Hybrid architecture of LPV dynamical systems in the context of cybersecurity

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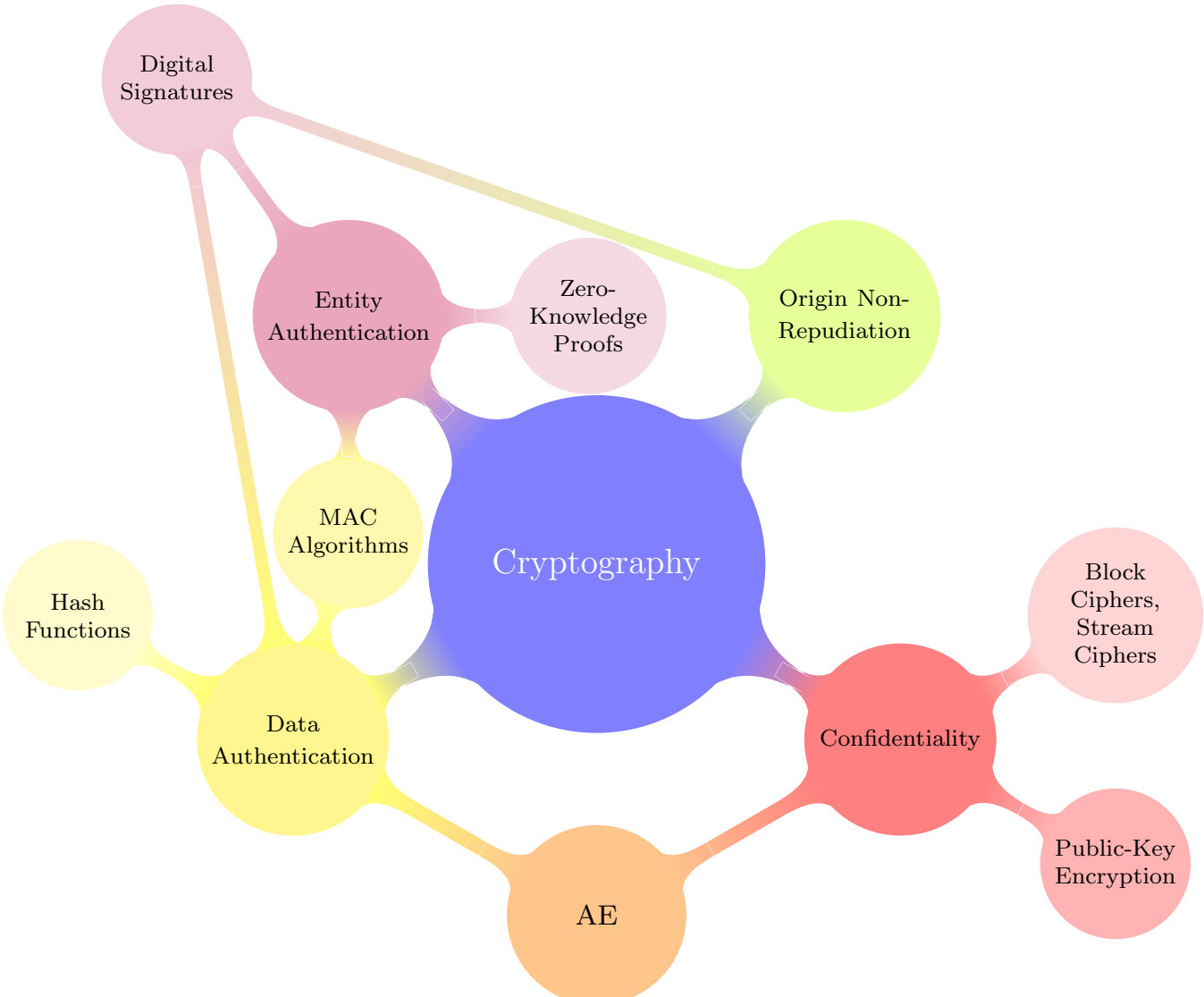
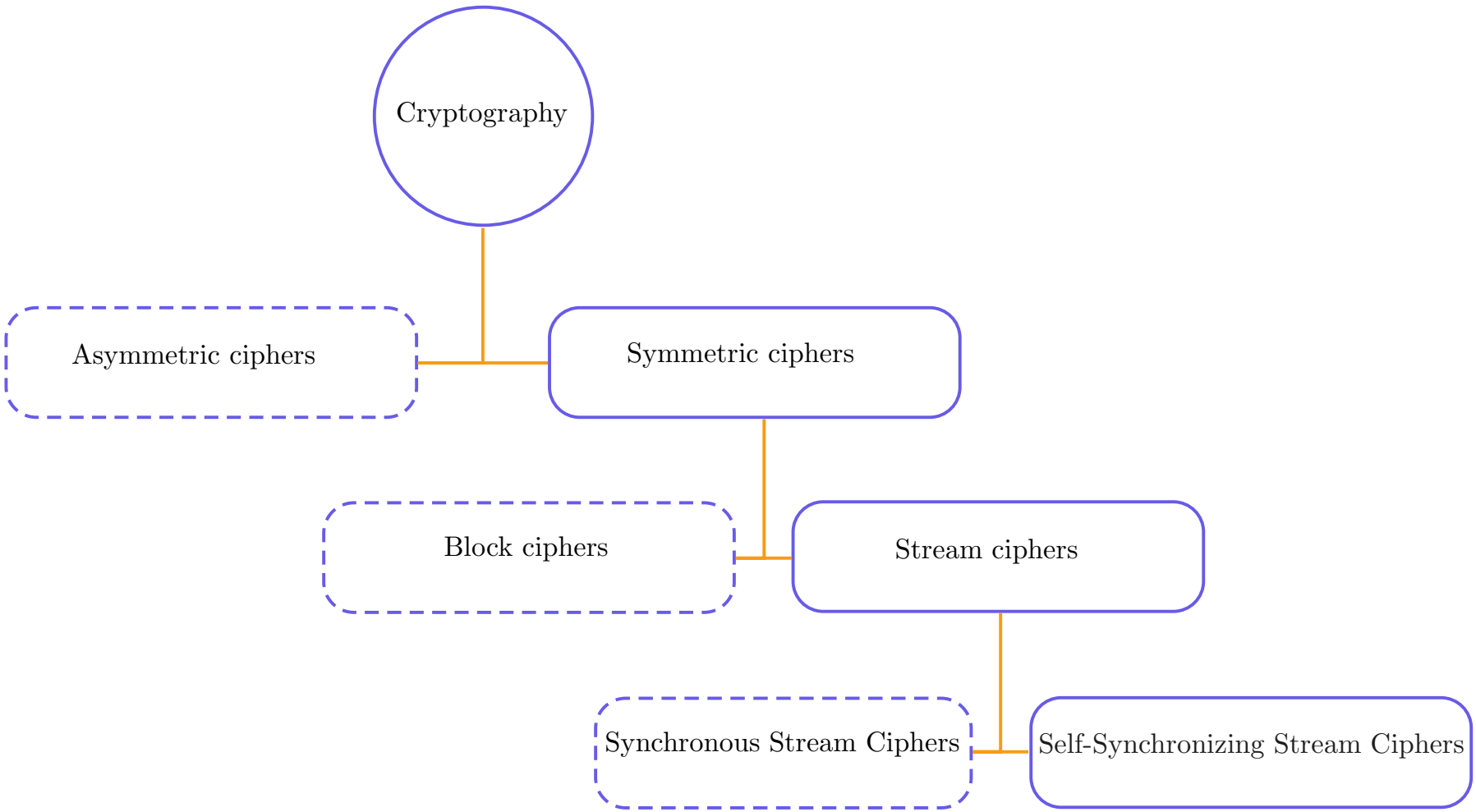
Overview

1. Cryptography
2. Generalities on stream ciphers
3. Self-synchronizing stream ciphers & Linear Parameter Varying (LPV) systems
4. Design of a hybrid architecture for a statistical Self-Synchronizing Stream Ciphers

Cryptography

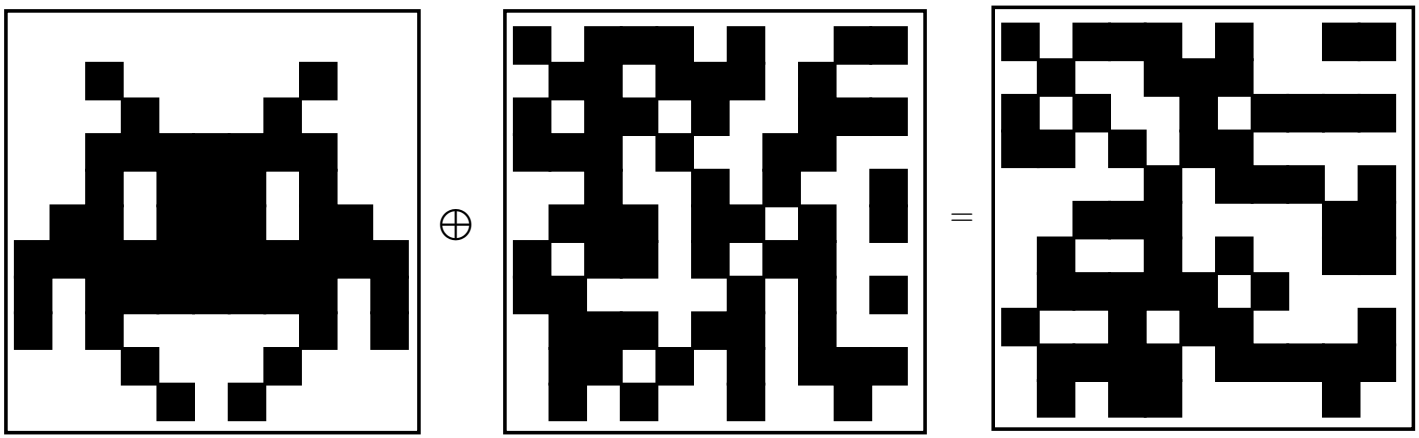


Stream ciphers



Vernam cipher (One Time Pad)

- An encryption technique that cannot be broken (encryption 100% sure).

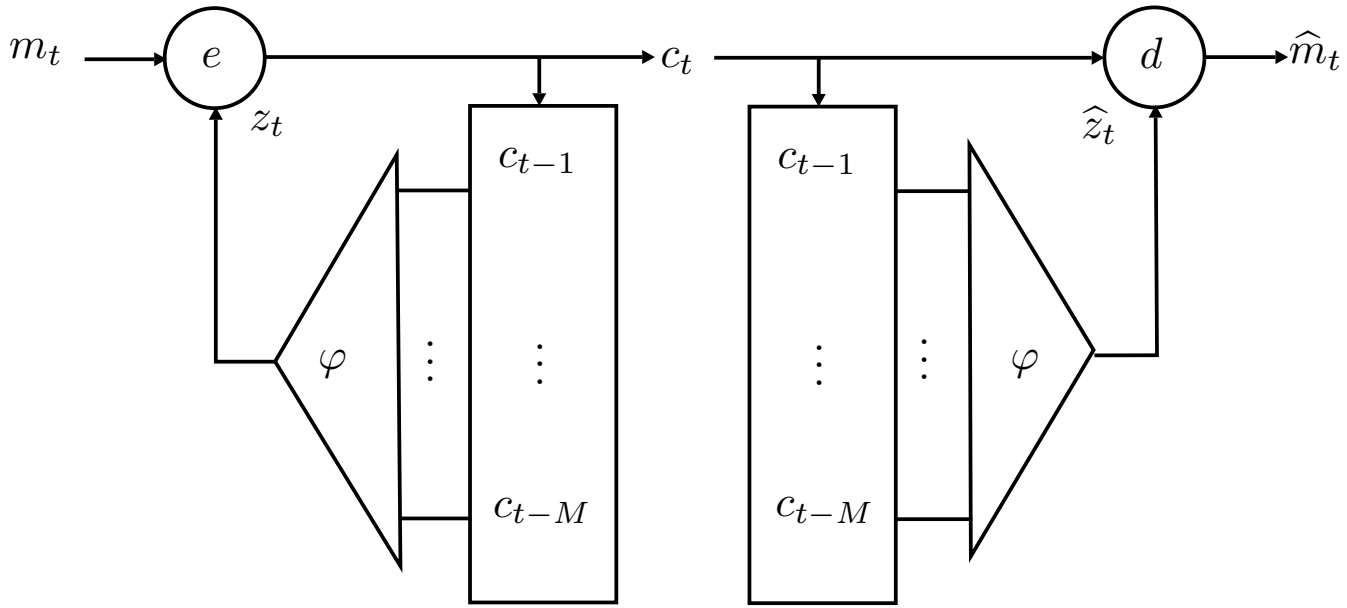


Plaintext Key Ciphertext

The key must:

- never be reused.
- be truly random.
- be at least as long as the plaintext.

Self-Synchronizing Stream Cipher (SSSC) [Maurer. 1991]



- Ciphering equation:

$$\begin{cases} z_t = \varphi(c_{t-l-M+1}, \ldots, c_{t-l}), \\ c_t = e(z_t, m_t) \end{cases}$$

- Deciphering equation:

$$\begin{cases} \widehat{z}_t = \varphi(c_{t-l-M+1}, \ldots, c_{t-l}), \\ \widehat{m}_t = d(\widehat{z}_t, c_t) \end{cases}$$

- HBB [Palash Sarkar. 2003], SSS [P.Hawkes, M.Paddon et al. 2004], Moustique [J.Daemen and P.Kitsos. 2005], Stanislas [G.Millerioux, M.Minier et al. 2020].

Flatness

$$S : \begin{cases} x_{t+1} = f(x_t, m_t), \\ c_t = h(x_t, m_t) \end{cases}$$

Definition

The system (S) is said to be flat if there exists a variable c_t, called a flat output, such that all system variables can be expressed as a function of the flat outputs and a finite number of its backward and forward shifts. In other words, there exist two functions F and G such that:

$$\begin{cases} x_t = F(c_{t+s_1}, \ldots, c_{t+s'_1}), \\ m_t = G(c_{t+s_2}, \ldots, c_{t+s'_2}) \end{cases}$$

where s_1 , s'_1 , s_2 and $s'_2 \in \mathbb{Z}$.

LPV system (Linear Parameter Varying) Single Input Single Output

- Ciphering Side:

$$\Sigma_\rho \begin{cases} x_{t+1} = A_{\rho(t)} x_t + B_{\rho(t)} m_t \\ c_t = C_{\rho(t)} x_t + D_{\rho(t)} m_t \end{cases} \tag{1}$$

where m_t, $c_t \in \mathbb{F}$ and $\mathbb{F}$ a finite field.

- A is the dynamical matrix:

$$A_{\rho(t)} = \begin{pmatrix} & 1 & & & & 1 & & \\ 1 & \rho^1(t) & 1 & & & & \rho^2(t) & \\ & & & 1 & & & & \\ & & & & \rho^3(t) & & & \\ 1 & \rho^4(t) & \rho^5(t) & & & 1 & & \rho^6(t) \\ & & & & & & 1 & \\ & & & & & & & 1 \\ \rho^7(t) & 0 & \rho^8(t) & 1 & 0 & \rho^9(t) & 0 & 0 \end{pmatrix}$$

Stanislas[G.Millerioux, M.Minier et al. 2020]: $\rho^i(t) \rightarrow$ S-box$(c_t, c_{t-1}, \ldots, c_{t-s}, K_i)$.

Characterization of flatness

- Ciphering equation:

$$\begin{cases} x_{t+1} = A_{\rho(t)} x_t + B_{\rho(t)} m_t, \\ c_{t+1} = C_{\rho(t)} x_t \end{cases}$$

- Deciphering equation:

$$\begin{cases} \widehat{x}_{t+1} = P_{\rho(t)} \widehat{x}_t + B c_t, \\ \widehat{m}_t = C_{\rho(t)} A_{\rho(t)} \widehat{x}_t + c_t \end{cases}$$

Where

$$P_{\rho(t)} = A_{\rho(t)} - BC \prod_{l=t+K}^{t} A_{\rho(l)}$$

- Synchronization equation: $\varepsilon_t = \hat{x}_{t+r} - x_t$

$$\varepsilon_{t+M} = P_{\rho(t+M)} P_{\rho(t+M-1)} \cdots P_{\rho(t)} \cdot \varepsilon_t$$

- We can show that if an LPV system is flat then: $\exists M \in \mathbb{N}, t \geq 0, \varepsilon_{t+M} = 0$ then

$$P_{\rho(t+M)} P_{\rho(t+M-1)} \cdots P_{\rho(t)} = 0$$

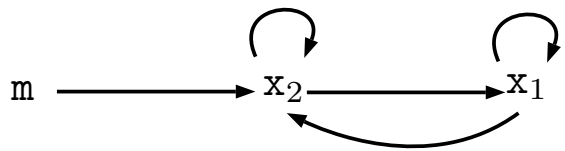
So, $x_t = \hat{x}_t$ and $m_t = \hat{m}_t$

How to construct a flat system ?

- Issue: Mortality problem (NP-Hard).
- Graph-oriented approach.

Complexity and security

- Graph-oriented approach:



- State space equation of the LPV system:

$$\Sigma_\rho \begin{cases} x_{t+1} = A_{\rho(t)} x_t + B_{\rho(t)} m_t \\ c_t = C_{\rho(t)} x_t + D_{\rho(t)} m_t \end{cases} \qquad A_\rho(t) = \begin{pmatrix} 1 & 1 \\ 1 & \rho(t) \end{pmatrix}, B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

- Security $\rightarrow$ diffusion delay.

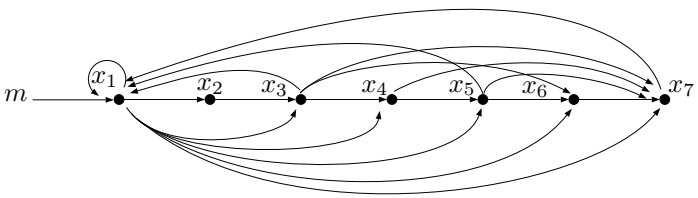
Definition (Diffusion delay)

Let p denotes the power of a matrix $Z \in M_n(\mathbb{F})$. Diffusion delay is the smallest value, denoted by d_0, of p such that Z^p does not have any zero coefficient.

The more the integers n (dimension) and n_a (number of edges), the more the security.

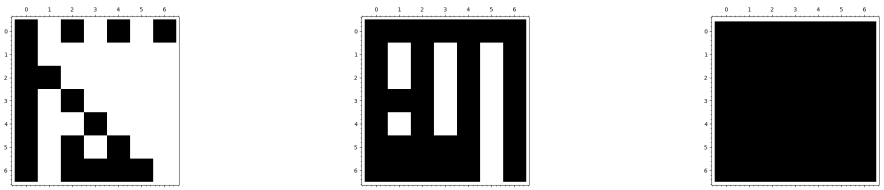
Diffusion delay

- The graph corresponds to a flat structured LPV system. (dimension: $n = 7$, edges: $na = 15$)



$$A = \begin{pmatrix} 1 & 0 & \rho(t) & 0 & \rho(t) & 0 & \rho(t) \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \rho(t) & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

- Powers of A^p with $p \in [1,3]$.

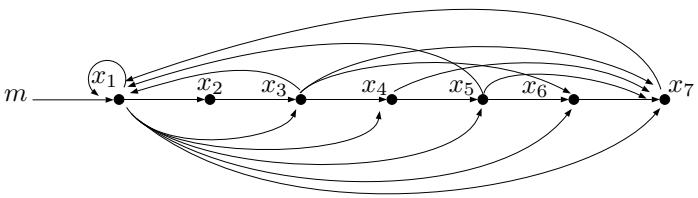


(a) A (b) A^2 (c) A^3

- Black squares represent diffusion $\rightarrow$ good diffusion.

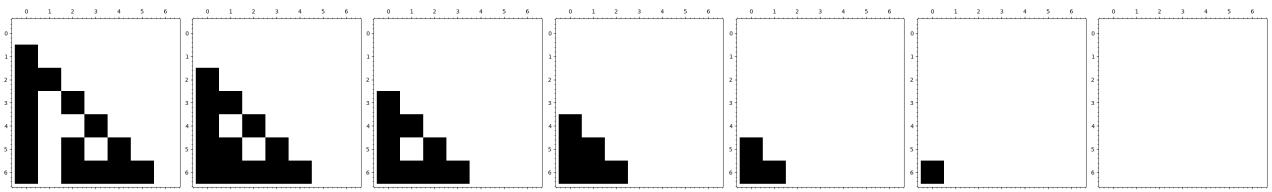
Diffusion delay

- The graph corresponds to a flat structured LPV system. (dimension: $n = 7$, edges: $na = 15$)



$$P = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \rho(t) & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

- Powers of P^p with $p \in [1, 7]$.

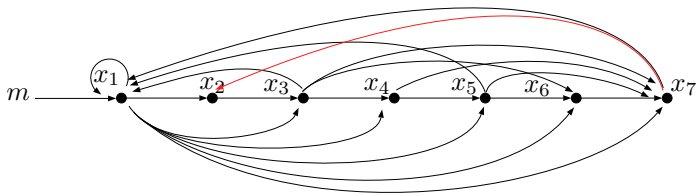


(a) P (b) P^2 (c) P^3 (d) P^4 (e) P^5 (f) P^6 (g) P^7

- Black squares represent diffusion $\rightarrow$ bad diffusion & triangular.

Diffusion delay

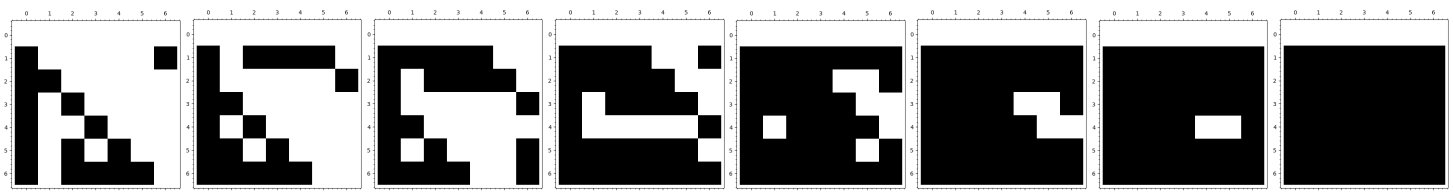
- The graph corresponds to a not-flat structured LPV system. (dimension: $n = 7$, edges: $na = 15$)



$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \rho(t) & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \rho(t) & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

A P

- Powers of P^p with $p \in [1, 7] \rightarrow$ good diffusion & non-triangular.

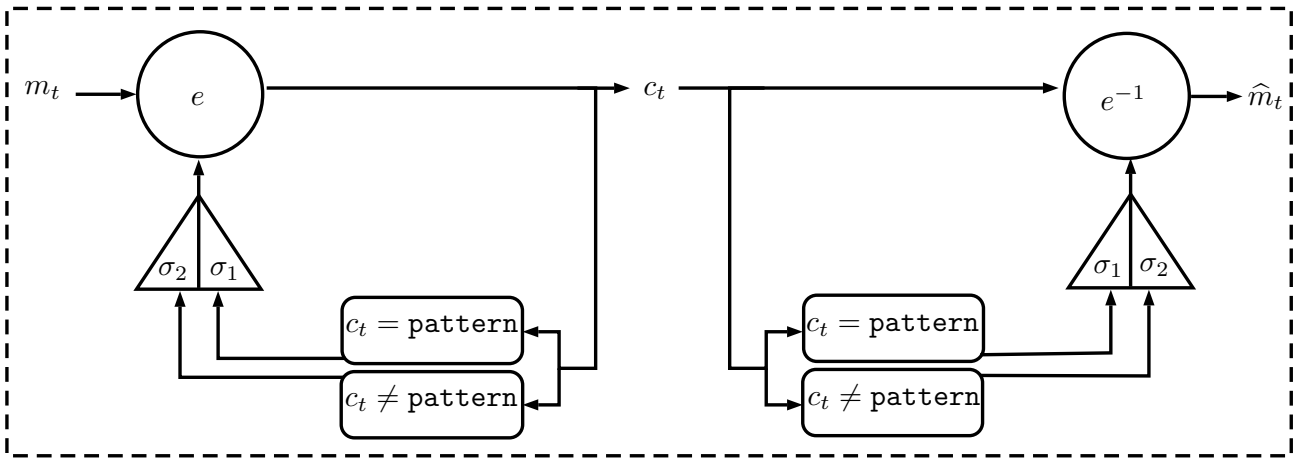


(a) P (b) P^2 (c) P^3 (d) P^4 (e) P^5 (f) P^6 (g) P^7 (h) P^8

A statistical self-synchronizing hybrid architecture

[H.boukerrou, G.Millerioux and M.Minier. LPVS'21]

The hybrid architecture with switching rule σ obeys the following state space representation at the:



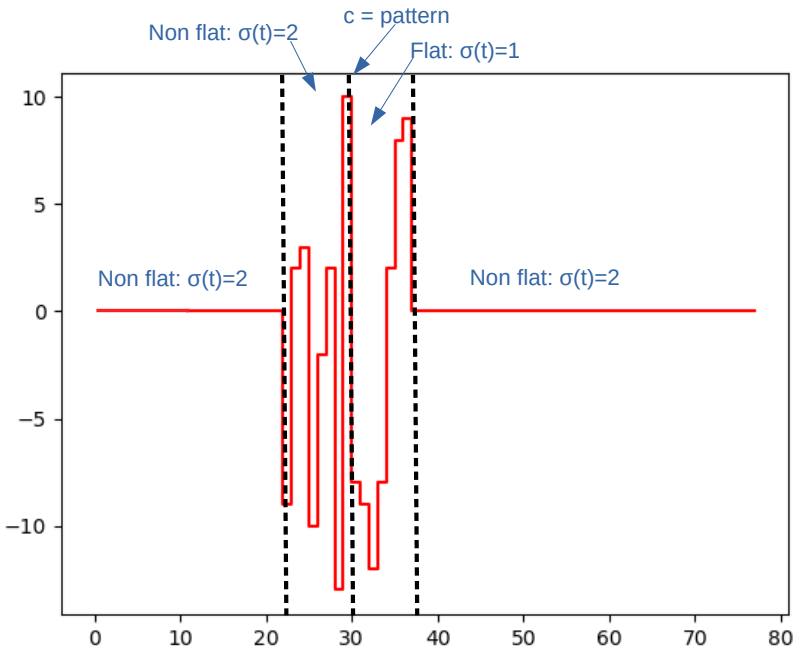
$\sigma(t) = 1$:

- the dynamical system is flat.
- the system gets the self-synchronization property.
- diffusion is not optimal.

$\sigma(t) = 2$:

- the dynamical system is not flat.
- the system does not get the self-synchronization property.
- diffusion is much more significant.

Proof-of-Concept Example

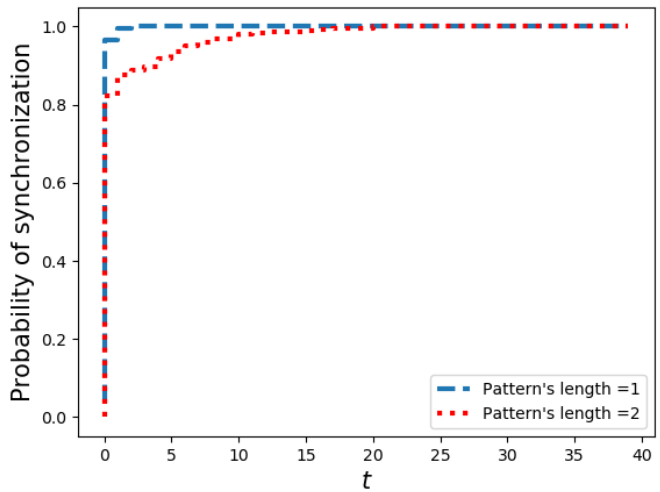


Time evolution of the error $m_t - \hat{m}_{t+1}$

- $t > 0$: synchronization $\rightarrow$ cipher and the decipher $\rightarrow \sigma(t) = 2$.
- $t = 21$: introduce a disturbance into the cryptogram.
- $t = 30$: $c_t = \textit{pattern} \rightarrow \sigma(t) = 1$.
- $t > 37$: both the cipher and the decipher switch on mode $\sigma(t) = 2$.

A statistical self-synchronizing hybrid architecture

- An LPV system is statistical self-synchronizing if ,
$\lim_{t \to +\infty} Pr[x_t = \hat{x}_t] = 1$



successful resynchronizations after a time t

- Statistical VS Deterministic:

	Deterministic	Statistical
Synchronization	++	+
Security	+	++

Conclusion & further work: Multiple Input Multiple Output (MIMO)

In progress...

- Construction of MIMO
- Security of MIMO

Thank You!