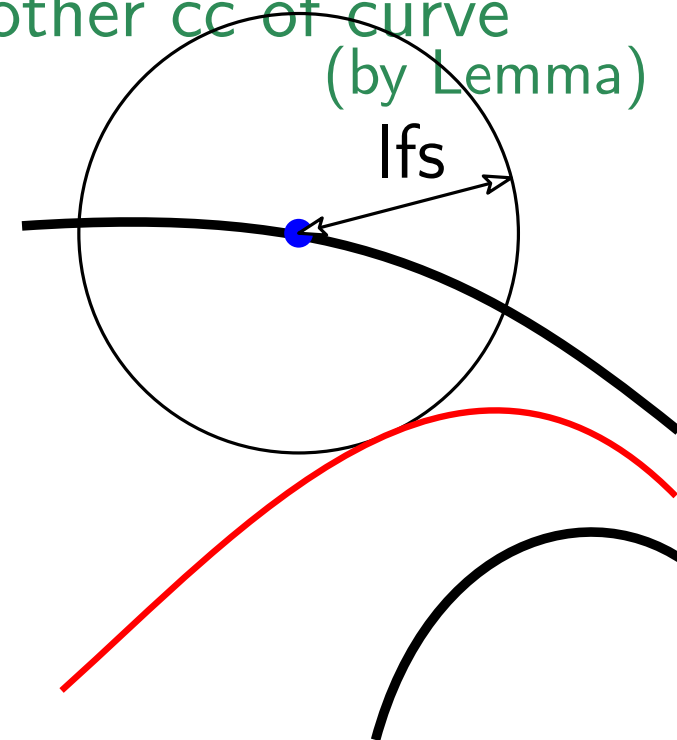
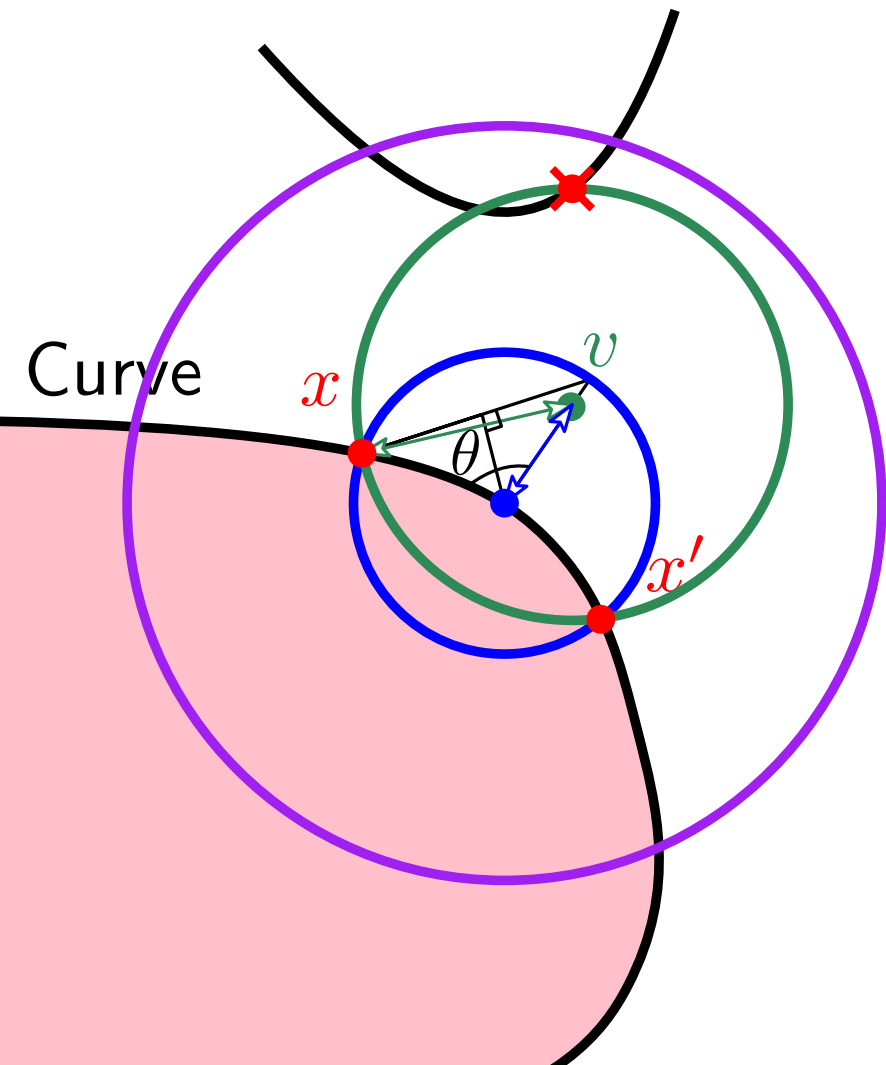
Circle thru x and x' centered on Curve

By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

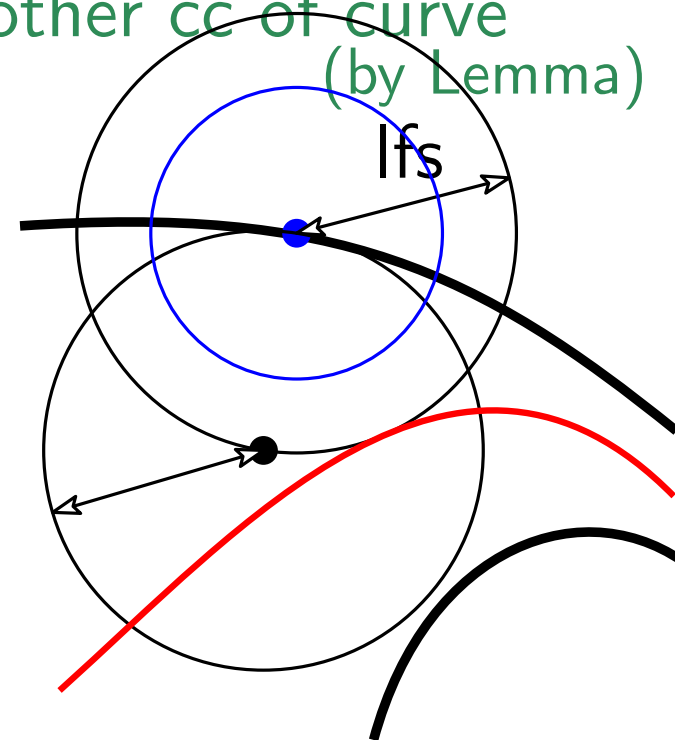
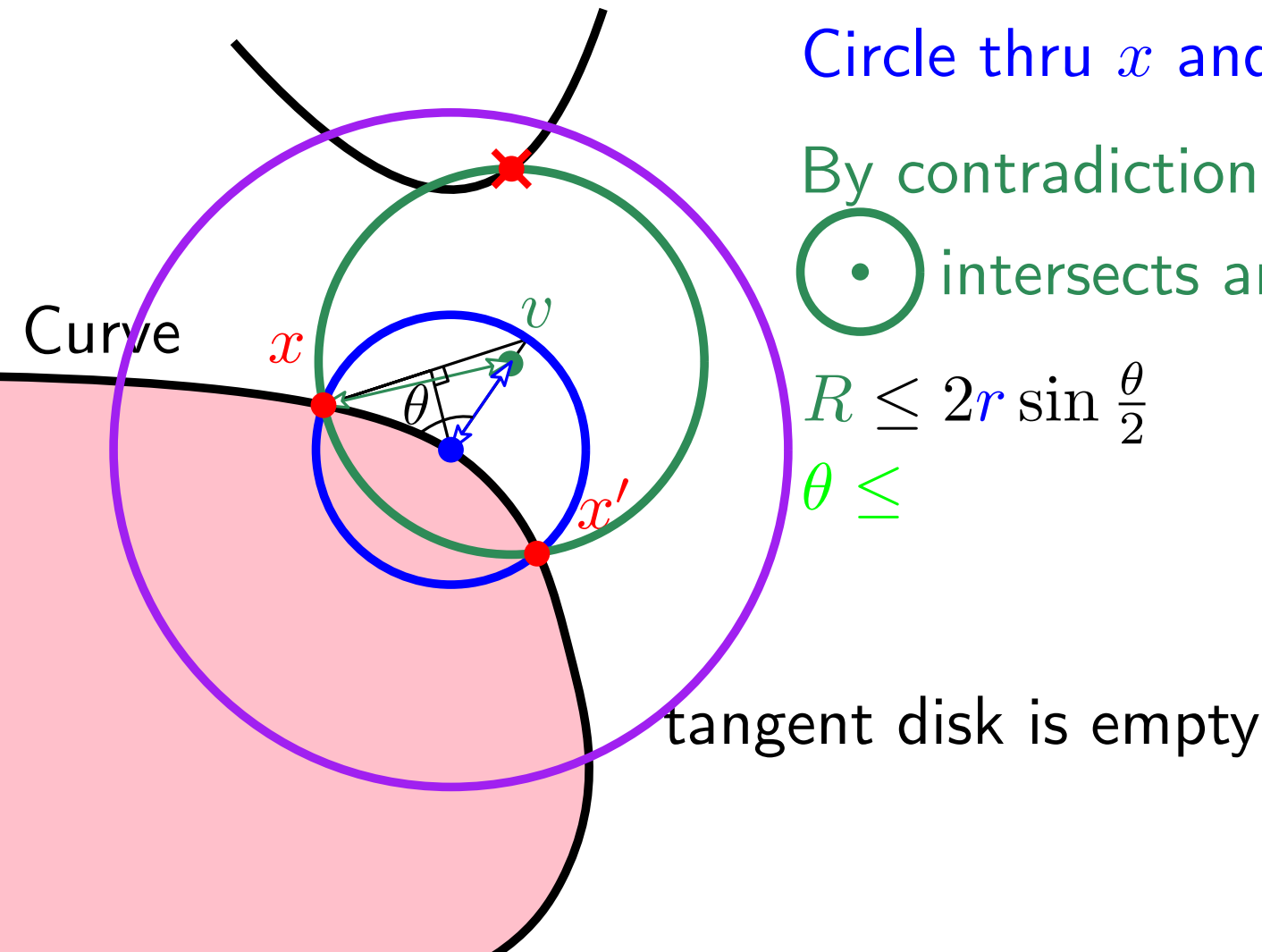
Circle thru x and x' centered on Curve

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$$\theta \leq$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

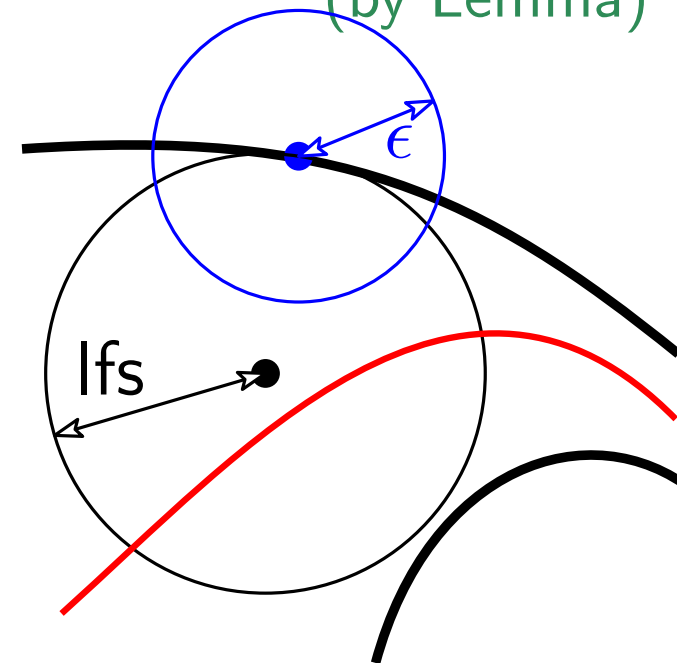
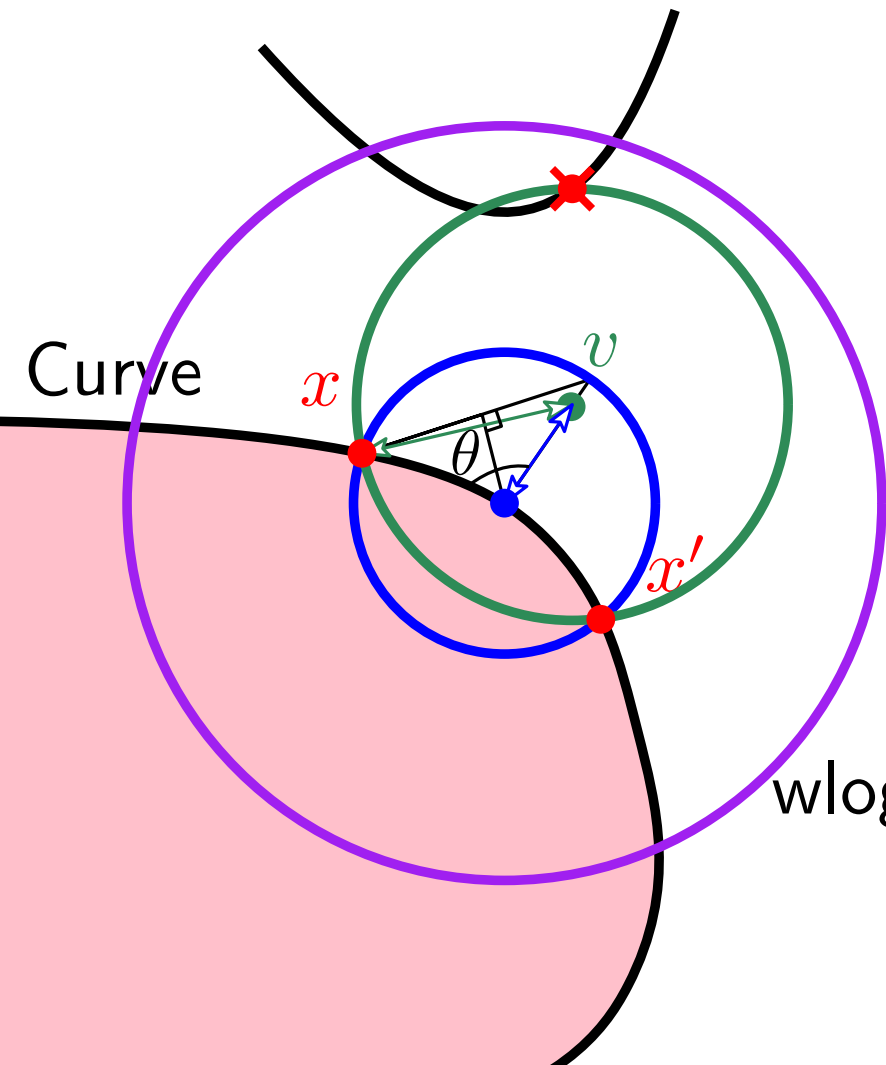
By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq$$

wlog lfs=1 and $r \leq \epsilon$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

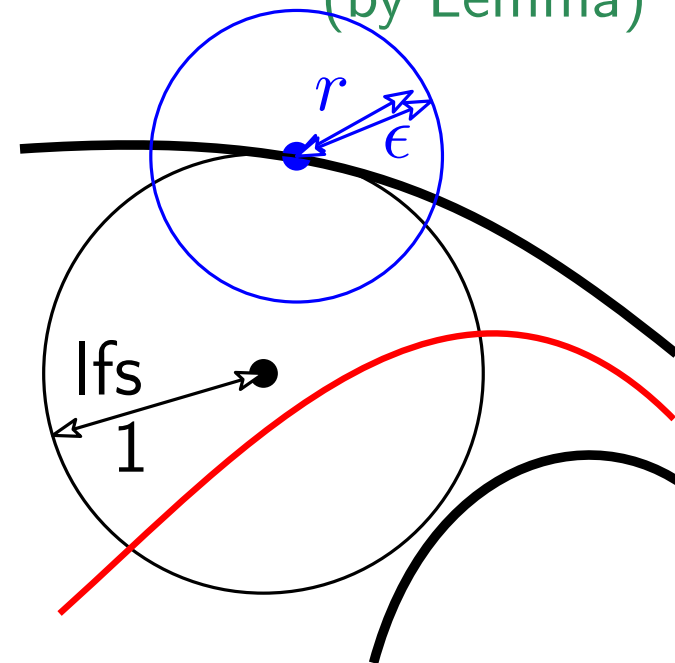
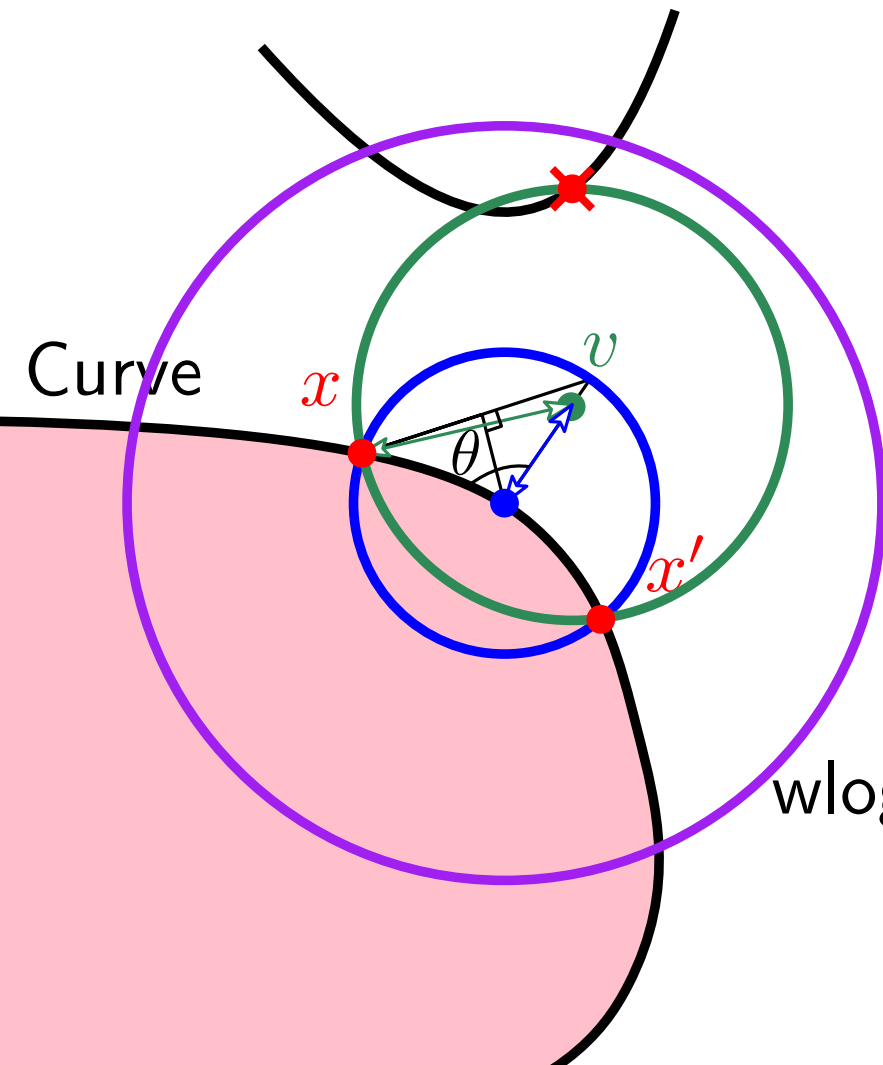
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$$\theta \leq$$

wlog lfs=1 and $r \leq \epsilon$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

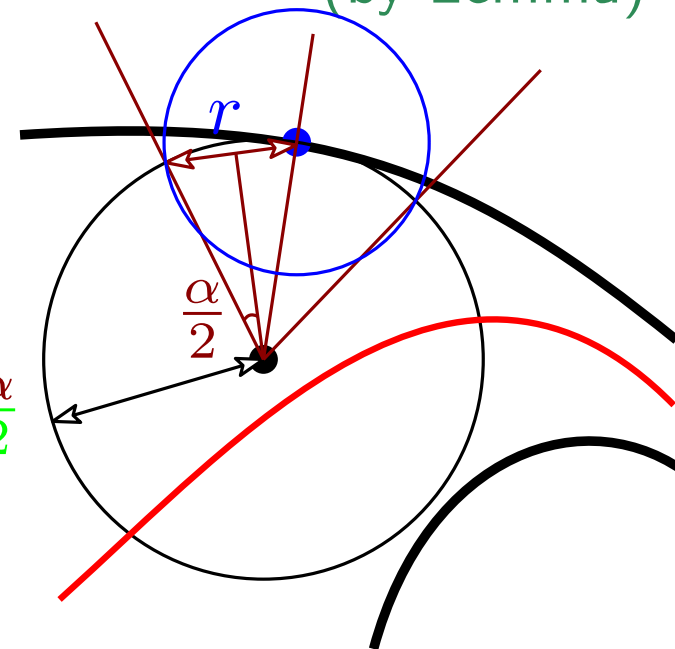
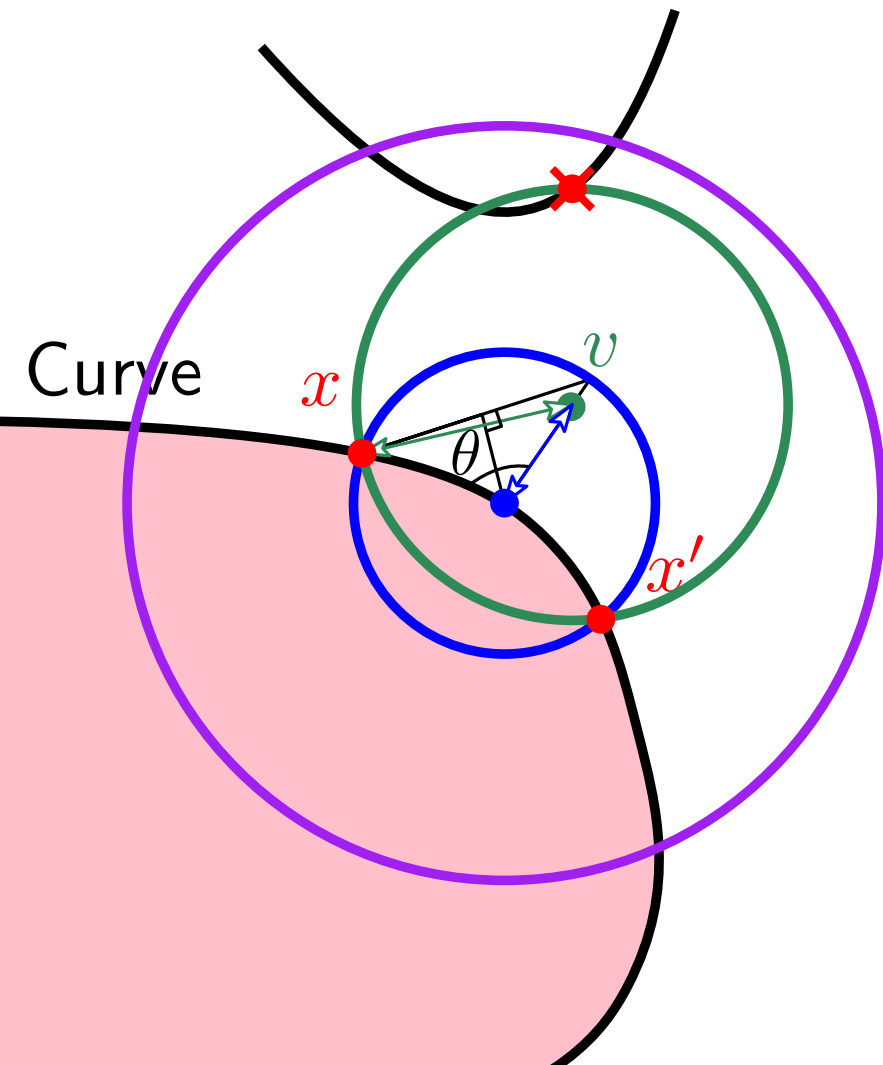
By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq$$

$$r = 2 \sin \frac{\alpha}{2}$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

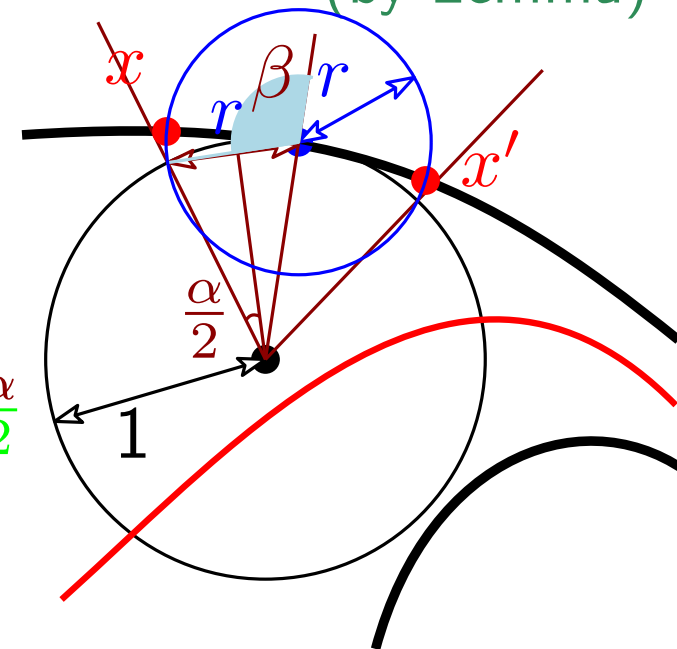
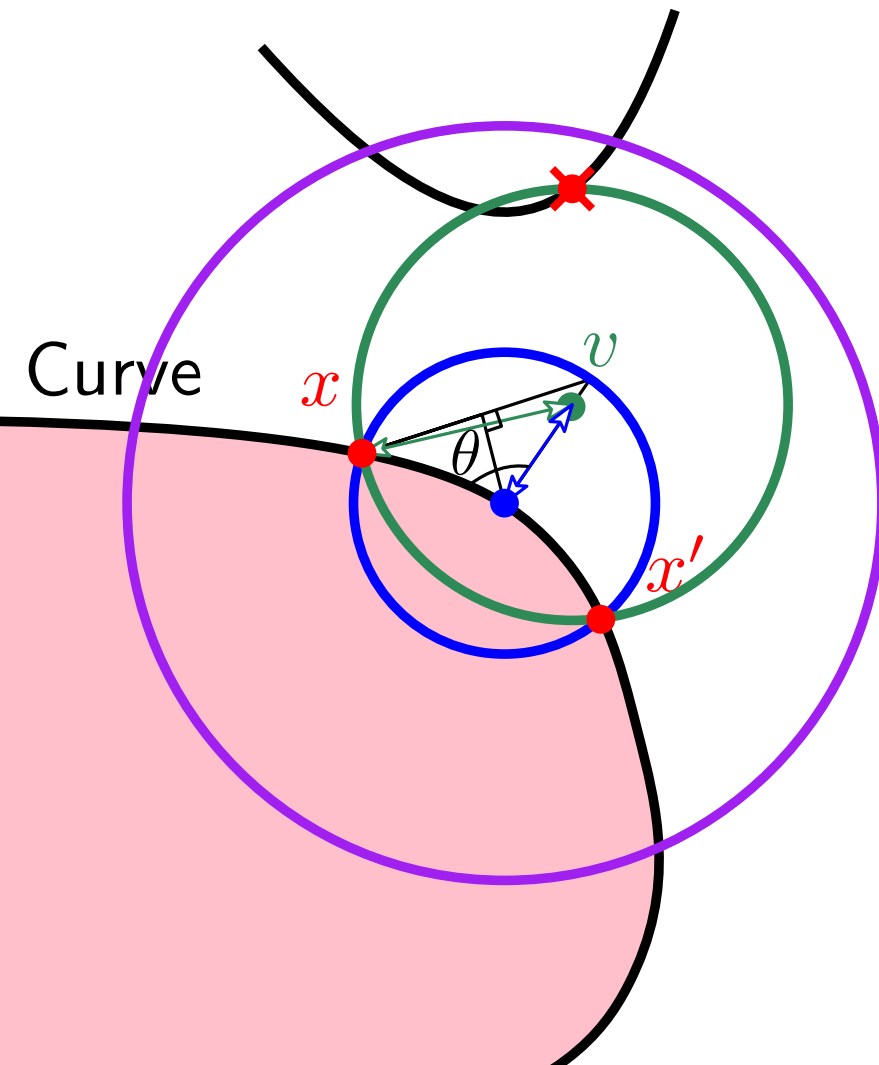
x, x' two neighboring points on Curve
 Circle thru x and x' centered on Curve

By contradiction assume $v \in \odot$
 \odot intersects another cc of curve
 (by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq \beta = \pi - \frac{\pi - \alpha}{2}$$

$$r = 2 \sin \frac{\alpha}{2}$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve
 Circle thru x and x' centered on Curve

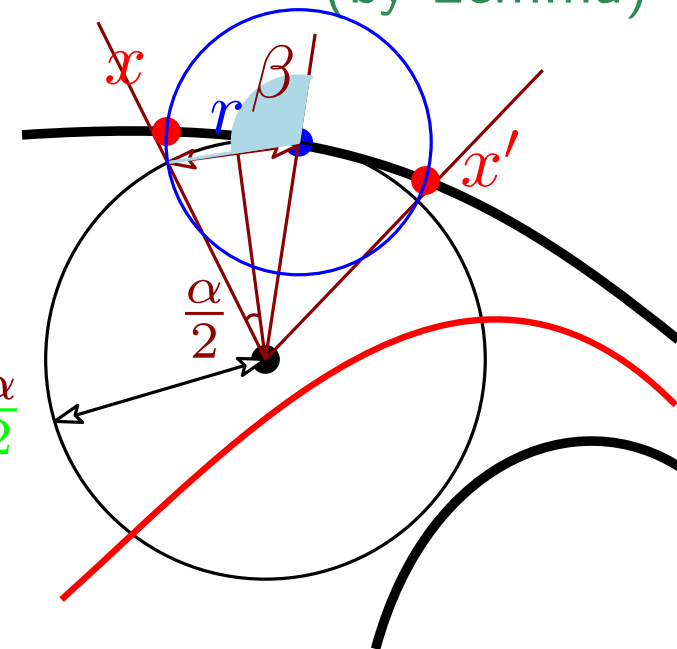
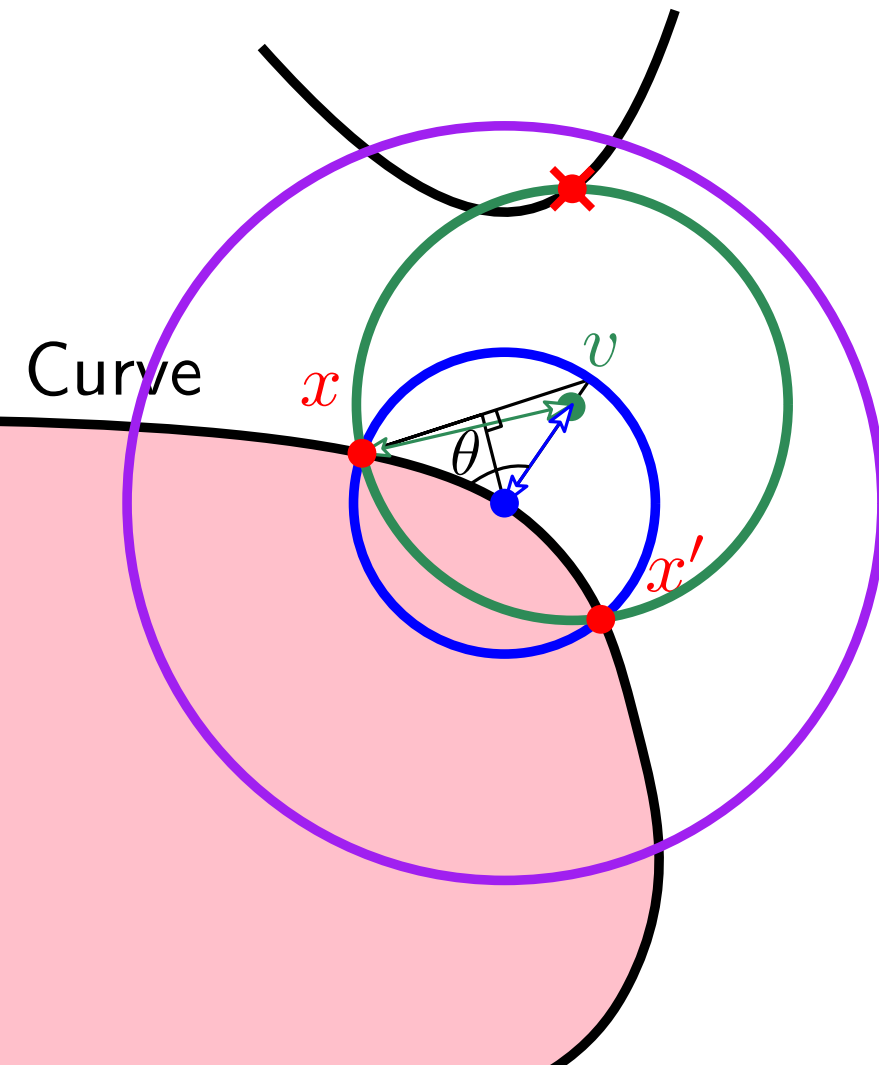
By contradiction assume $v \in \odot$
 \odot intersects another cc of curve
 (by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq \beta = \pi - \frac{\pi - \alpha}{2}$$

$$\leq \frac{\pi}{2} + \arcsin \frac{r}{2}$$

$$r = 2 \sin \frac{\alpha}{2}$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

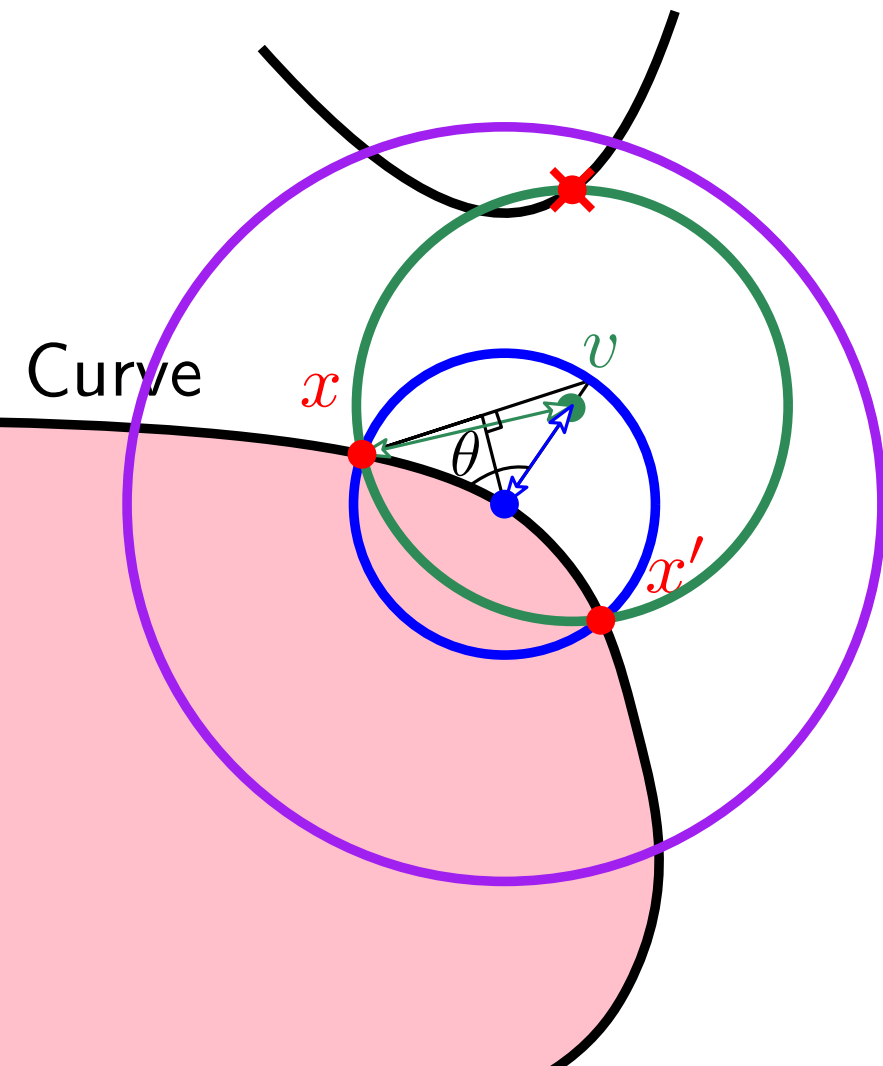
By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq \frac{\pi}{2} + \arcsin \frac{r}{2}$$

$$\| \bullet \times \| \leq \| \bullet \bullet \| + \| \bullet \times \|$$



Reconstruction

Crust 2D $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

Theorem: $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

x, x' two neighboring points on Curve

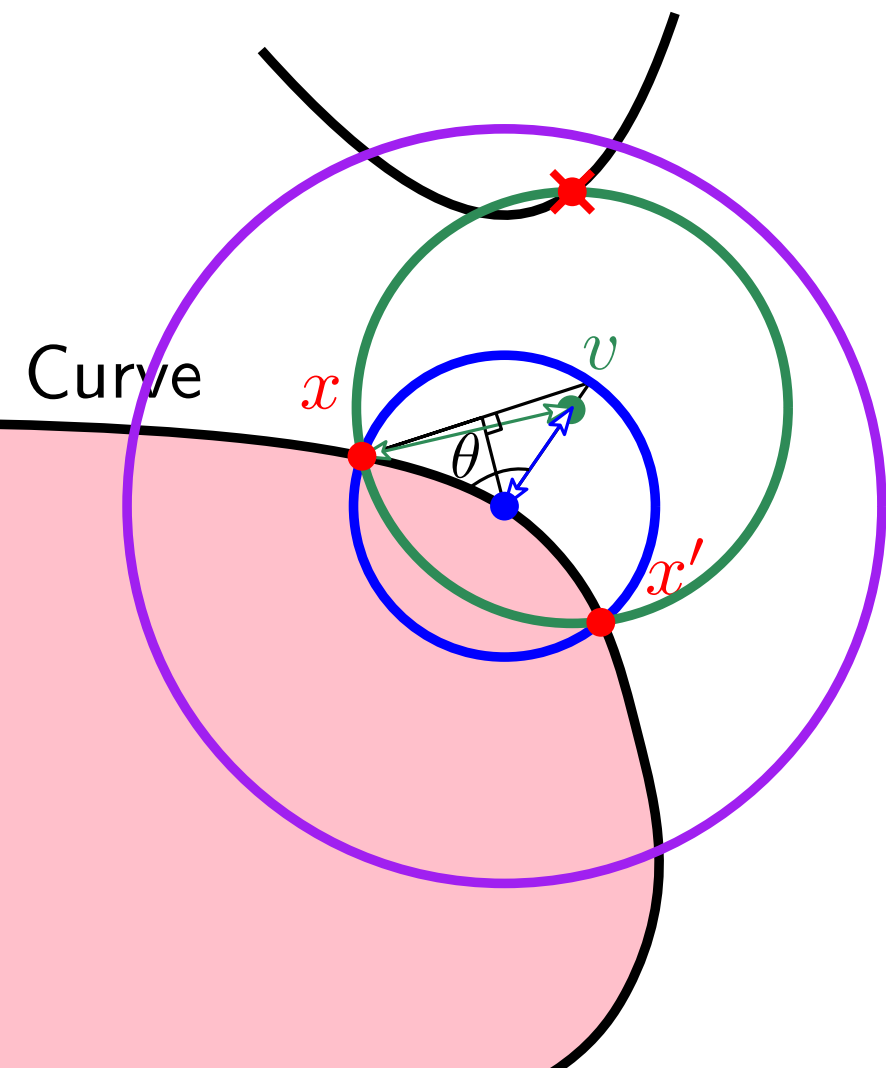
Circle thru x and x' centered on Curve

By contradiction assume $v \in \odot$
 \odot intersects another cc of curve
 (by Lemma)

$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq \frac{\pi}{2} + \arcsin \frac{r}{2}$$

$$\begin{aligned} \|\bullet \times\| &\leq \|\bullet \bullet\| + \|\bullet \times\| \\ &\leq r + 2r \sin \left(\frac{\pi}{4} + \frac{1}{2} \arcsin \frac{r}{2} \right) \end{aligned}$$



Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust

x, x' two neighboring points on Curve

Circle thru x and x' centered on Curve

By contradiction assume $v \in$ 

 intersects another cc of curve
(by Lemma)

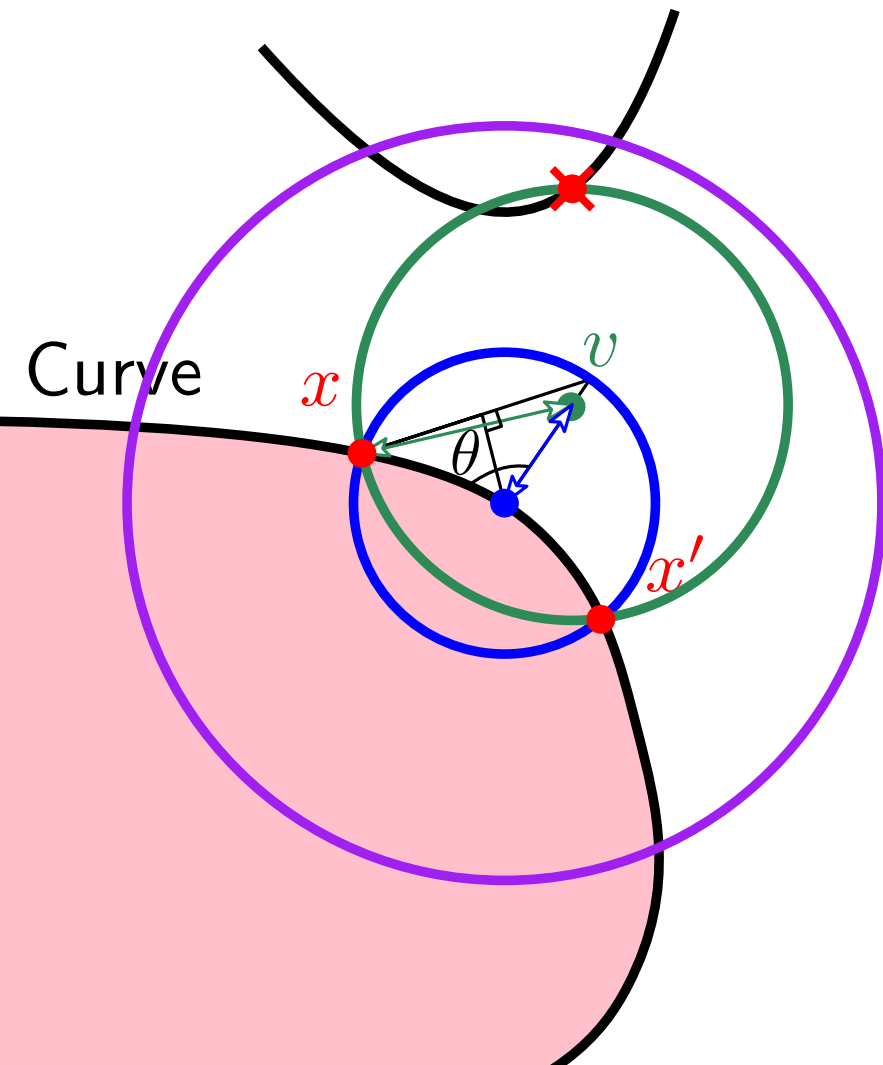
$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq \frac{\pi}{2} + \arcsin \frac{r}{2}$$

$$\| \bullet \times \| \leq \| \bullet \bullet \| + \| \bullet \times \|$$

$$\leq r + 2r \sin \left(\frac{\pi}{4} + \frac{1}{2} \arcsin \frac{r}{2} \right)$$

if $\| \bullet \times \| \leq \text{fs} = 1$ contradiction is reached

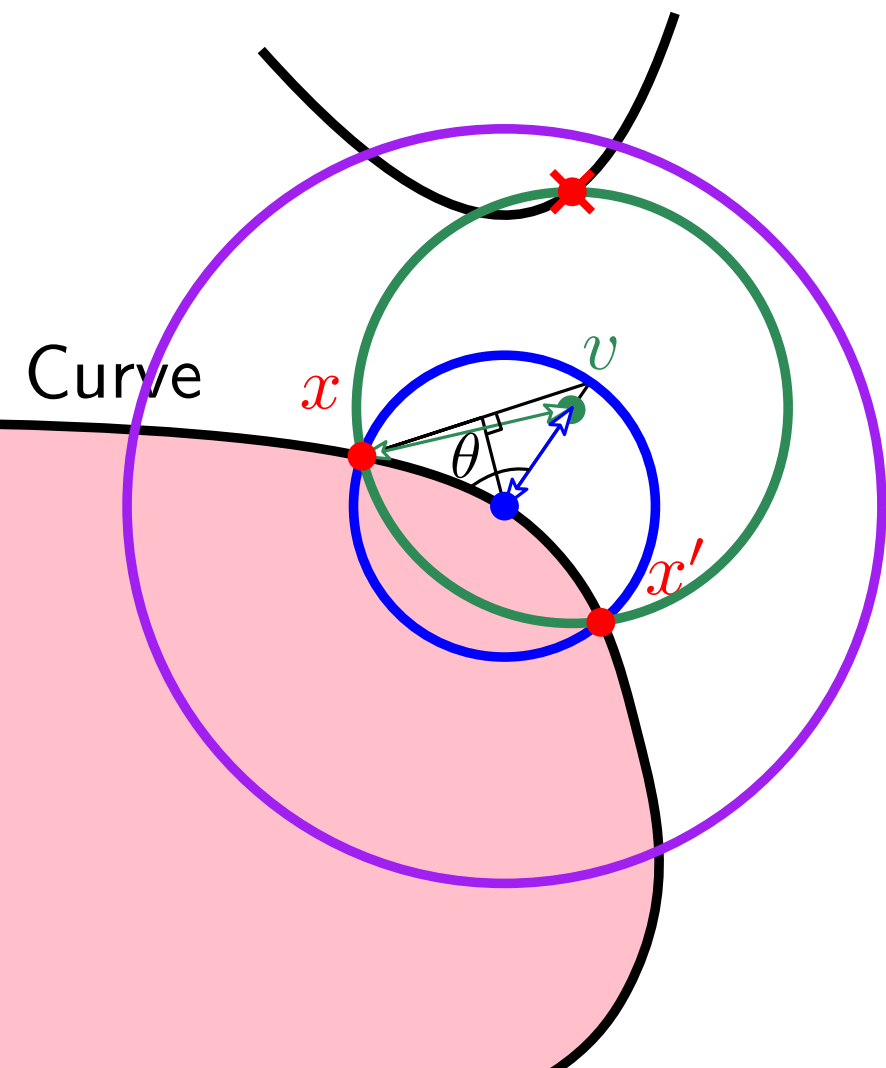


Reconstruction

Crust 2D

0.4 sample \Rightarrow wanted result \subset crust

Theorem: 0.4 sample \Rightarrow wanted result \subset crust



x, x' two neighbors

Circle thru x

By contradiction

Two circles intersect

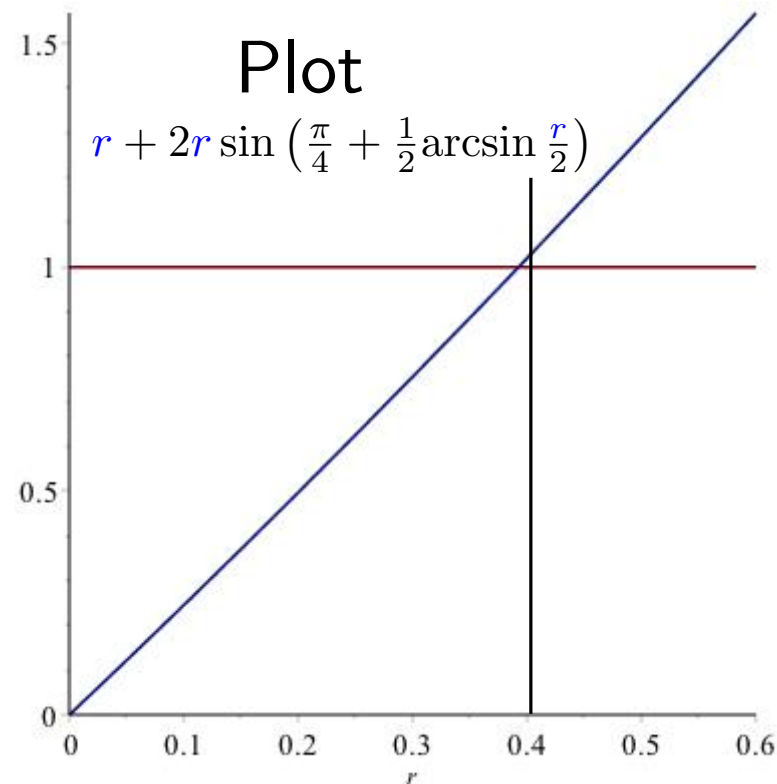
$$R \leq 2r \sin \frac{\theta}{2}$$

$$\theta \leq \frac{\pi}{2} + \arcsin \frac{r}{2R}$$

$$\| \bullet \times \| \leq \| \bullet \bullet \| + \| \bullet \times \|$$

$$\leq r + 2r \sin \left(\frac{\pi}{4} + \frac{1}{2} \arcsin \frac{r}{2} \right)$$

if $\| \bullet \times \| \leq 1$ contradiction is reached



Reconstruction

Crust 2D $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

Theorem: $0.4 \text{ sample} \Rightarrow \text{wanted result} \subset \text{crust}$

Reconstruction

Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

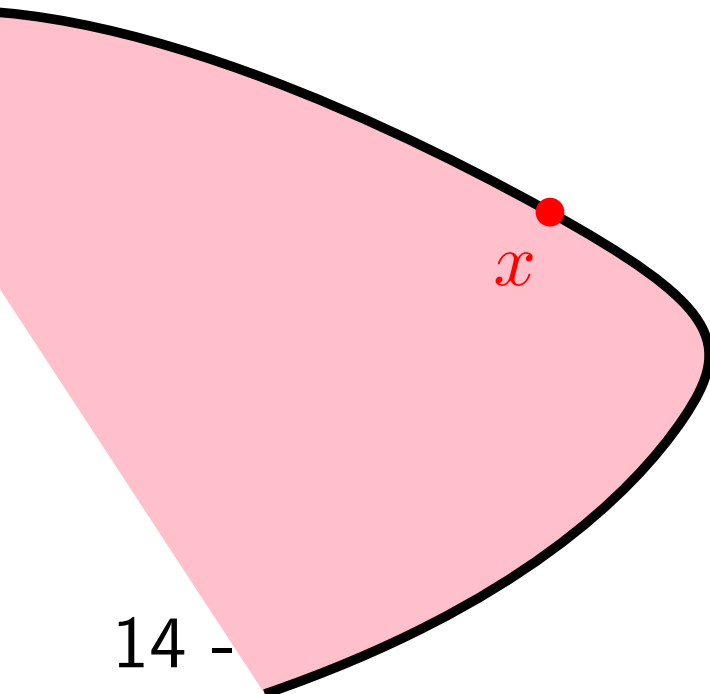
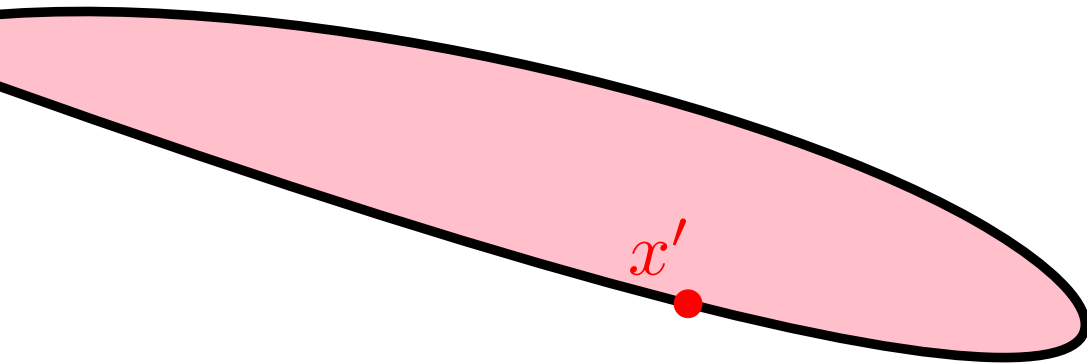
Theorem: 0.25 sample \Rightarrow crust \subset wanted result

Reconstruction

Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

Theorem: 0.25 sample \Rightarrow crust \subset wanted result

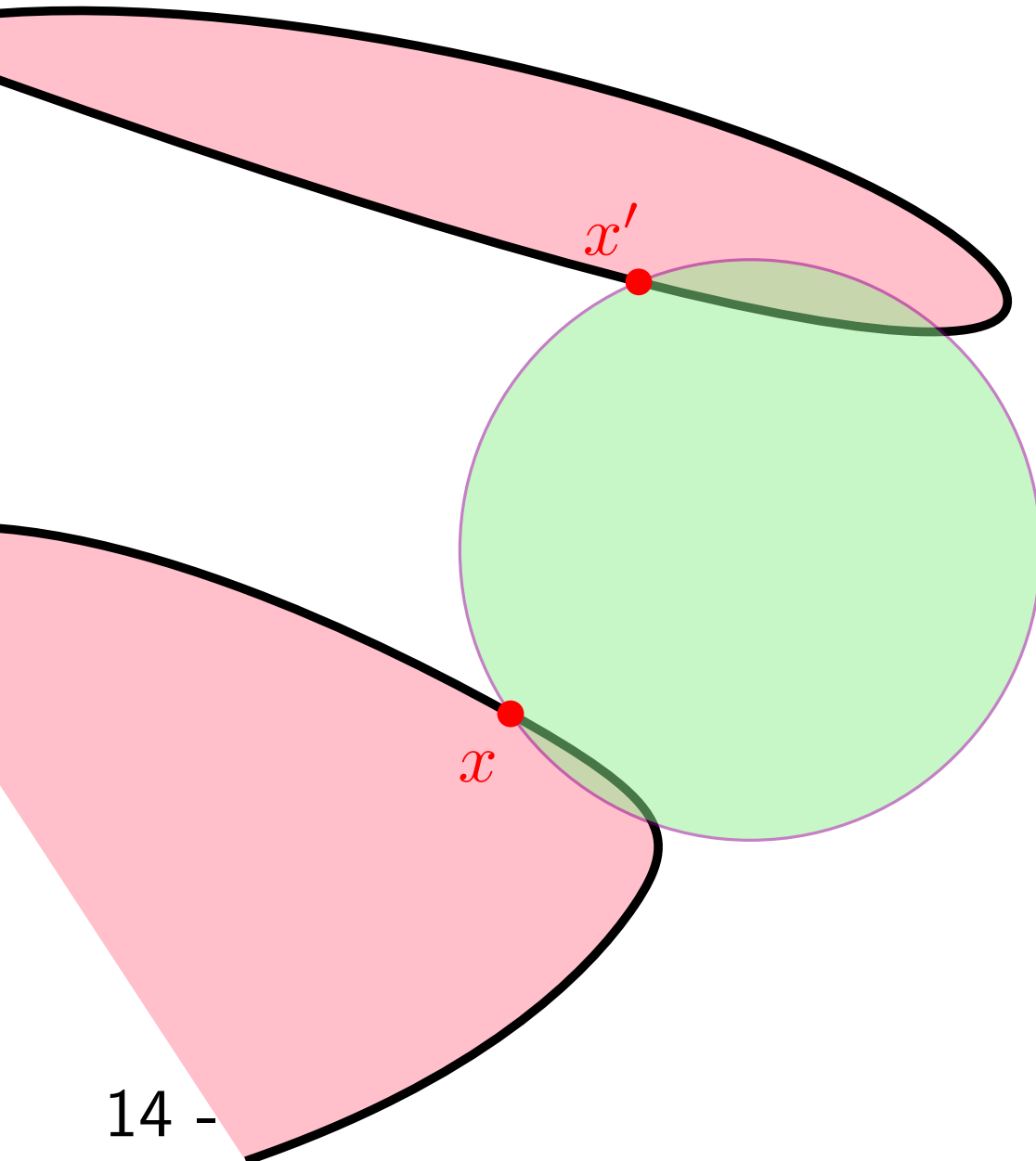


Reconstruction

Crust 2D $0.25 \text{ sample} \Rightarrow \text{crust} \subset \text{wanted result}$

Theorem: $0.25 \text{ sample} \Rightarrow \text{crust} \subset \text{wanted result}$

Assume empty circle



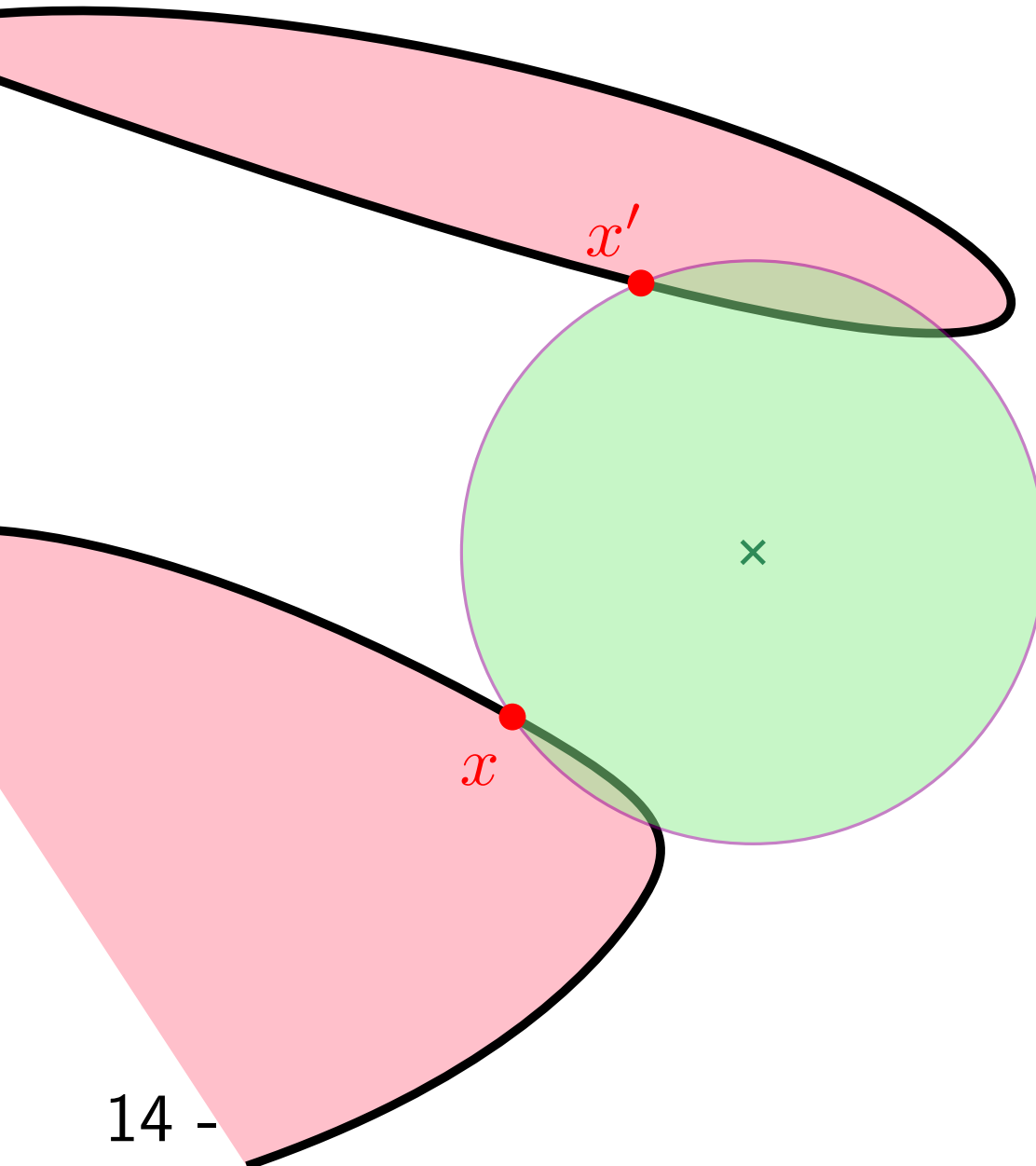
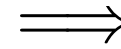
Reconstruction

Crust 2D $0.25 \text{ sample} \Rightarrow \text{crust} \subset \text{wanted result}$

Theorem: $0.25 \text{ sample} \Rightarrow \text{crust} \subset \text{wanted result}$

Assume empty circle

No Voronoi vertices there



Reconstruction

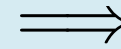
Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

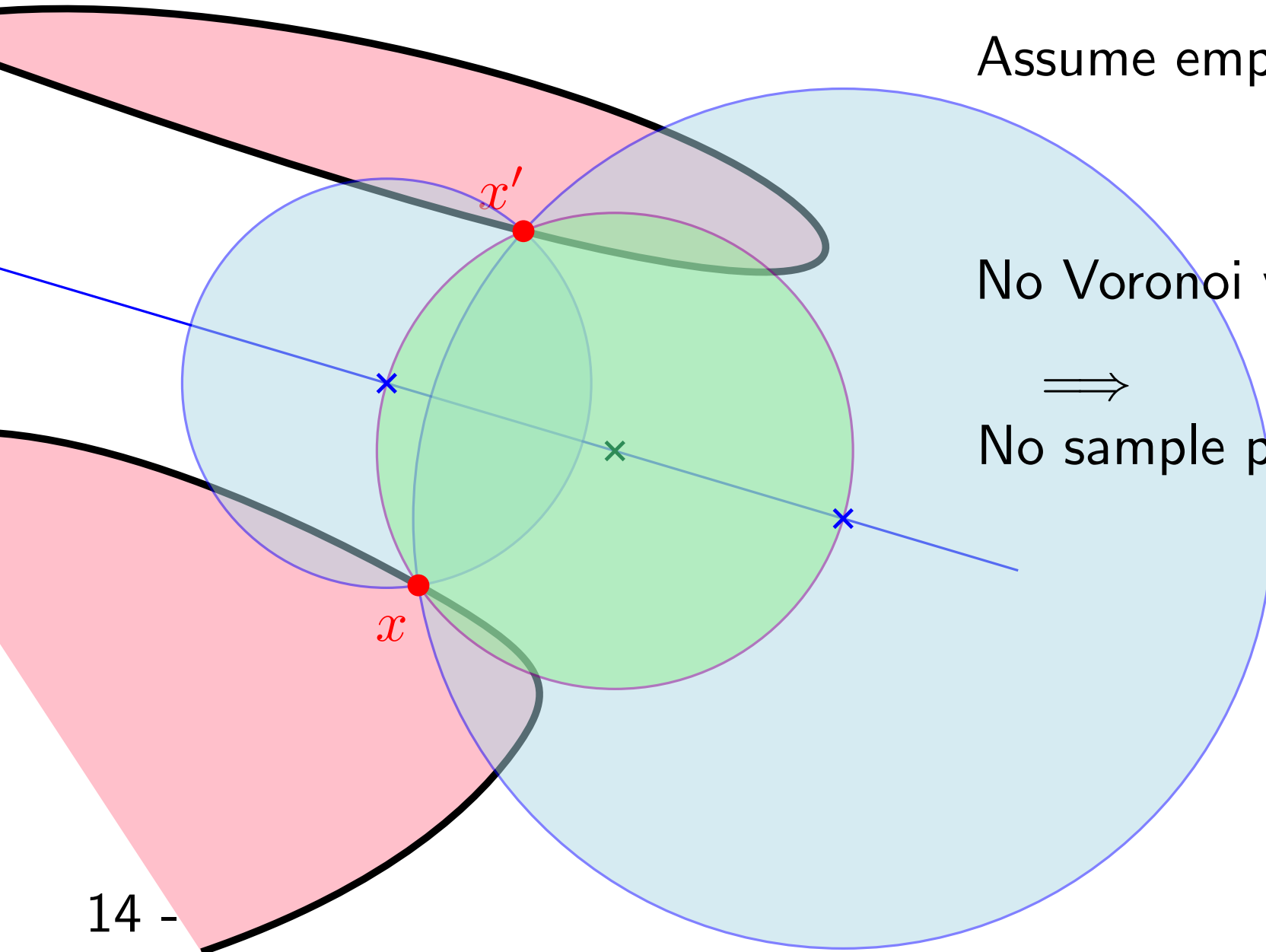
Theorem: 0.25 sample \Rightarrow crust \subset wanted result

Assume empty circle

No Voronoi vertices there



No sample points there



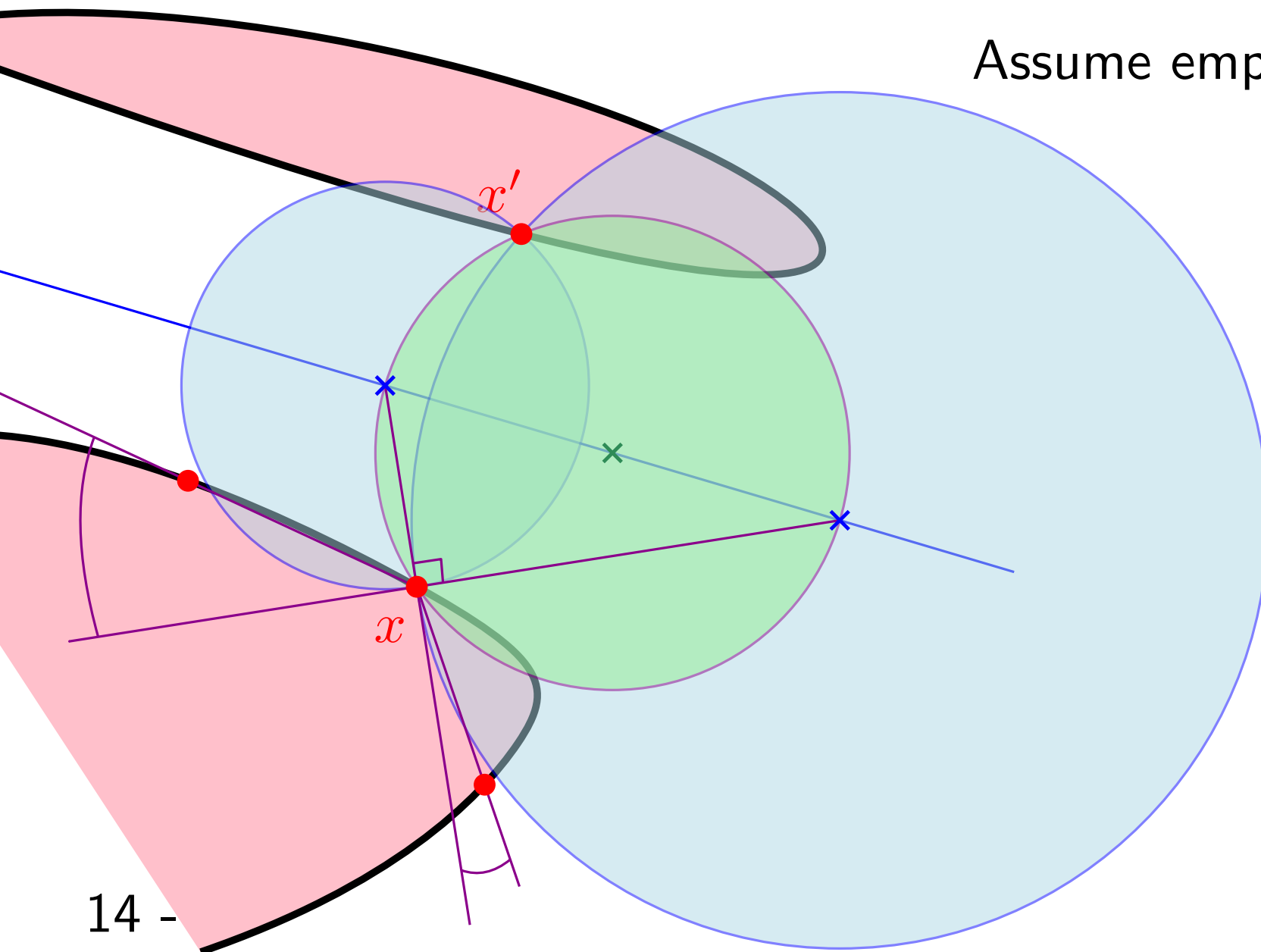
Reconstruction

Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

Theorem: 0.25 sample \Rightarrow crust \subset wanted result

Assume empty circle

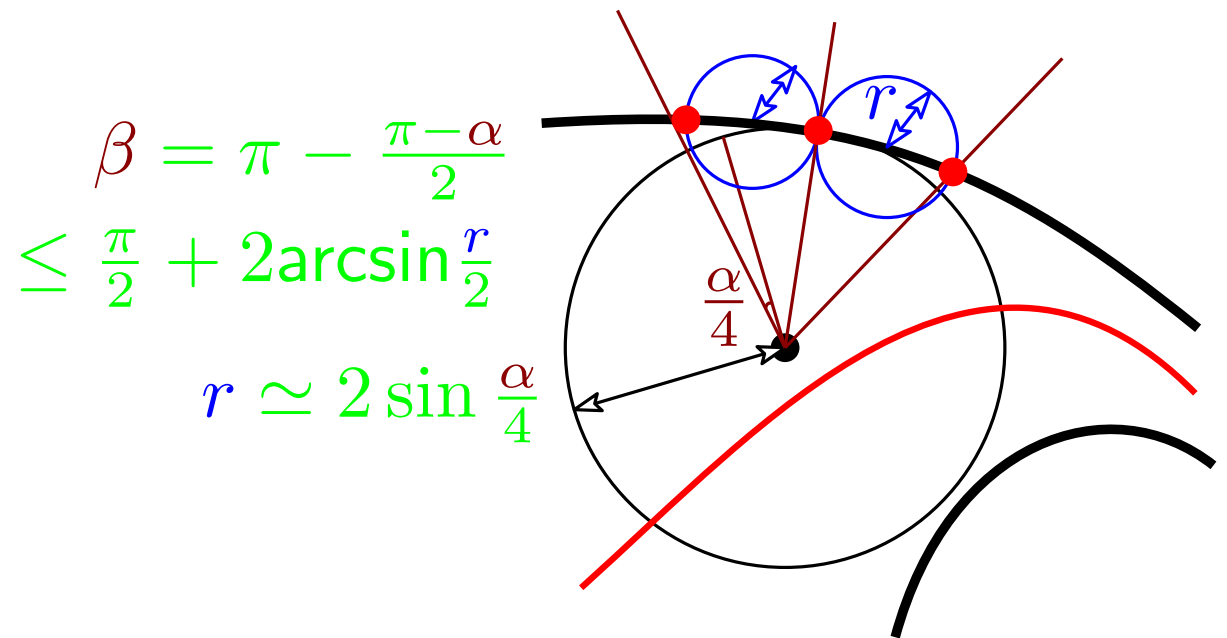


Reconstruction

Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

Theorem: 0.25 sample \Rightarrow crust \subset wanted result

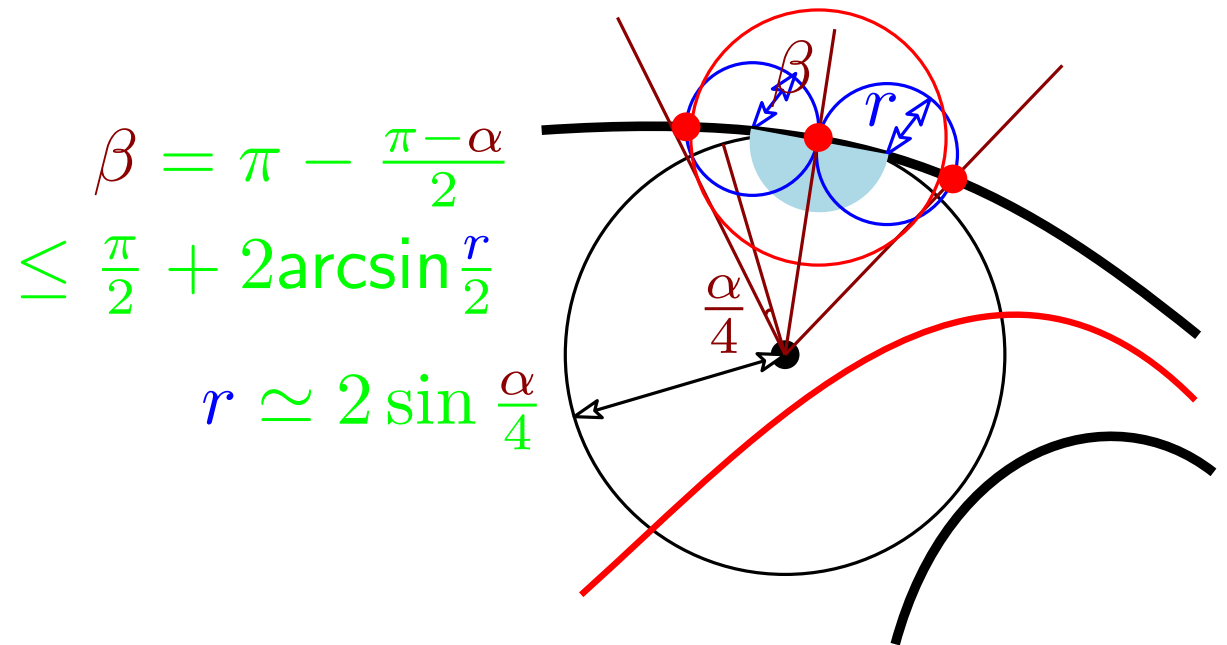


Reconstruction

Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

Theorem: 0.25 sample \Rightarrow crust \subset wanted result



Reconstruction

Crust 2D

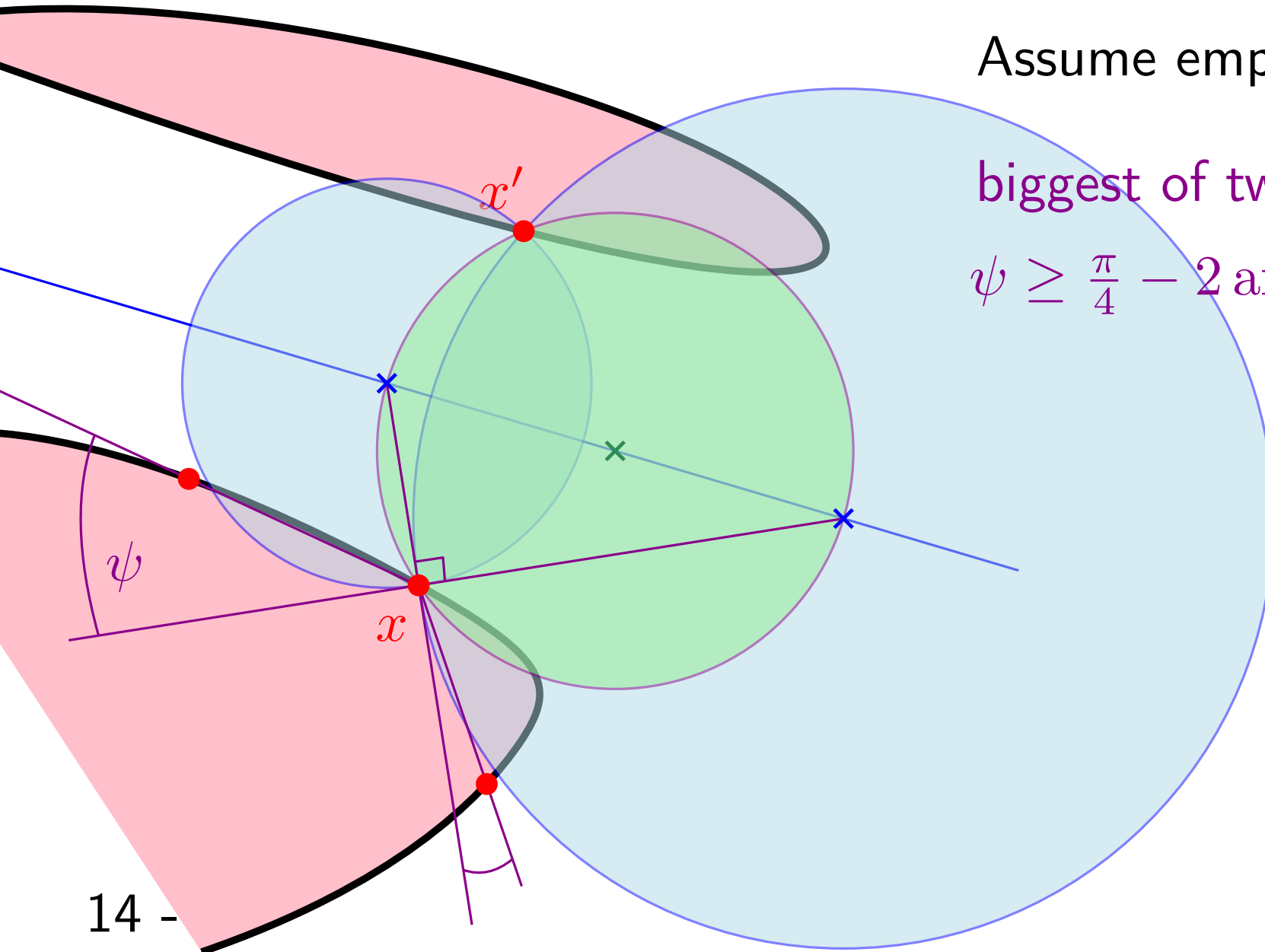
0.25 sample \Rightarrow crust \subset wanted result

Theorem: 0.25 sample \Rightarrow crust \subset wanted result

Assume empty circle

biggest of two angles

$$\psi \geq \frac{\pi}{4} - 2 \arcsin \frac{r}{2}$$



Reconstruction

Crust 2D

0.25 sample \Rightarrow crust \subset wanted result

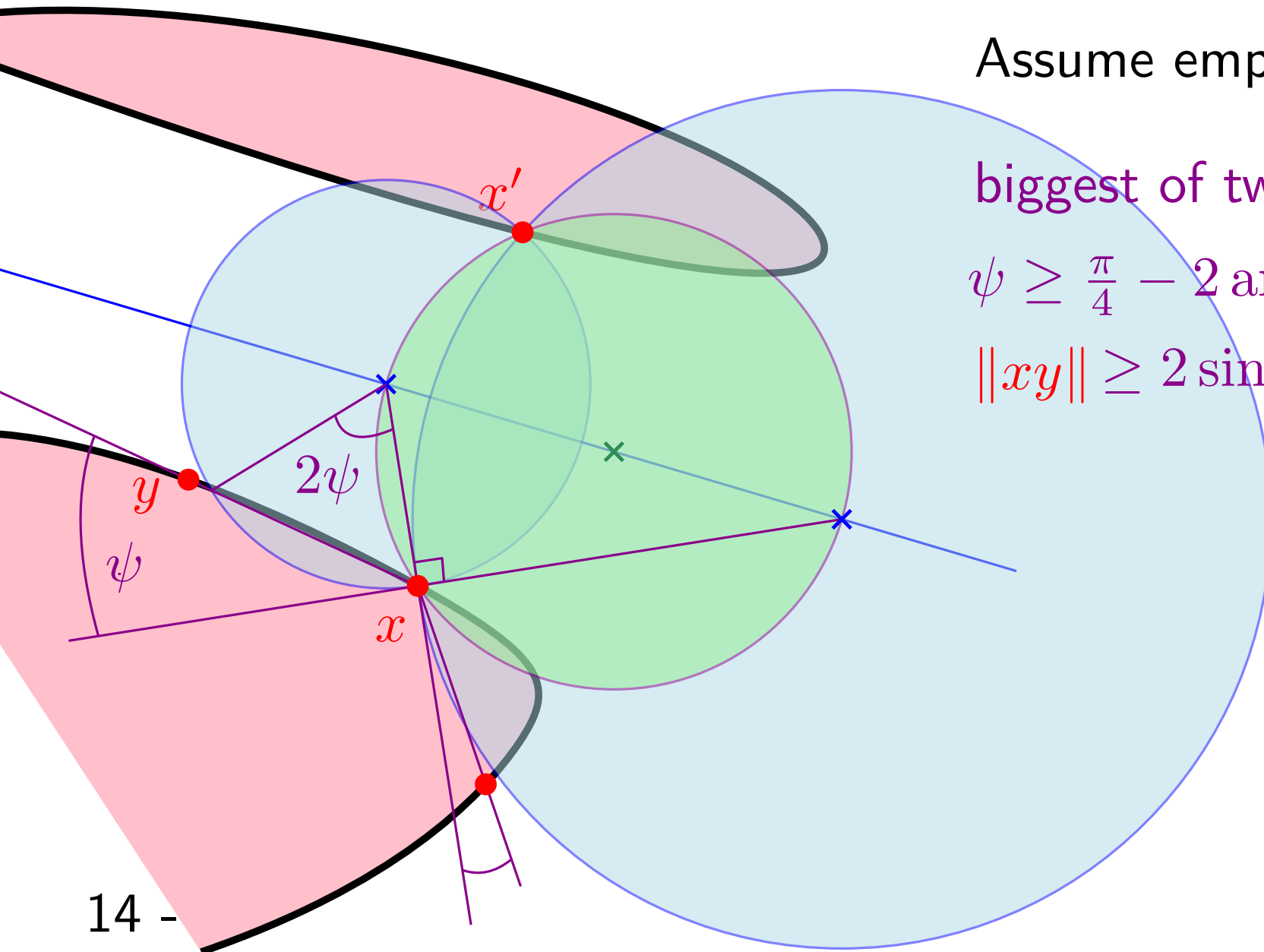
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$$\|xy\| \geq 2 \sin \psi$$



Reconstruction

Crust 2D

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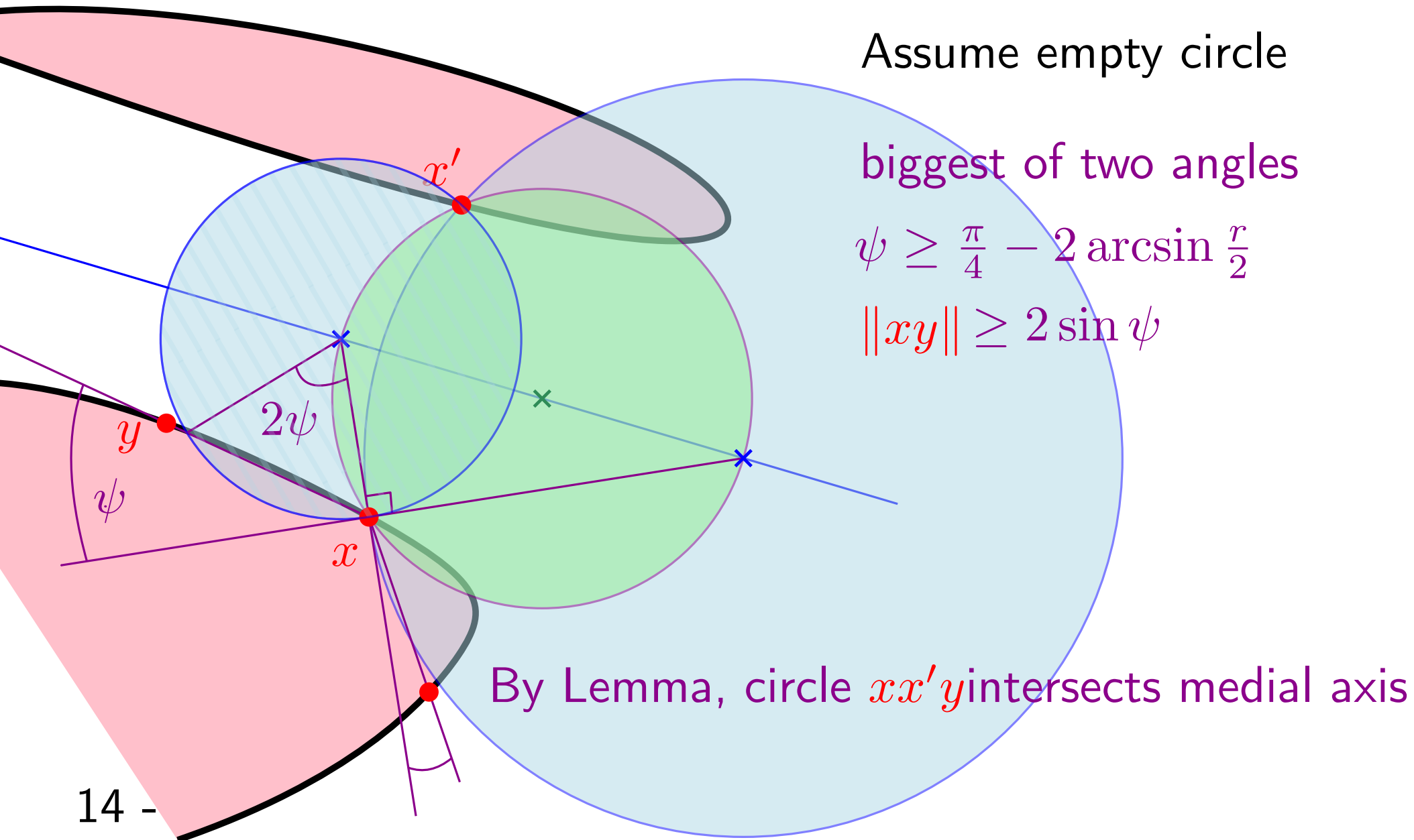
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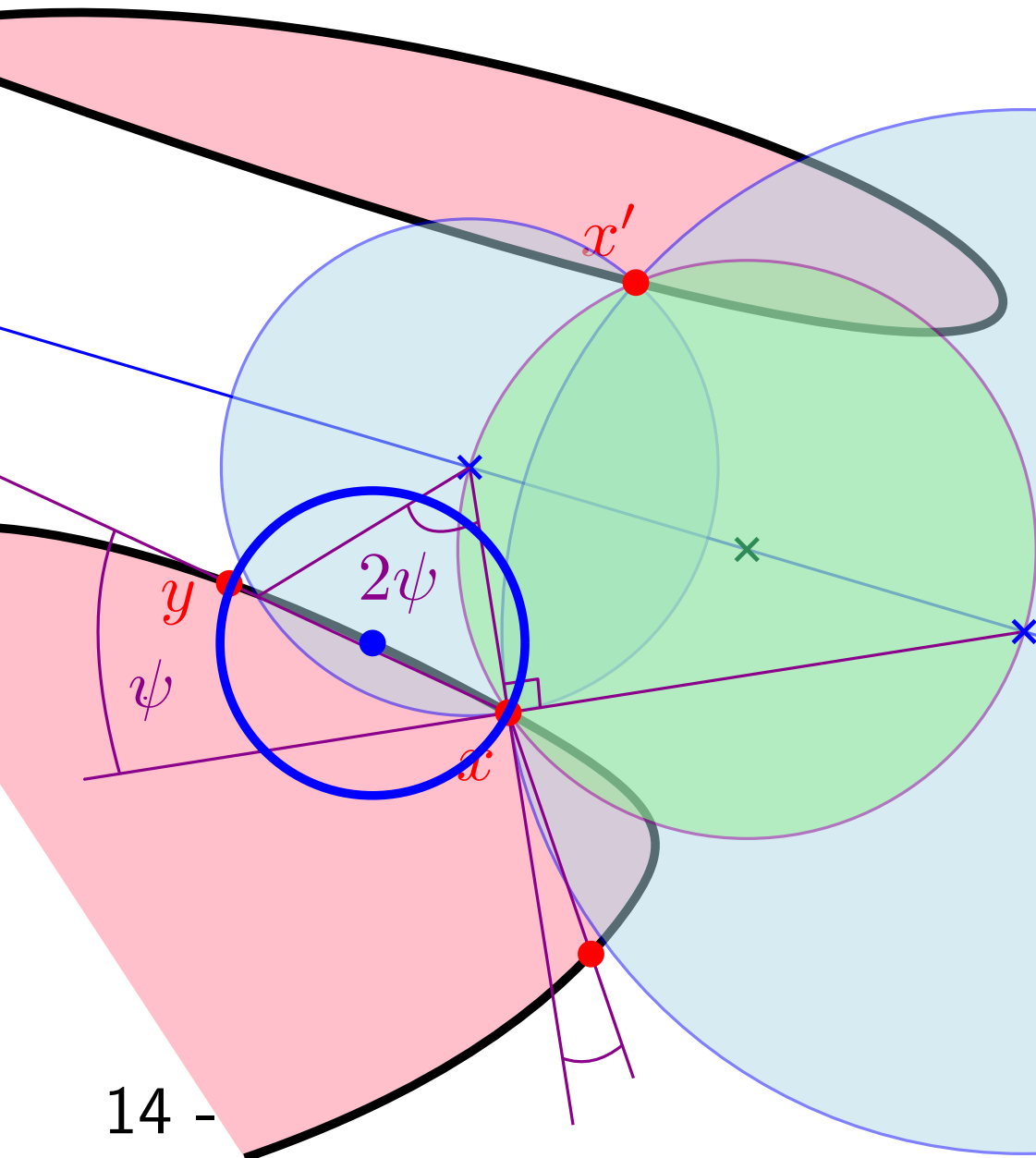
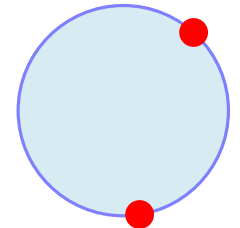
biggest of two angles

$$\psi \geq \frac{\pi}{4} - 2 \arcsin \frac{r}{2}$$

$$\|xy\| \geq 2 \sin \psi$$

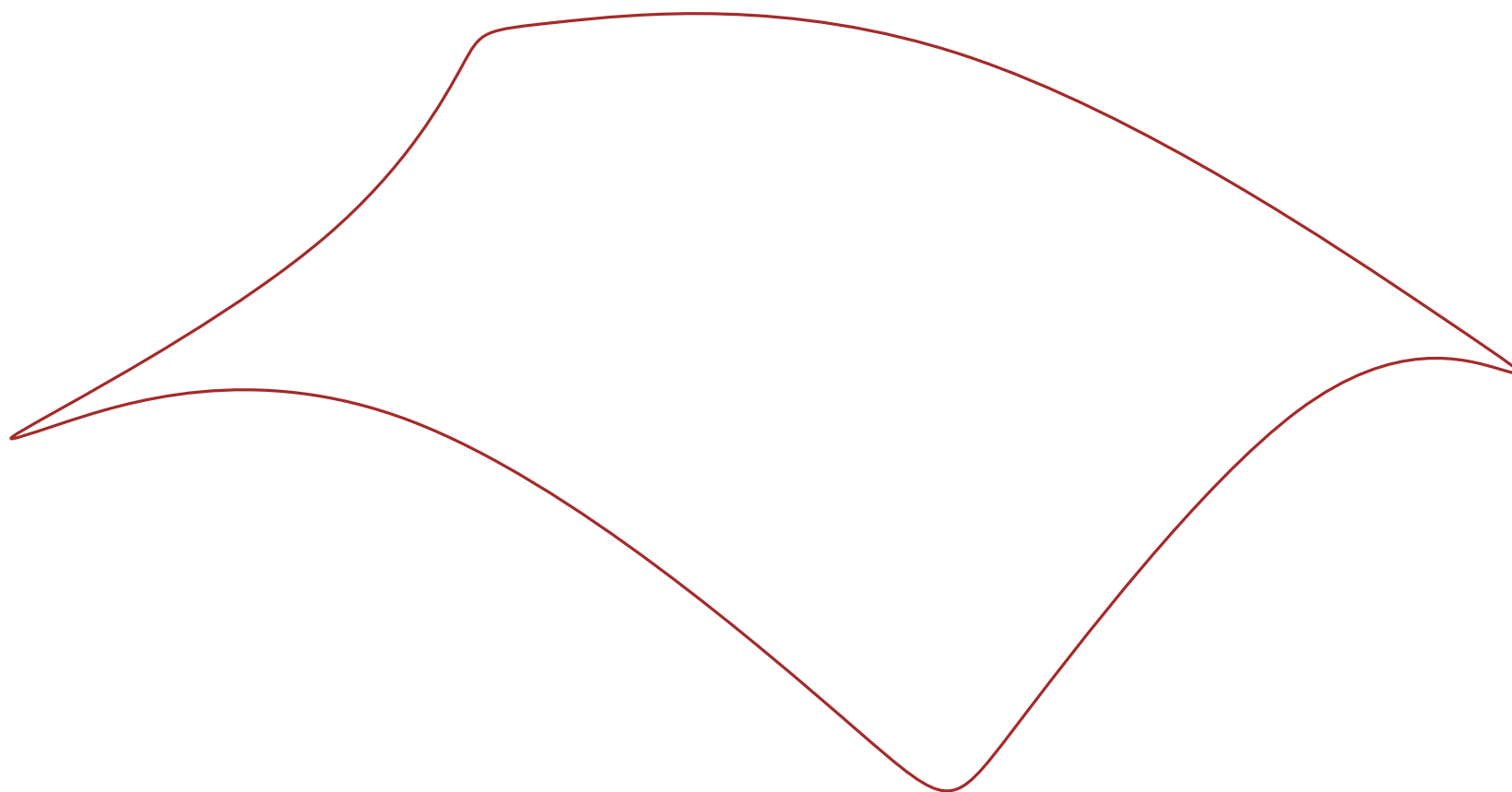
Compute ϵ to ensure that

$\frac{1}{\epsilon} \times$  encloses



Reconstruction

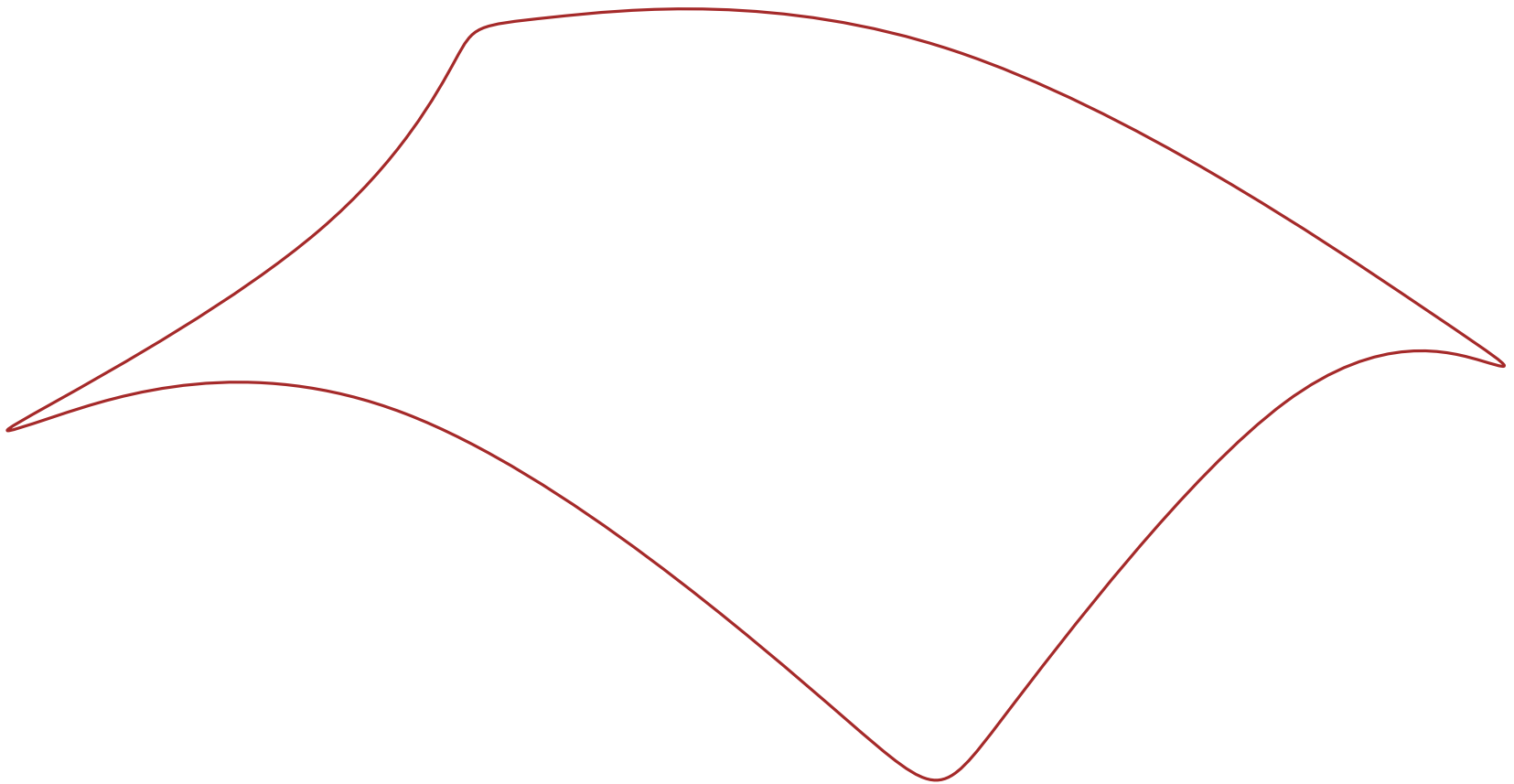
3D



Reconstruction

3D

Difficulty: sliver

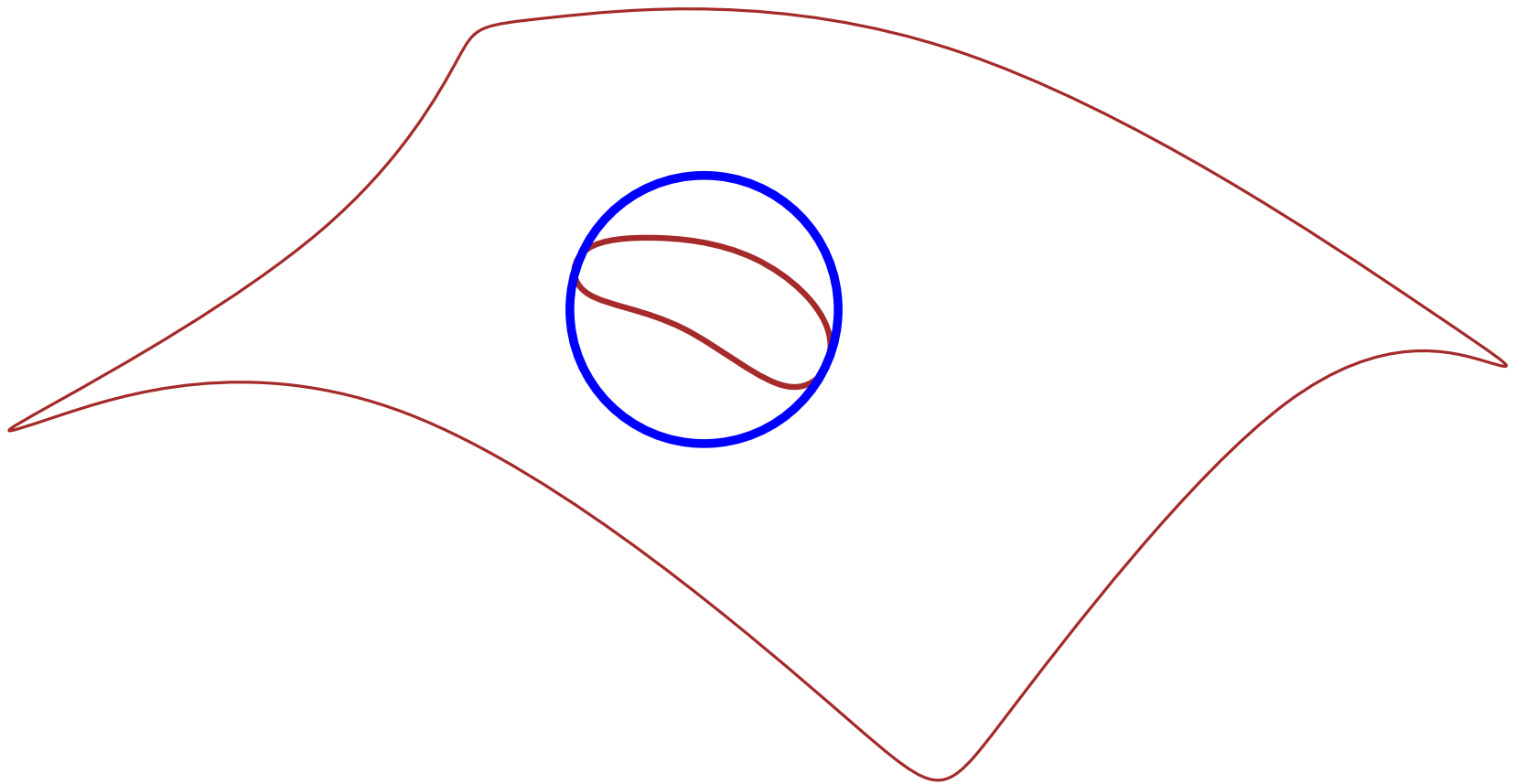


Reconstruction

3D

Difficulty: sliver

small sphere



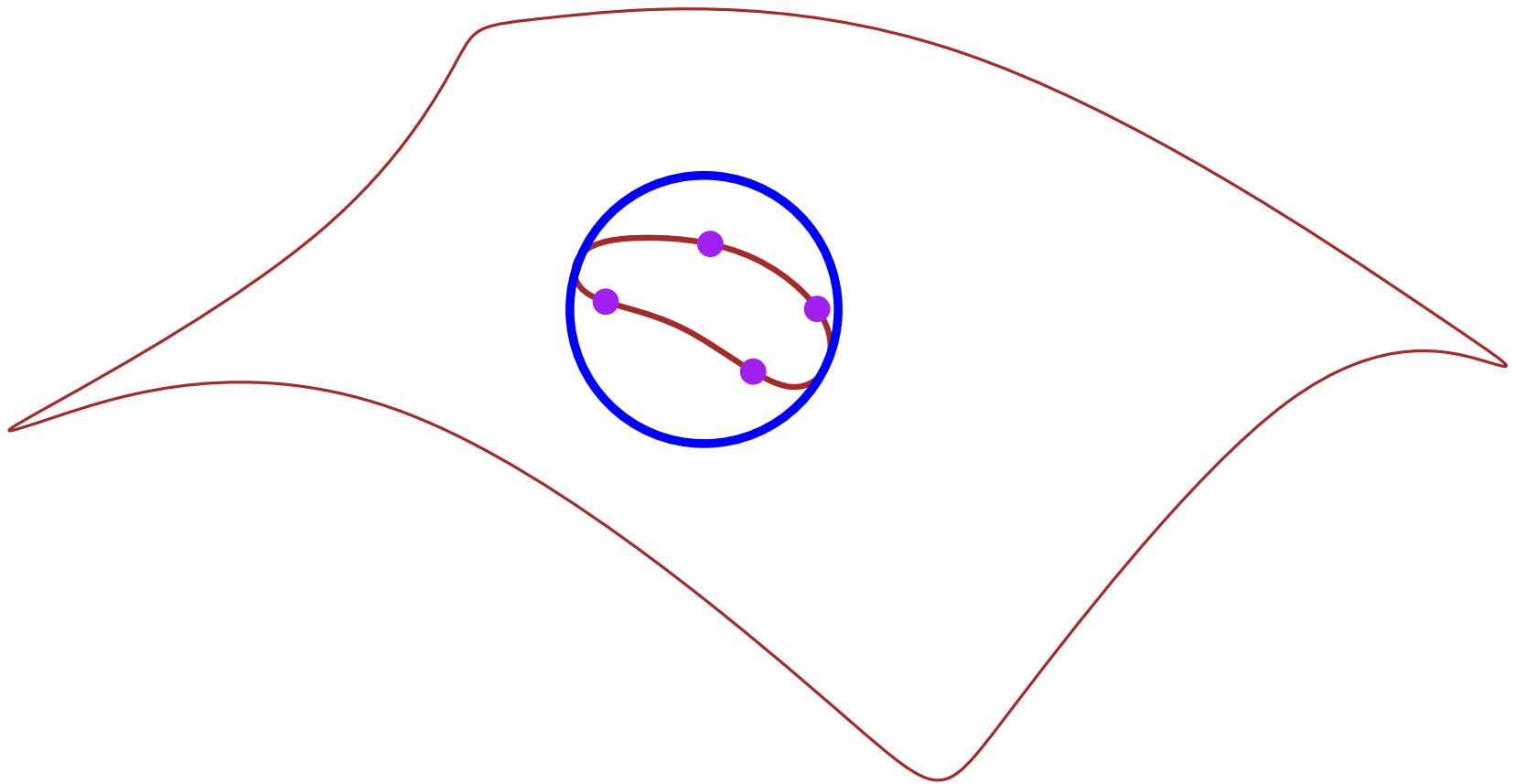
Reconstruction

3D

Difficulty: sliver

small sphere

four sample points



Reconstruction

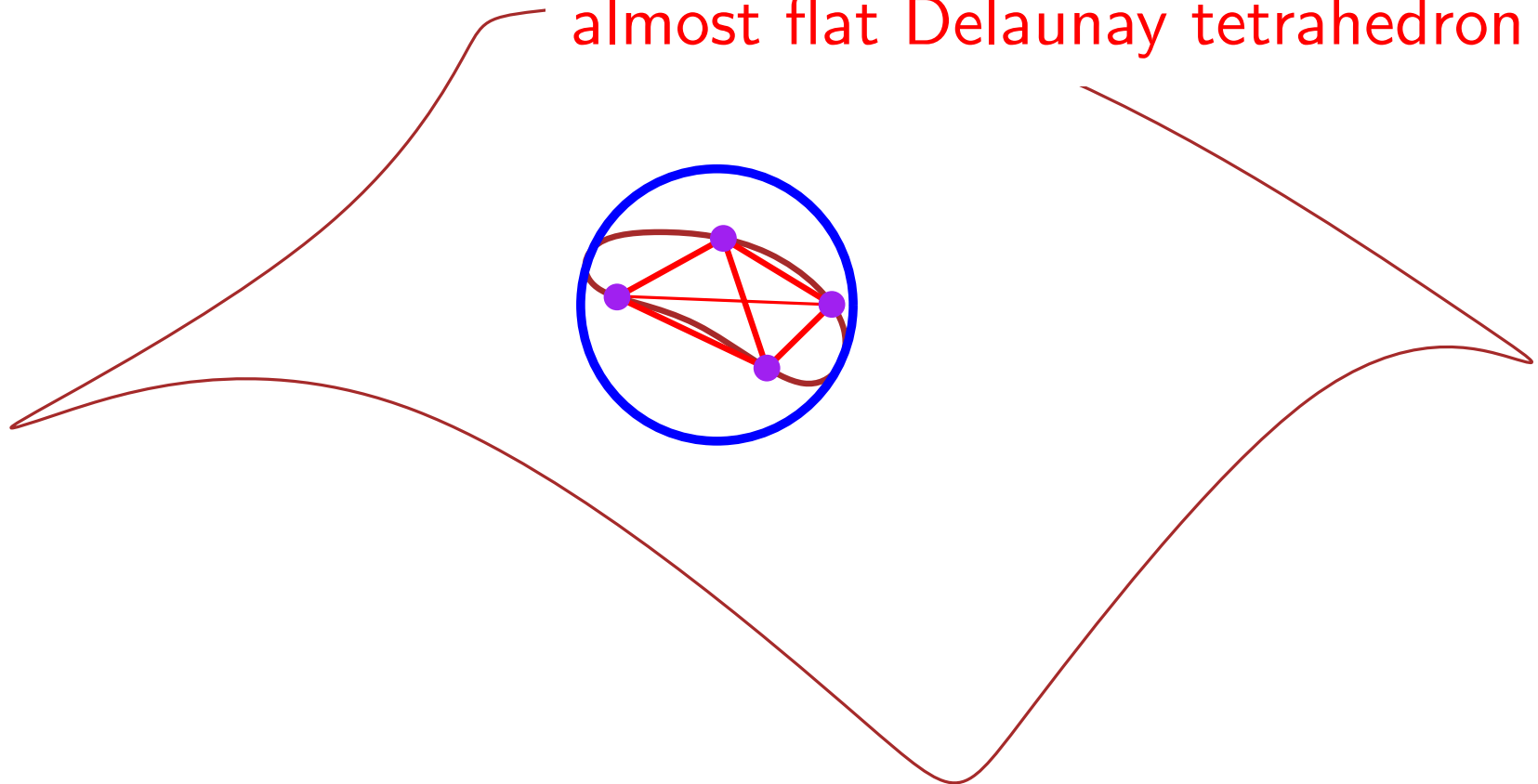
3D

Difficulty: sliver

small sphere

four sample points

almost flat Delaunay tetrahedron



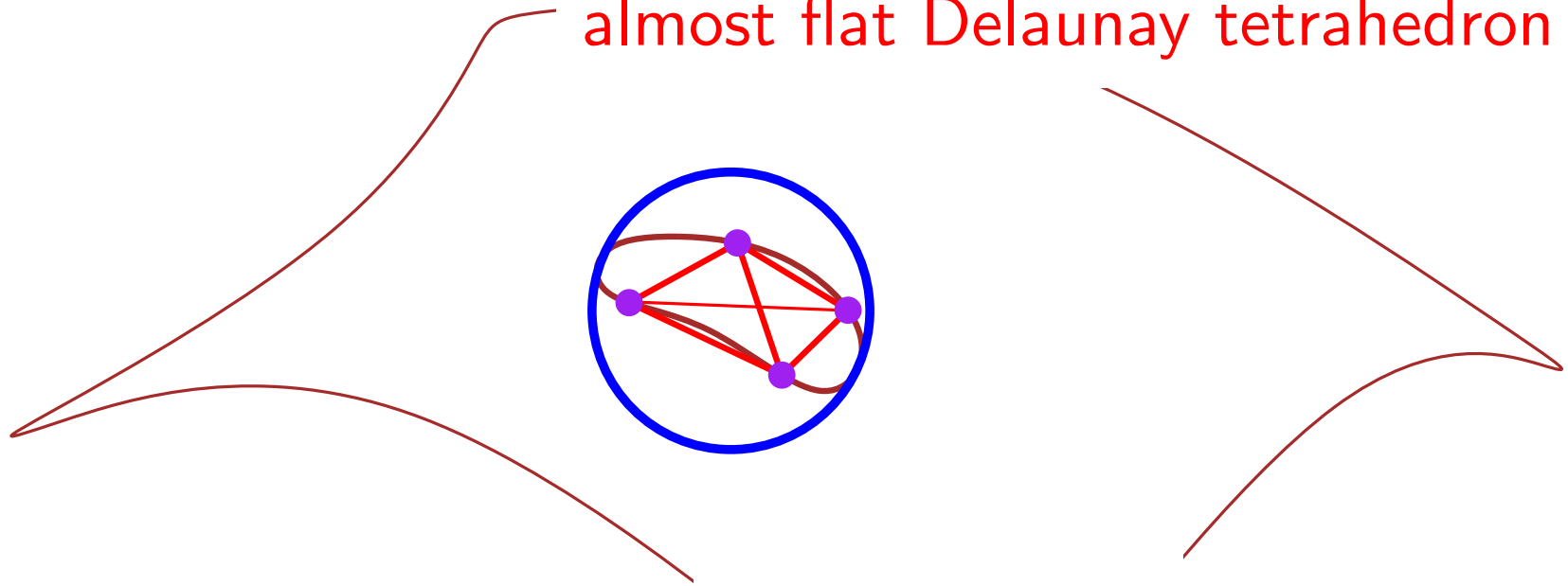
Reconstruction

3D

Difficulty: sliver

small sphere four sample points

almost flat Delaunay tetrahedron



Which triangle belongs to reconstruction ?

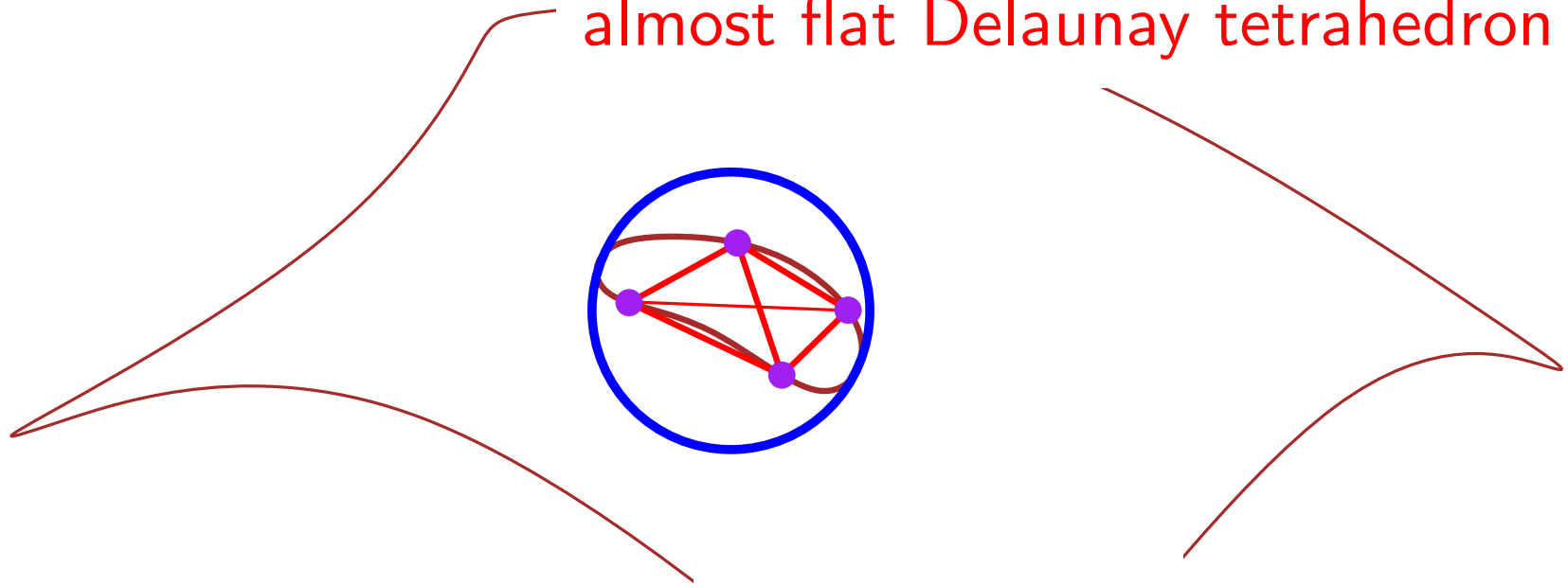
Reconstruction

3D

Difficulty: sliver

small sphere four sample points

almost flat Delaunay tetrahedron

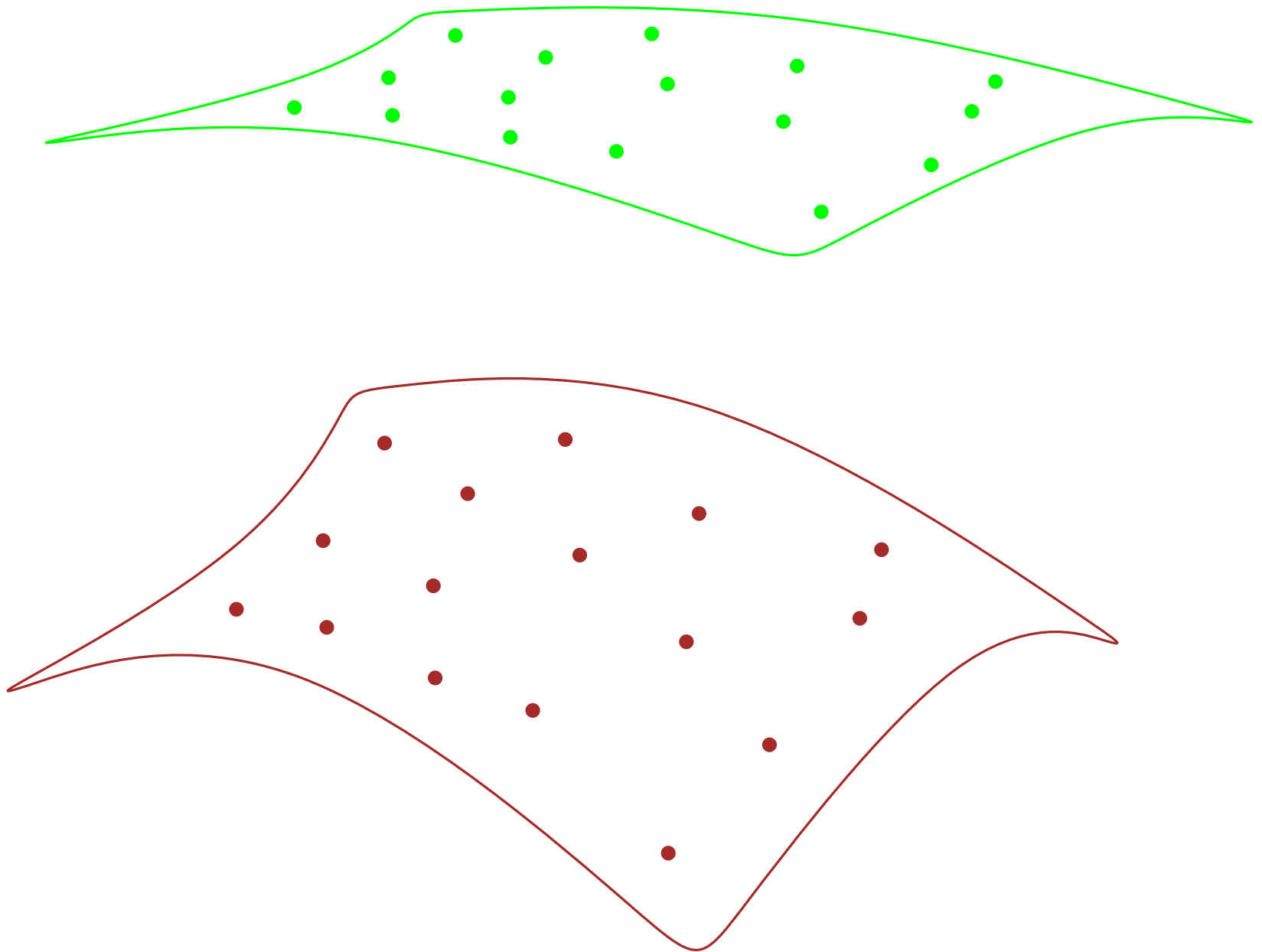


Which triangle belongs to reconstruction ?

Crust: Voronoi vertices may kill useful triangles

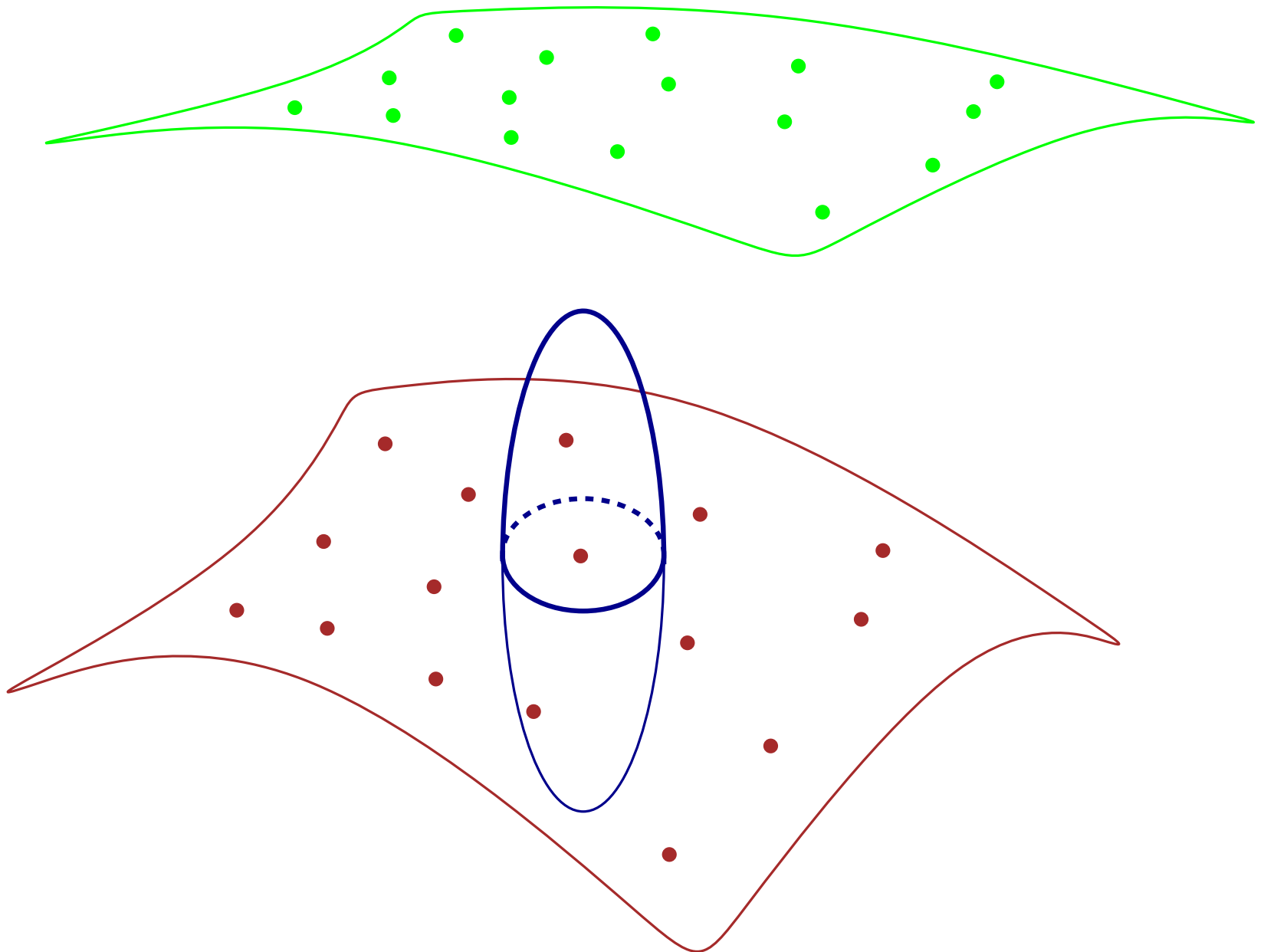
Reconstruction

3D



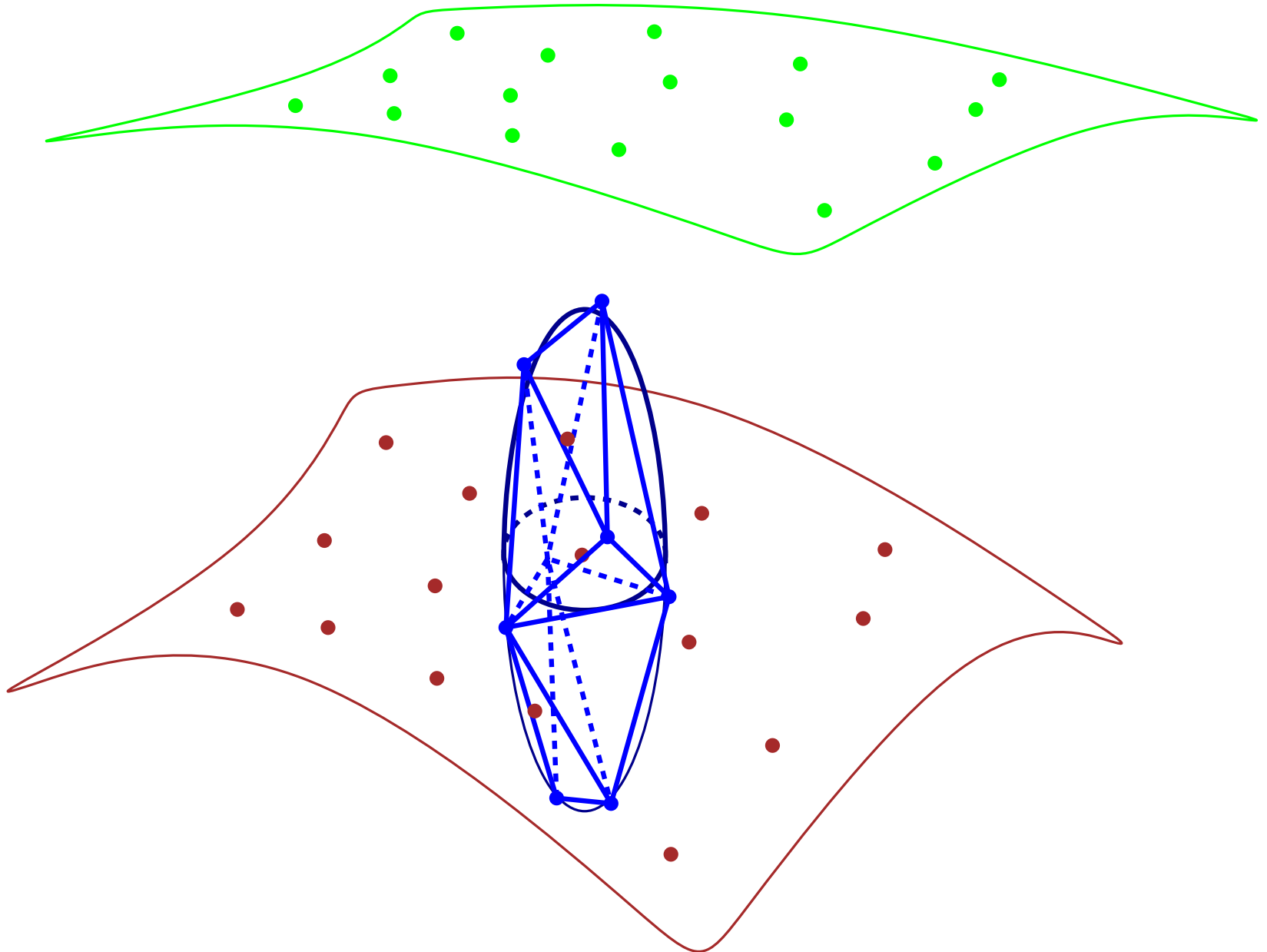
Reconstruction

3D



Reconstruction

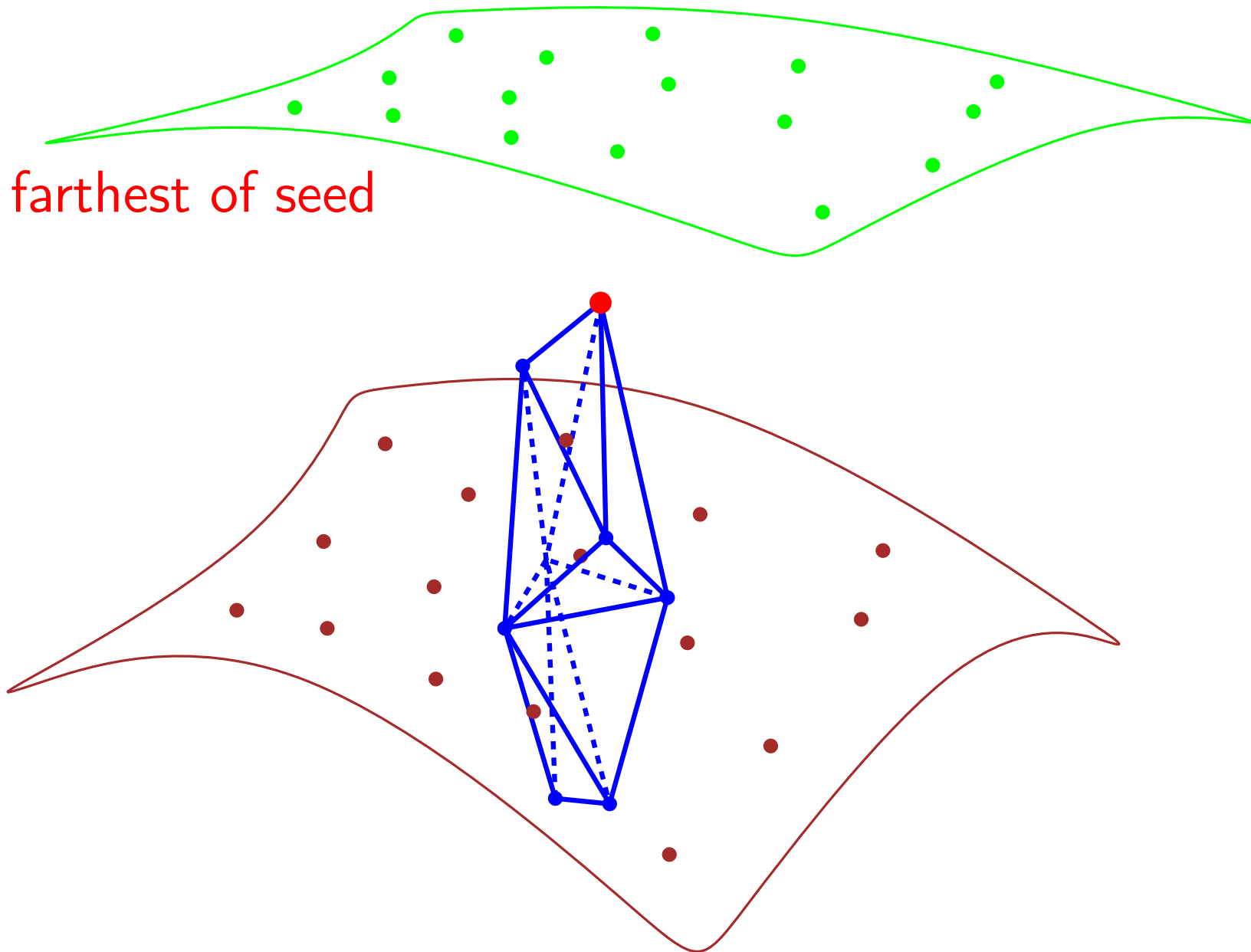
3D



Reconstruction

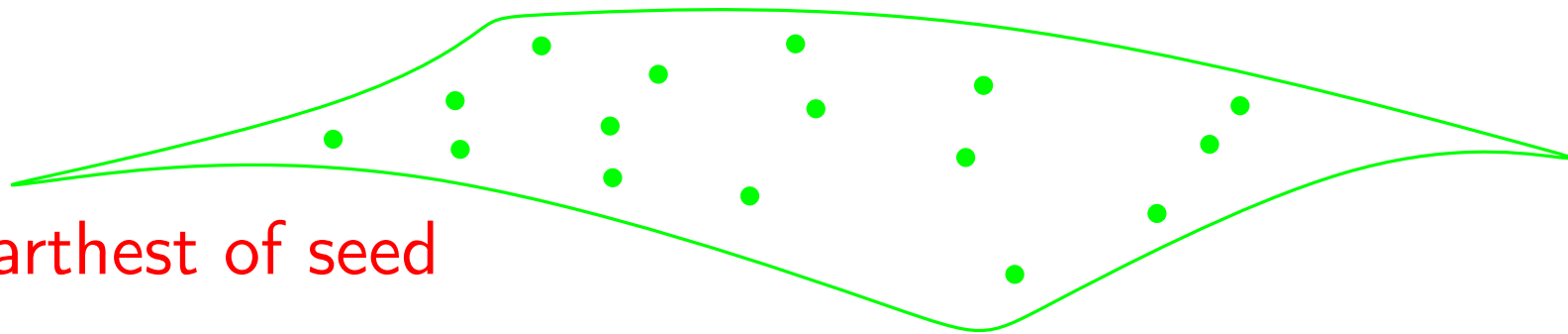
3D

Pole = farthest of seed



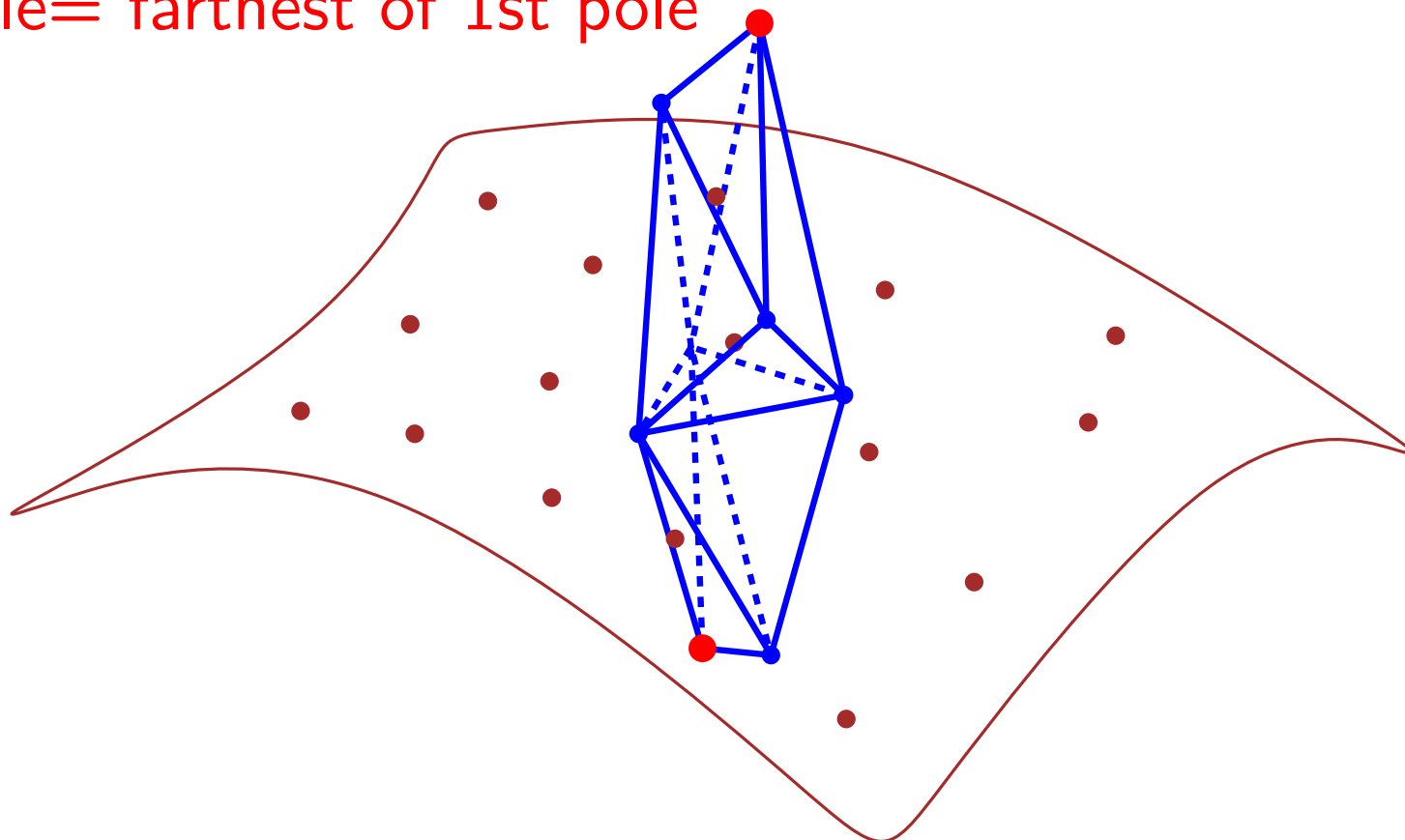
Reconstruction

3D



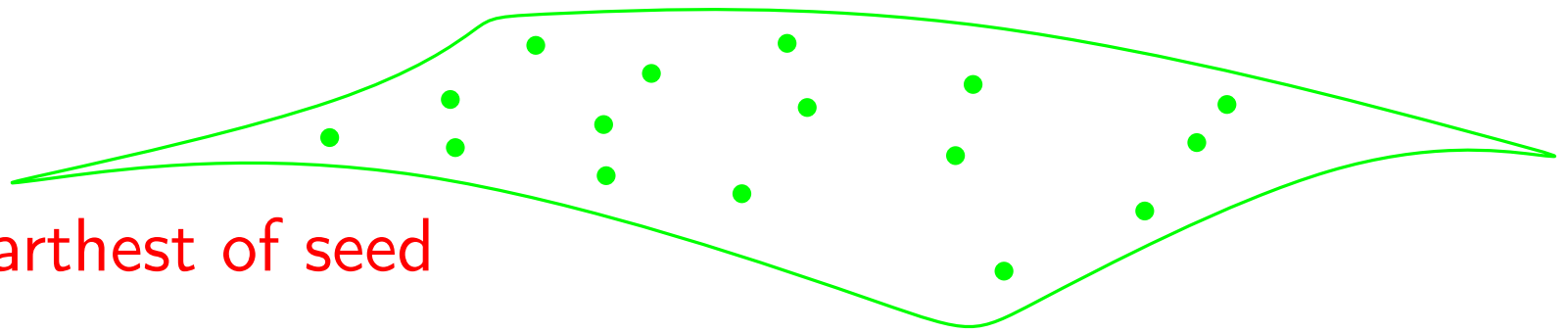
Pole = farthest of seed

2nd pole = farthest of 1st pole



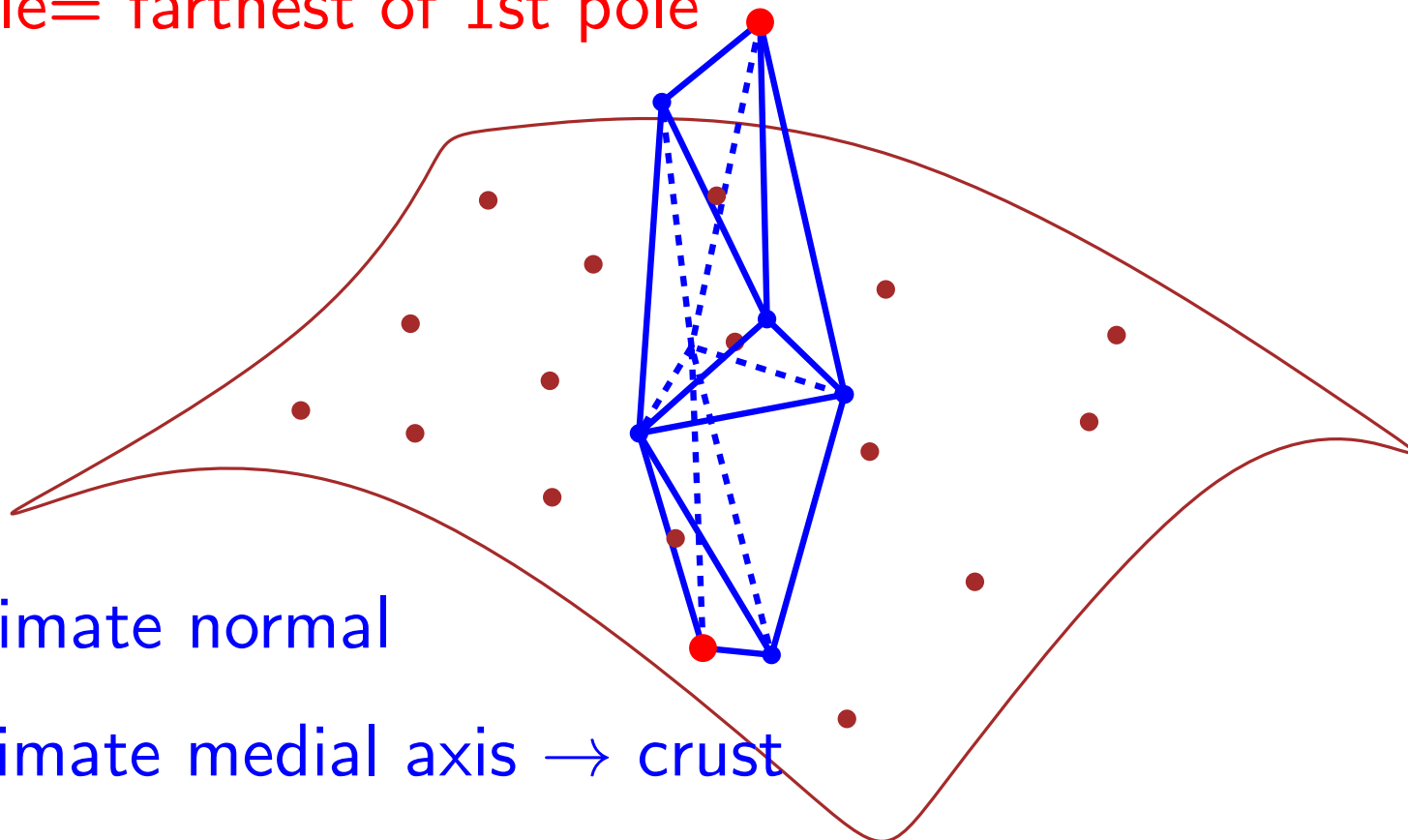
Reconstruction

3D



Pole = farthest of seed

2nd pole = farthest of 1st pole

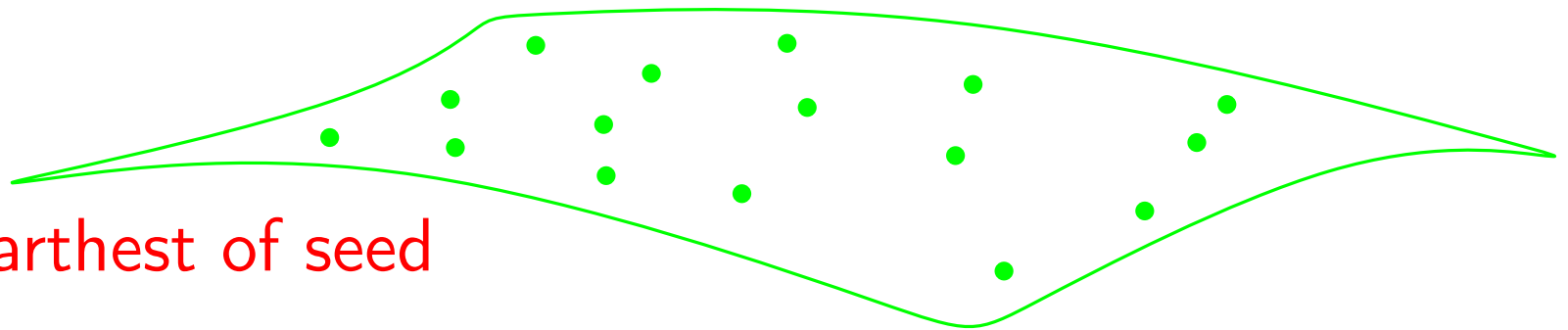


Approximate normal

Approximate medial axis → crust

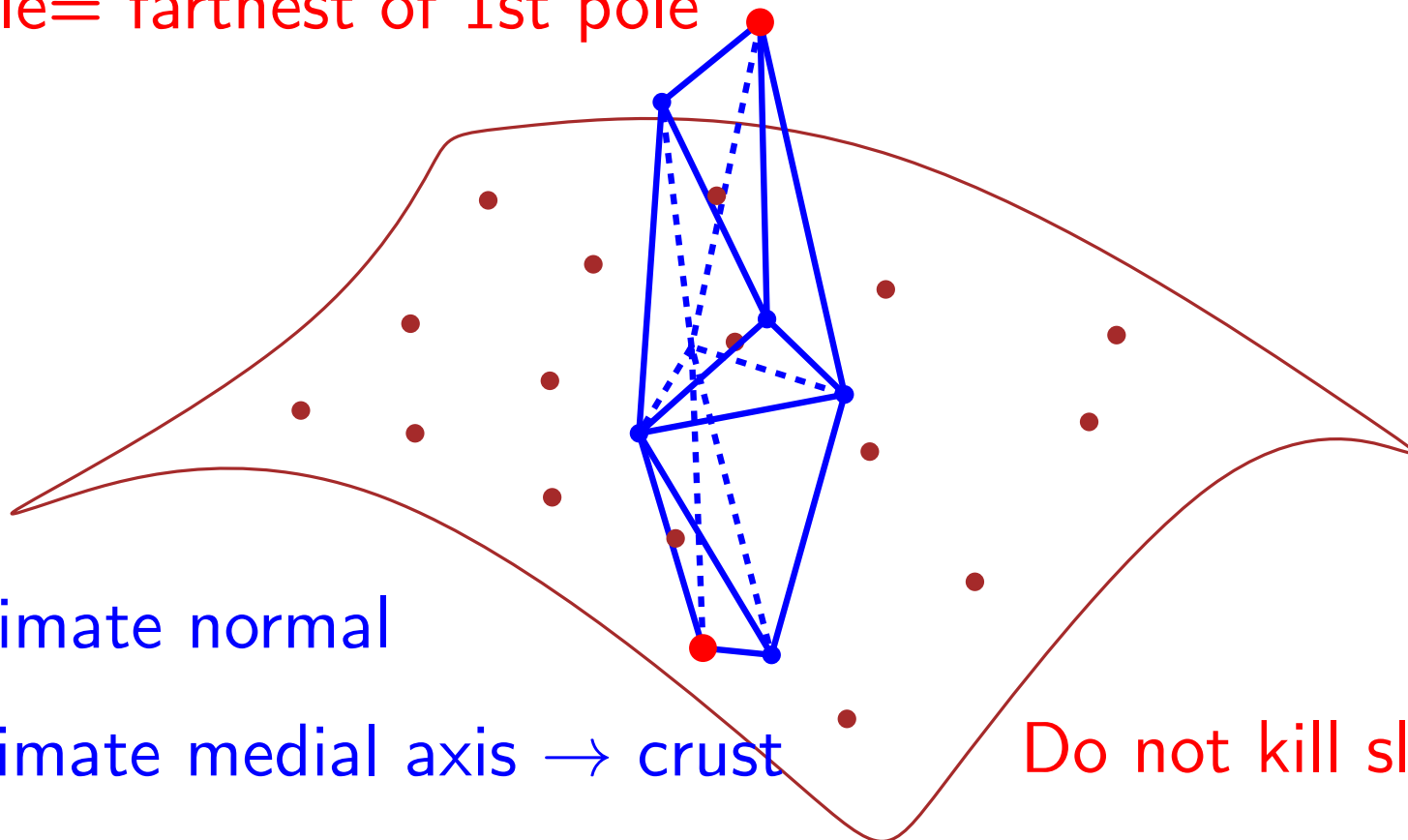
Reconstruction

3D



Pole = farthest of seed

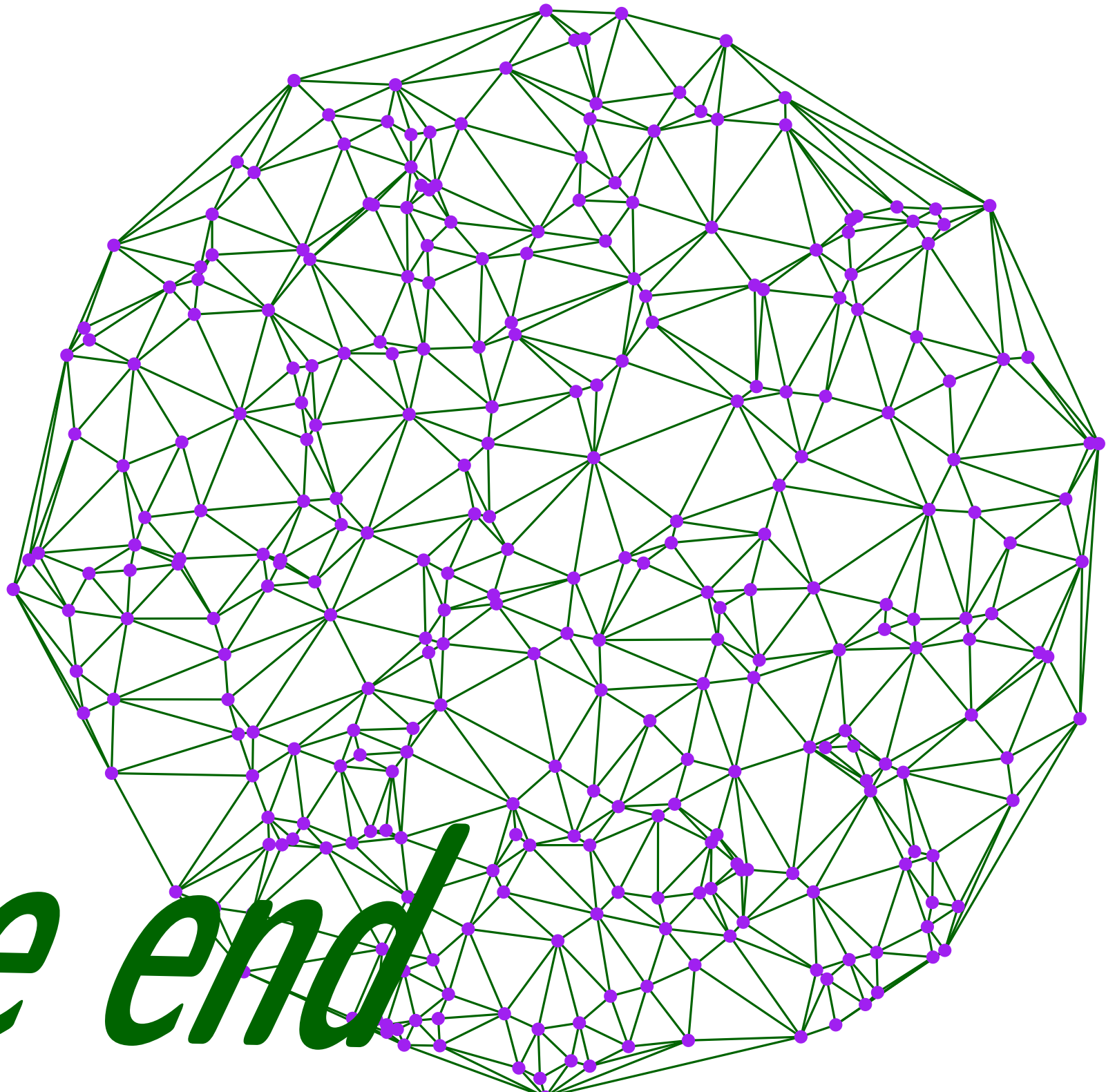
2nd pole = farthest of 1st pole



Approximate normal

Approximate medial axis → crust

Do not kill slivers



The end