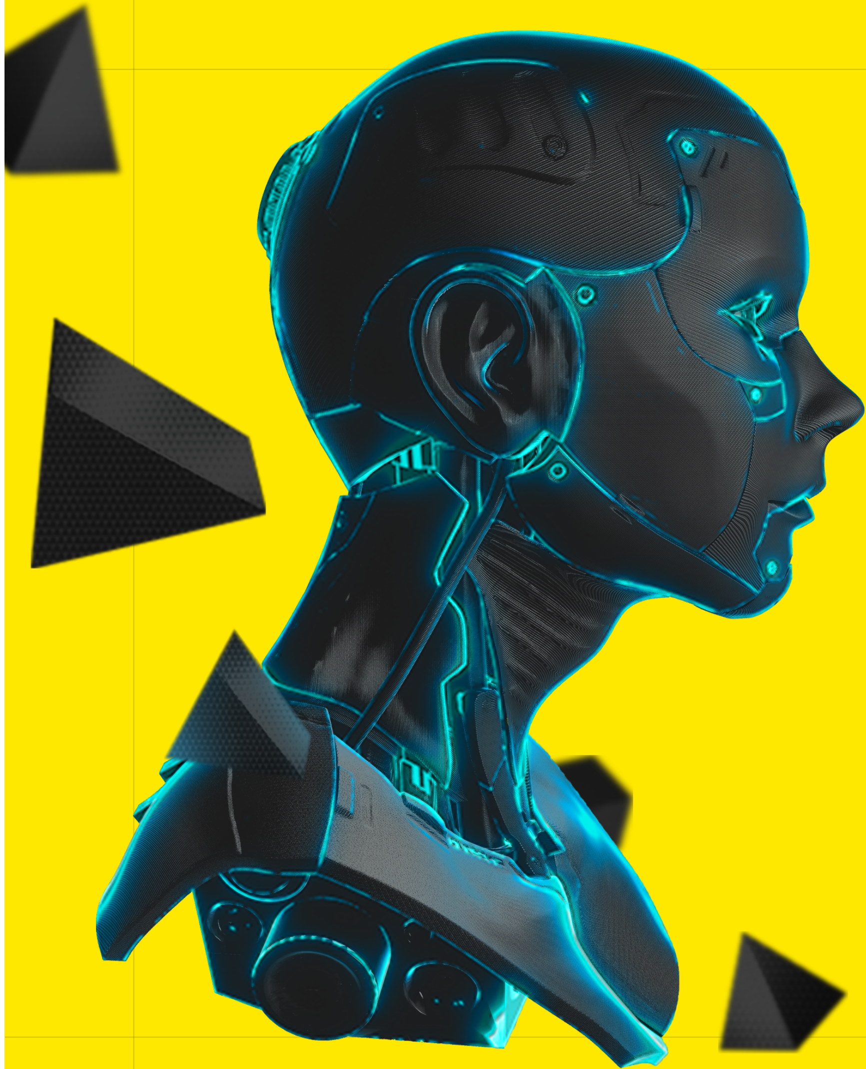

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FOLDOVER-FREE MAPS IN 50 LINES OF CODE

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L. KUDRYAVTSEVA, F. PROTAIS,
N. RAY, D. SOKOLOV.

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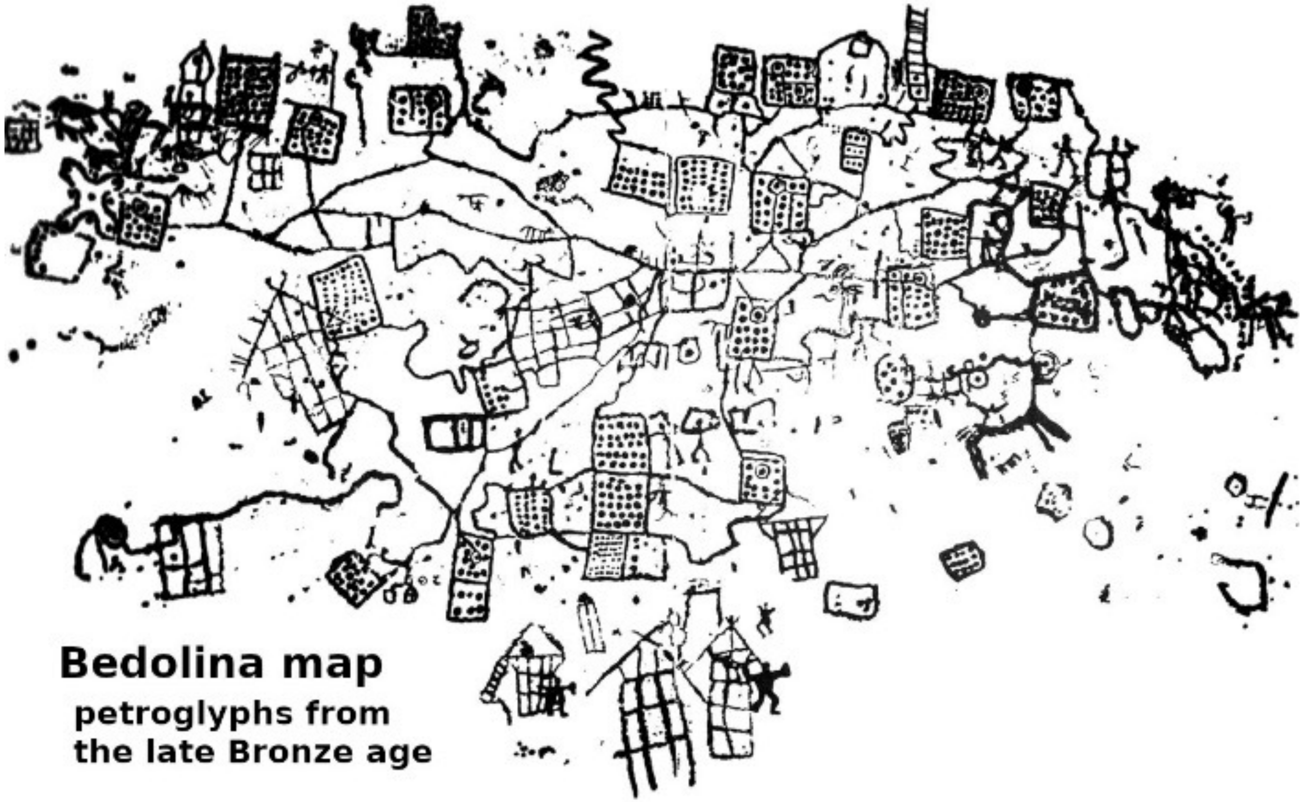
INTRODUCTION

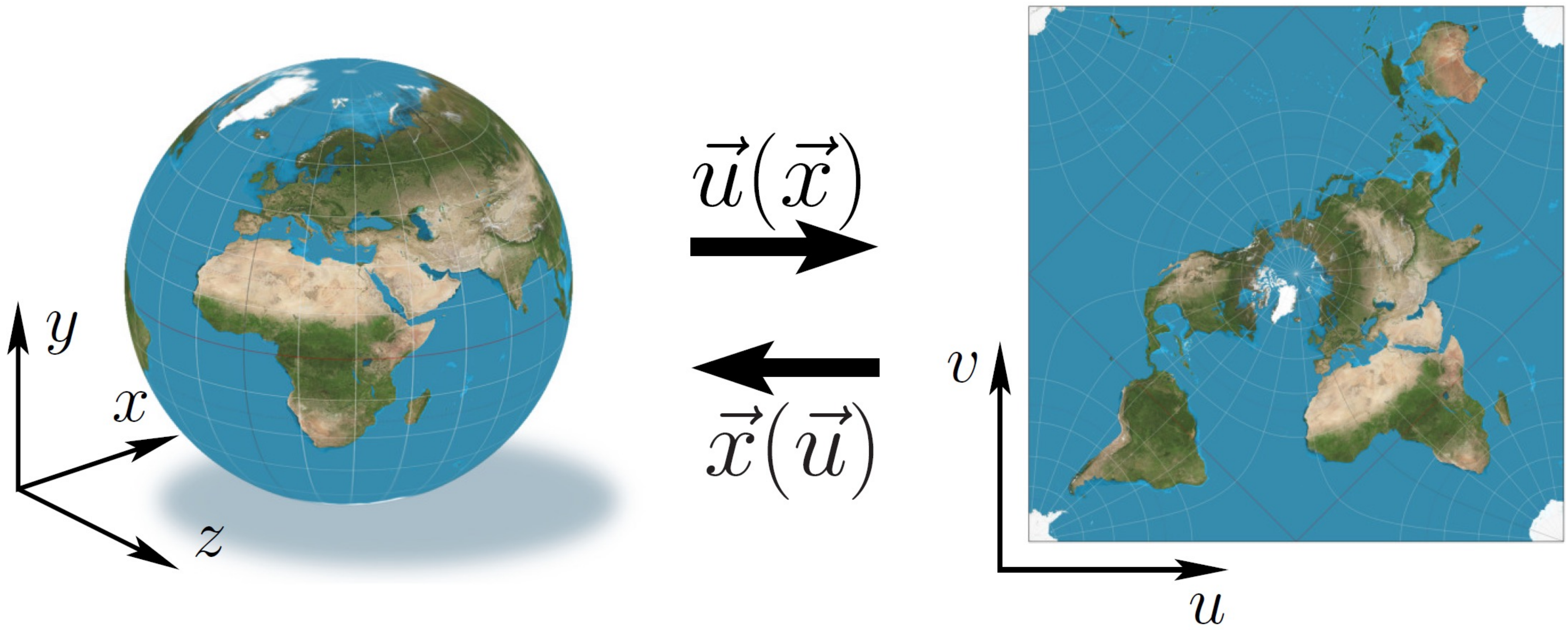
MAPS IN COMPUTER GRAPHICS

CREATING MAPS



Bedolina map
petroglyphs from
the late Bronze age

CREATING MAPS



CREATING MAPS

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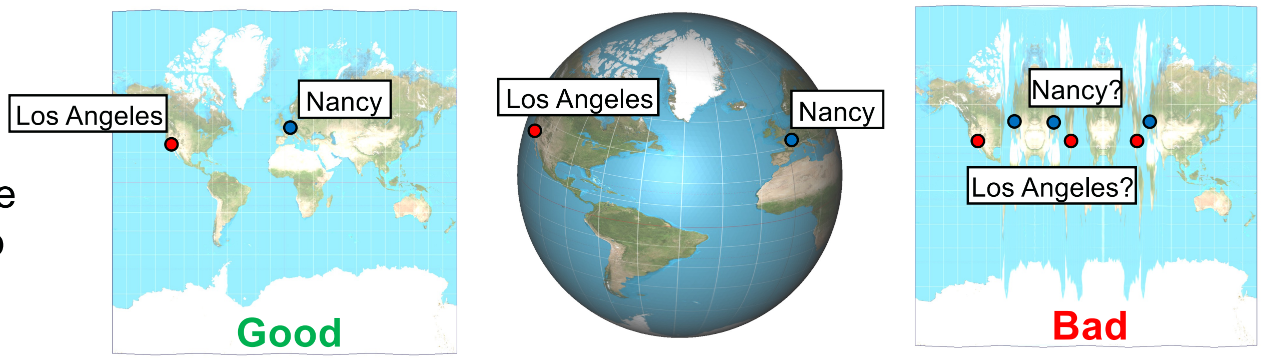
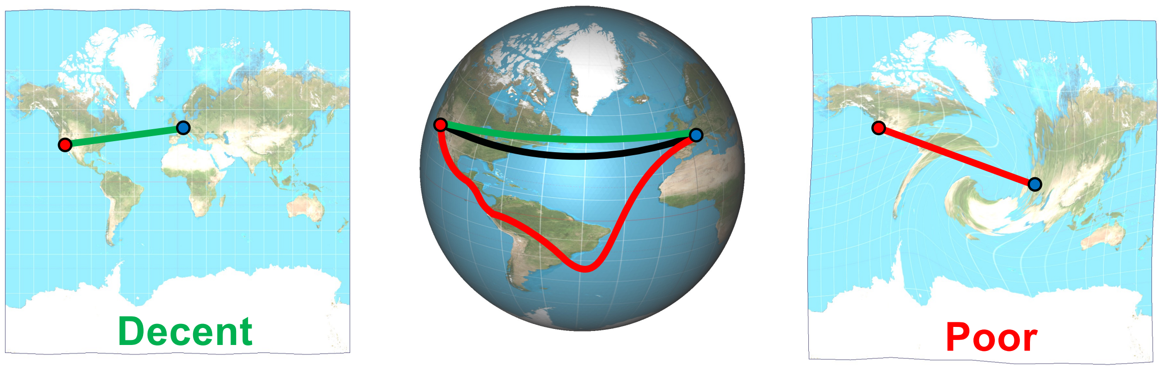
- **<u>Bijectivity</u>**

Non-ambiguous correspondence between the object and the map



- **<u>Control of distortion</u>**

Map should be usable (mesuring distances, angles, areas…)



FAST-FOWARD TO COMPUTER GRAPHICS

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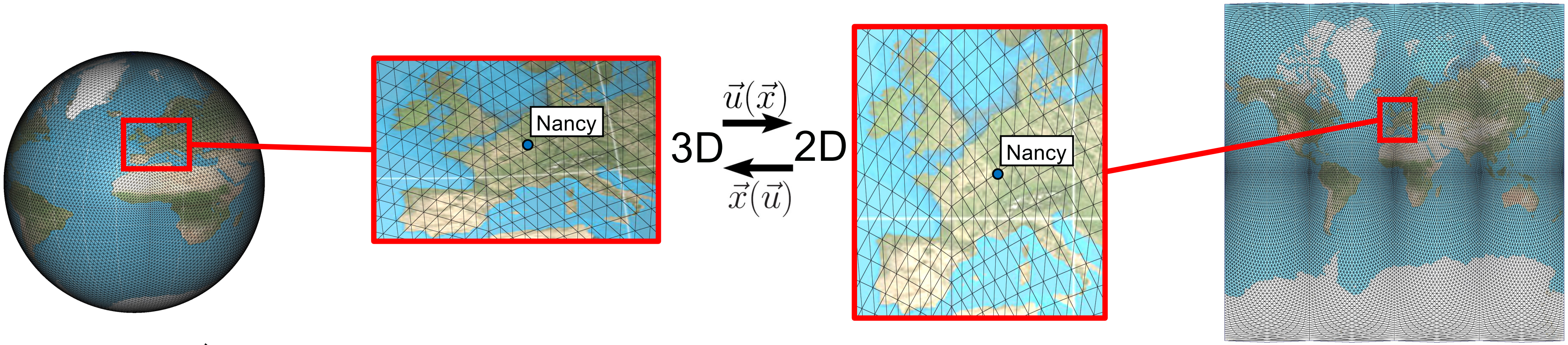
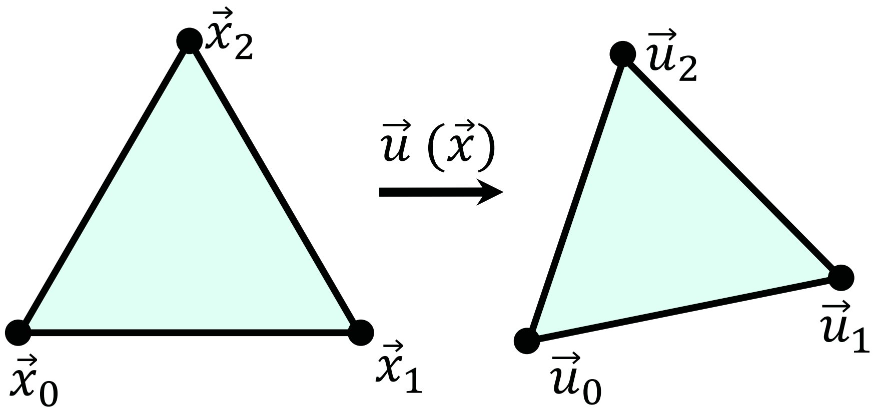




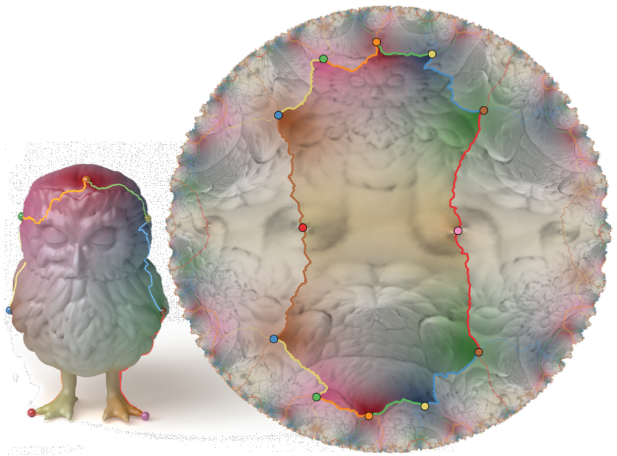
- $\vec{u}: \mathbb{R}^3 \to \mathbb{R}^2$ is linear on each triangle
- Bijectivity needs $\det\left(\frac{\partial \vec{u}}{\partial \vec{x}}\right) > 0$ on each triangle

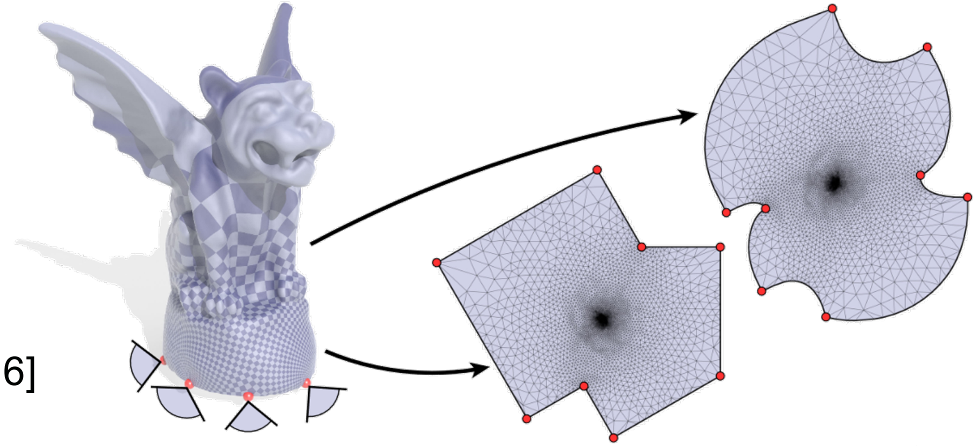
  $\frac{\partial \vec{u}}{\partial \vec{x}}$ = Jacobian matrix J

USES OF MAPS

Shape correspondance

[Aigerman & Lipman 2016]



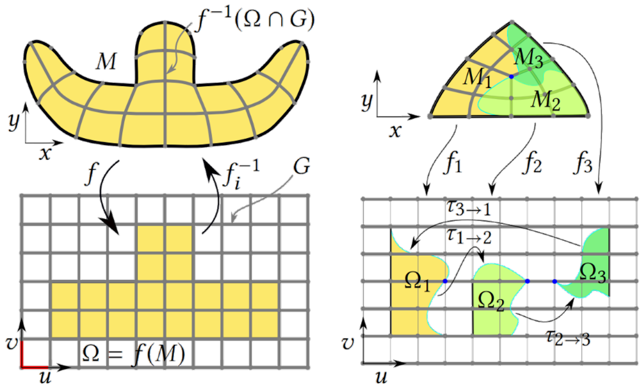


[Sawhney & Crane 2017]

Texturing

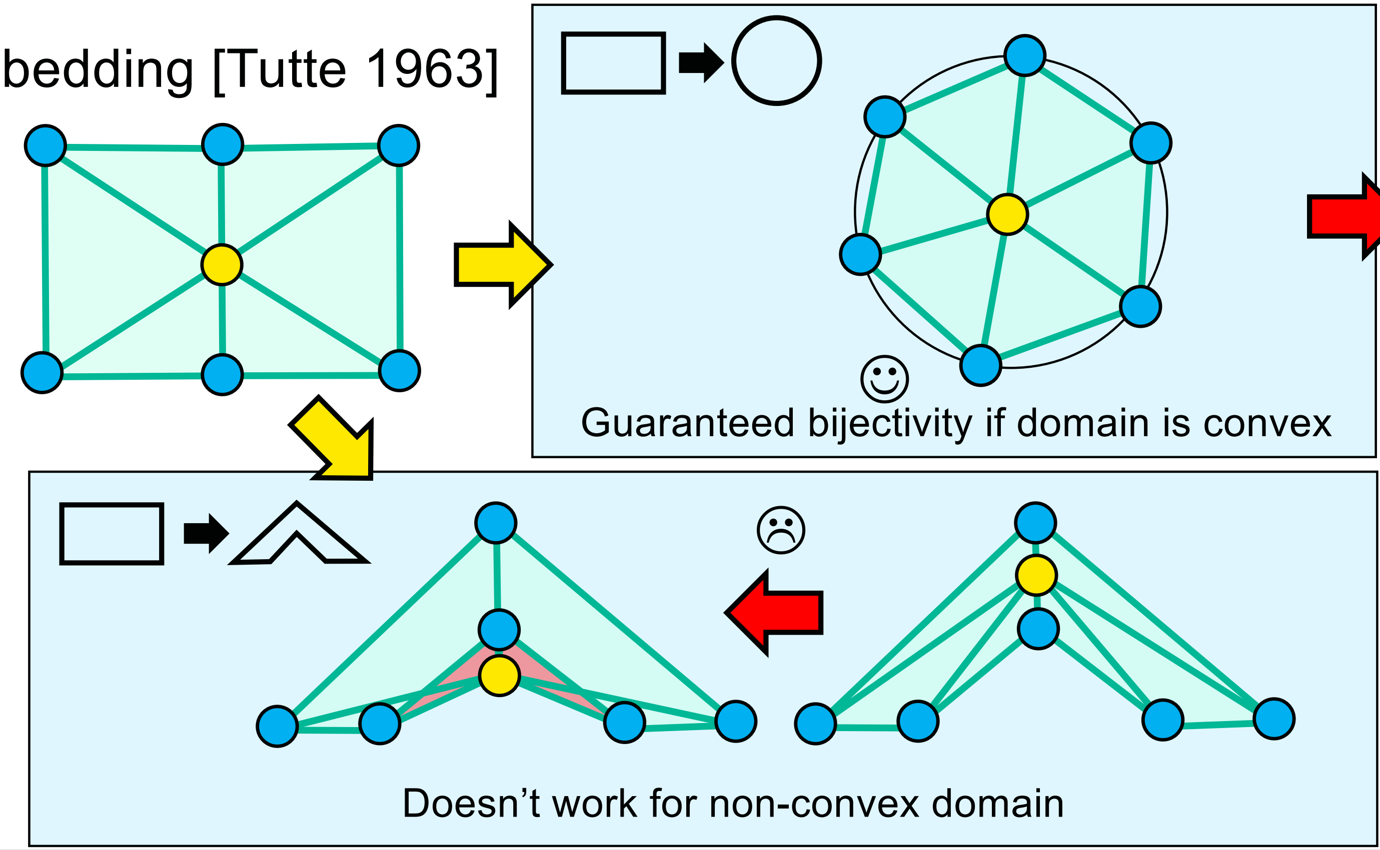
Re-meshing

[Liu *et al.* 2018]



GOLD STANDARD OF MAP COMPUTATION

- Tutte embedding [Tutte 1963]



Doesn't extend to 3D

☹

We would like:

$$\arg\min_{\vec{u}} \int_{\Omega} F(u)\, dx$$

RECENT TRENDS

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- shape distortion energy

$$f = \frac{\operatorname{tr} J^T J}{(\det J)^{2/d}}$$

MIPS

[IVANENKO 1988, HORMANN & GREINER 2000]

- area distortion energy

$$g = \|J\| + \|J^{-1}\| = \det J + \frac{1}{\det J}$$

[GARANZHA 2000]

$$\arg\min_{\vec{u}} \underbrace{\int_\Omega (f(J) + \lambda g(J))\, dx}_{E(\vec{u})}$$

Polyconvex energy [Ball1976] leading to a well-posed problem [Garanzha2014]

From hyperelasticity theory

ENERGY VARIATIONS

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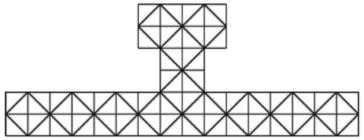
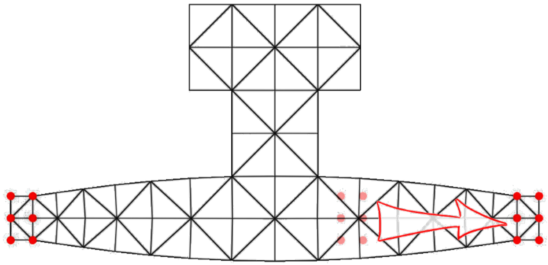
$$\int_{\Omega} \left(f(J) + \lambda g(J)\right)\, dx$$



Rest shape

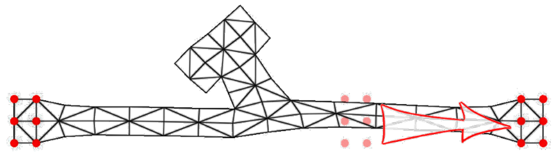
- $\lambda = 0$

  Quasi-conformal



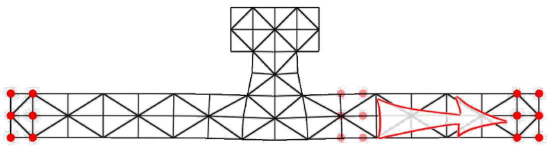
- $\lambda = 10^4$

  Quasi-area-preserving

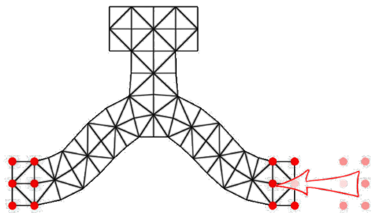


- $\lambda = 1$

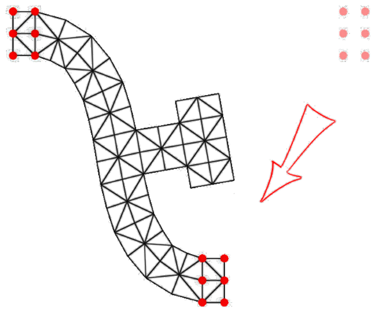
  Quasi-isometric



stretch



compress



bend

RECENT TRENDS

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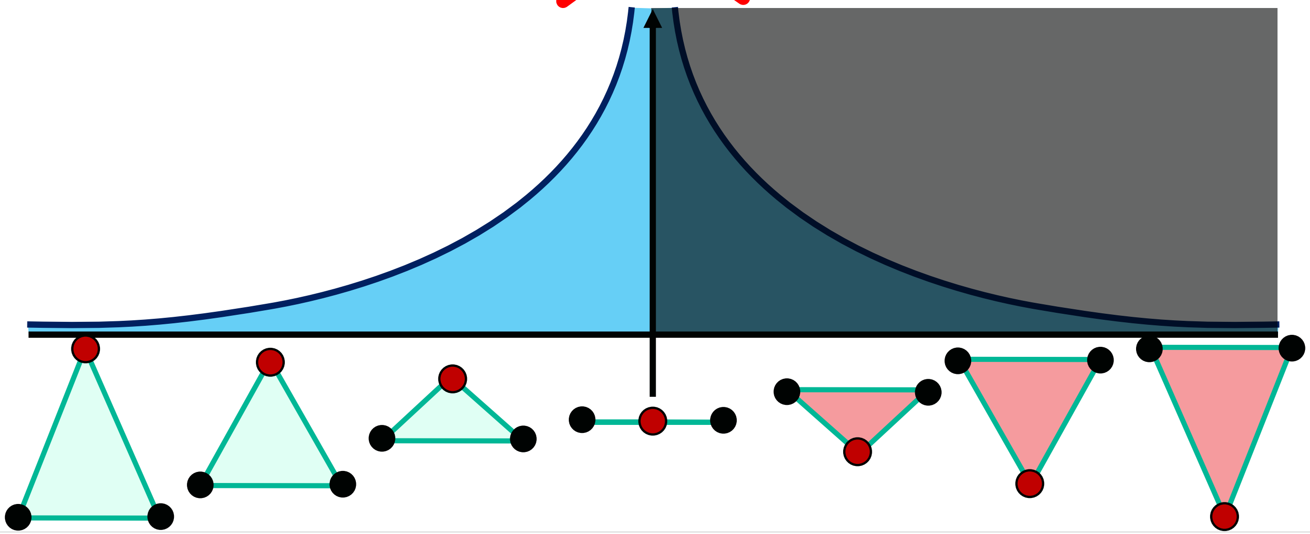
$$f = \frac{\operatorname{tr} J^T J}{\det J} + g = \det J + \frac{1}{\det J}$$

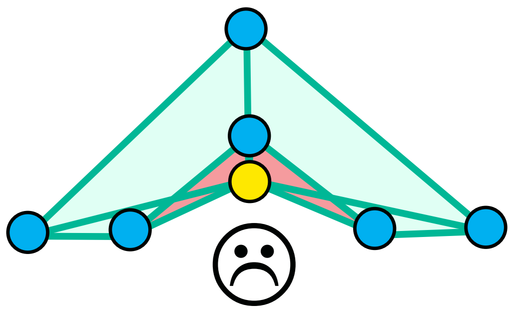
$$\cancel{\frac{1}{|\det J|}} \Rightarrow \frac{1}{\max\{0, \det J\}}$$



$$f(J) := \begin{cases} \frac{\operatorname{tr} J^\top J}{\det J}, & \det J > 0 \\ +\infty, & \det J \leq 0 \end{cases} = \frac{\operatorname{tr} J^\top J}{\max\{\det J, 0\}}$$

$$g(J) := \begin{cases} \det J + \frac{1}{\det J}, & \det J > 0 \\ +\infty, & \det J \leq 0 \end{cases} = \frac{(\det J)^2 + 1}{\max\{\det J, 0\}}$$

But what if «$\det J < 0$» from the start ?



$$E(\vec{u}) = \infty$$

RECENT ADVANCES

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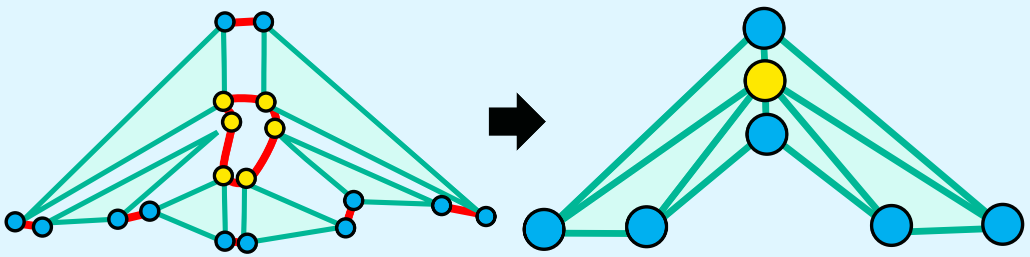
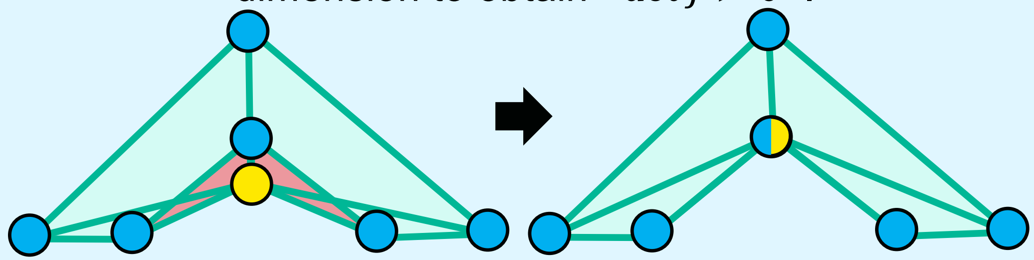
[Fu & Liu 2016]

Disassemble triangles to optimize $E(\vec{u})$



[Du *et al.* 2020]

Minimizing sum of unsigned area of simplices in higher dimension to obtain «$\det J > 0$».



Then use a standard energy

We propose an approach that:

- Is defined continuously (independent of simplices)
- Directly works with $E(\vec{u})$
- Is more reliable

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REGULARIZATION

OUR CONTRIBUTION

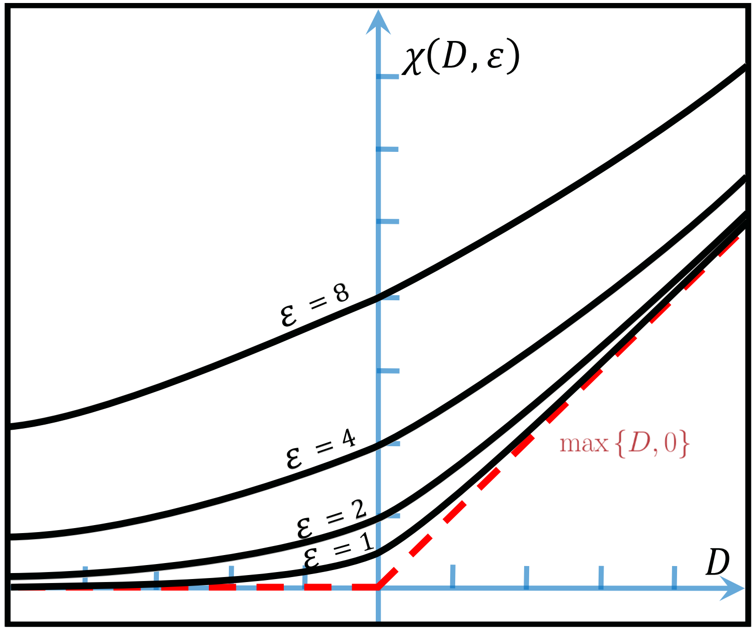
REGULARIZATION

[Garanzha 2000]

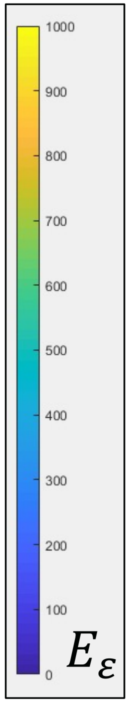
$$\chi(D,\varepsilon) = \frac{D+\sqrt{\varepsilon^2+D^2}}{2} \xrightarrow[\varepsilon \to 0]{} \max\{D,0\}$$

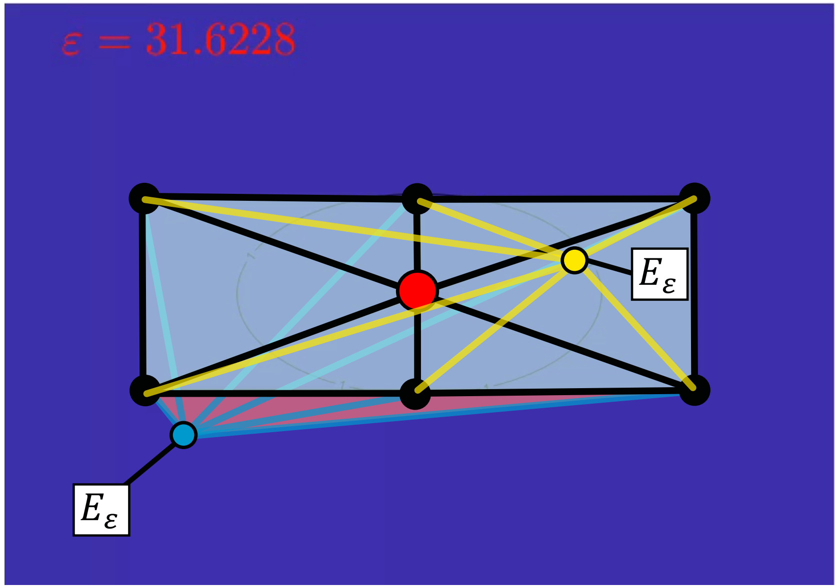
$$f_\varepsilon(J) = \frac{\operatorname{tr} J^T J}{\chi(\det J, \varepsilon)} + g_\varepsilon(J) = \frac{(\det J)^2+1}{\chi(\det J,\varepsilon)}$$

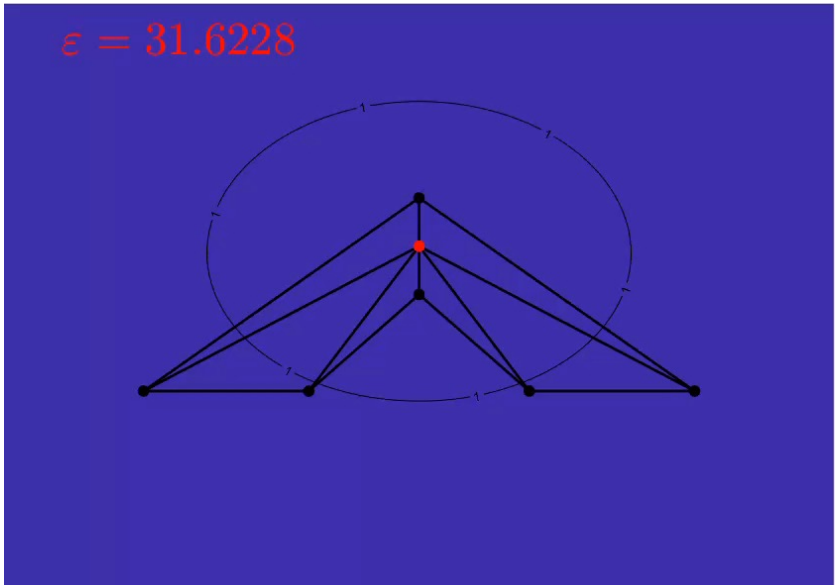
$$\lim_{\varepsilon\to 0^+} \arg\min_{\vec{u}} \underbrace{\int_\Omega \left(f_\varepsilon(J) + \lambda g_\varepsilon(J)\right)\, dx}_{E_\varepsilon(\vec{u}) \xrightarrow[\varepsilon \to 0]{} E(\vec{u})}$$



PLOTTING THE ENERGY







SCHEME

Algorithm

repeat
- Choose ε
- Minimise E_ε

until $\det J > 0$

- Requires a decreasing sequence
- Smaller ε :
  faster convergence but stiffer problems to solve
- We provide an ε sequence with theoretical guarantees! (see the paper)

- L-BFGS works very well
  https://github.com/ssloy/invertible-maps
- We use SPD part of Hessian for the stiffest problems

More theoretical results on untangling in the paper

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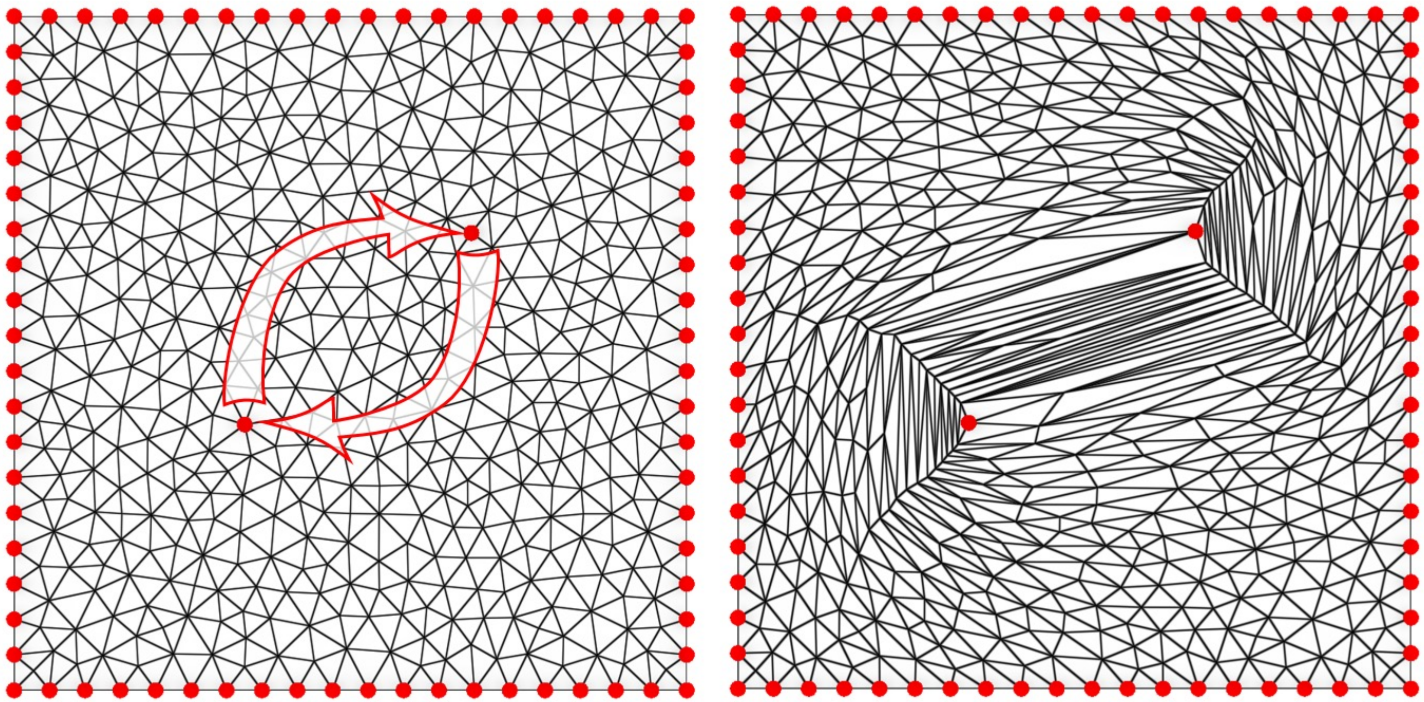
RESULTS

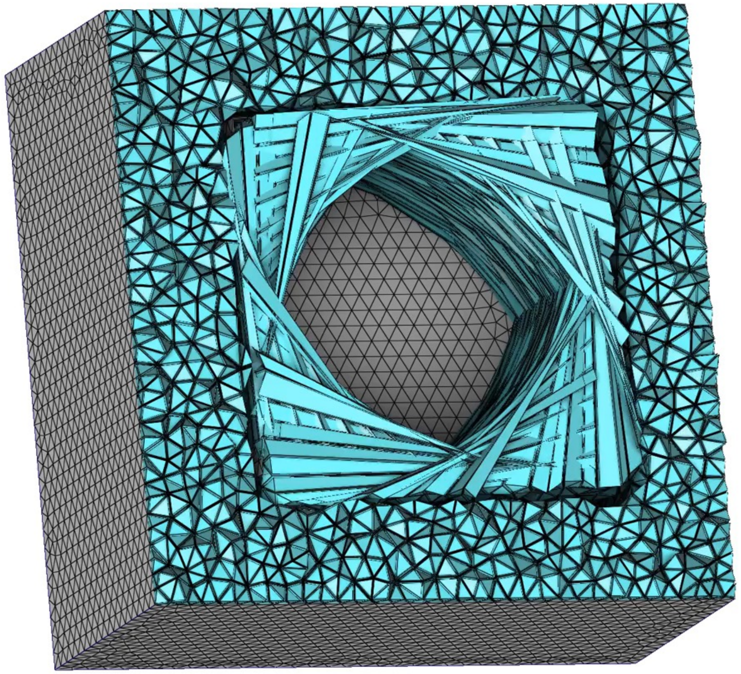
UNTANGLING AND QUALITY MAPPING

BASIC TEST CASE

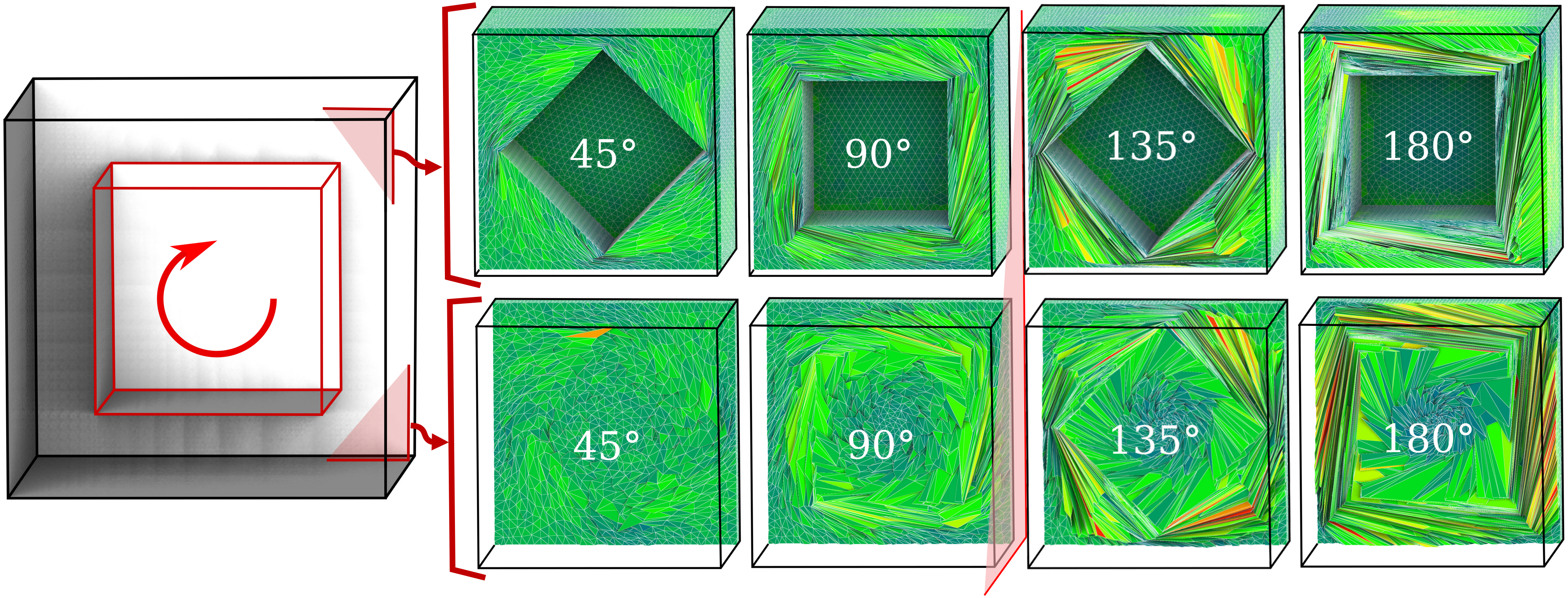


TRY IT OUT: https://github.com/ssloy/invertible-maps

STRESS TEST



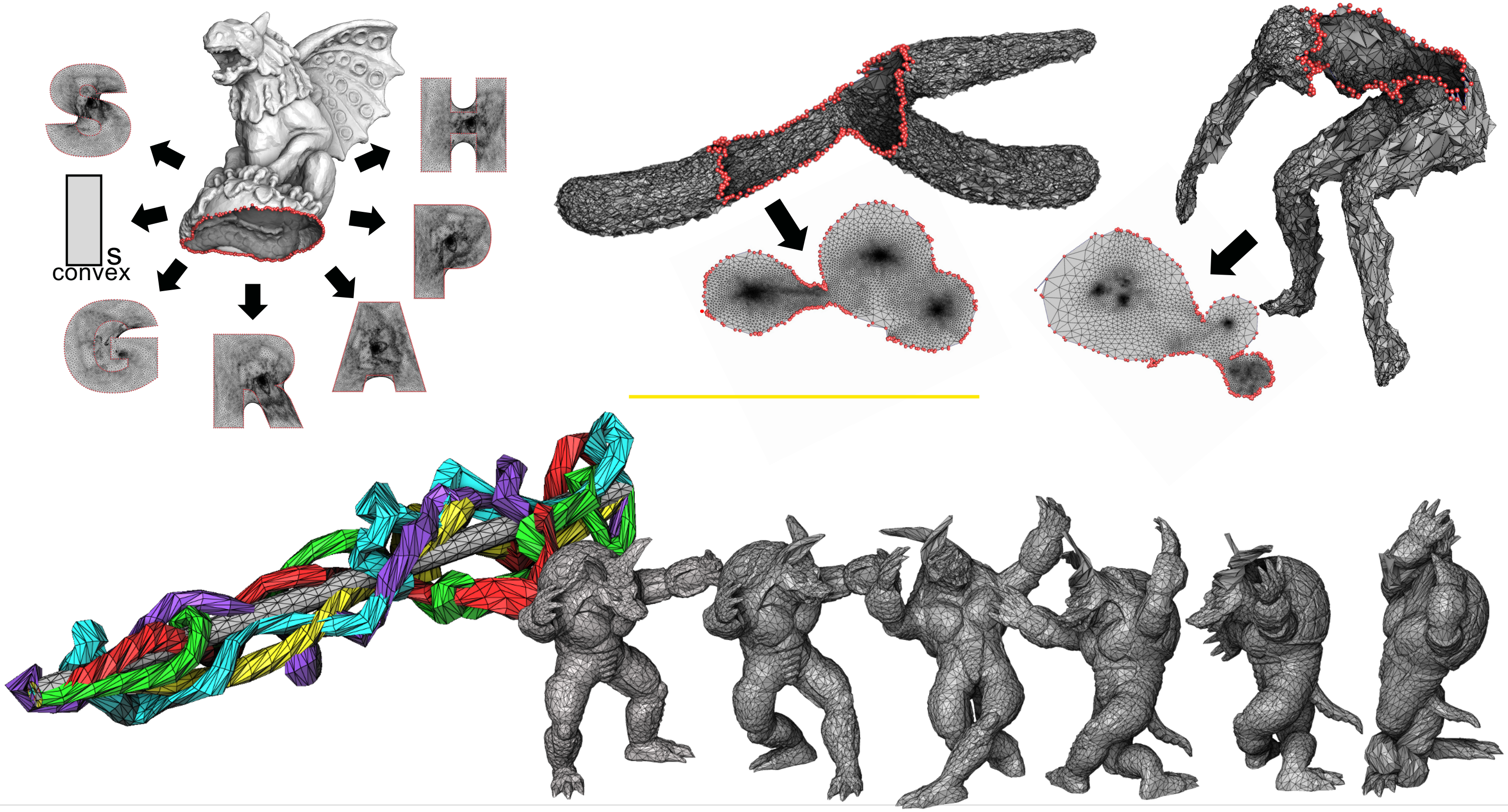
STRESS TEST

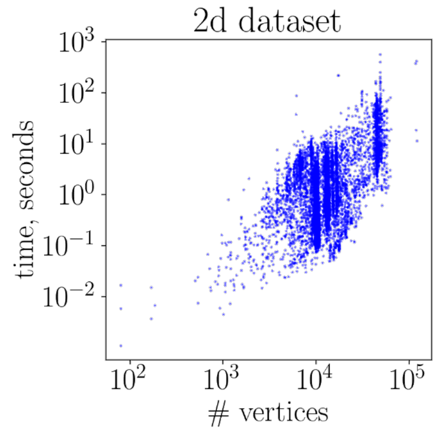


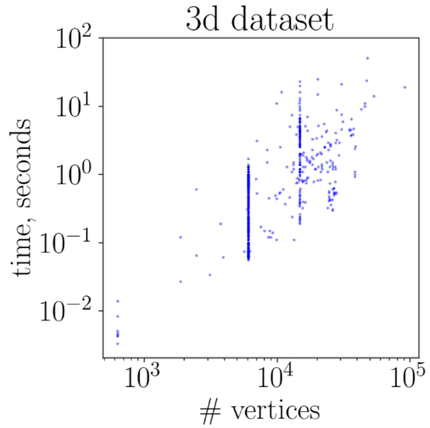
Limit of
[Du *et al* 2020]

LARGE SCALE TEST

https://github.com/duxingyi-charles/Locally-Injective-Mappings-Benchmark

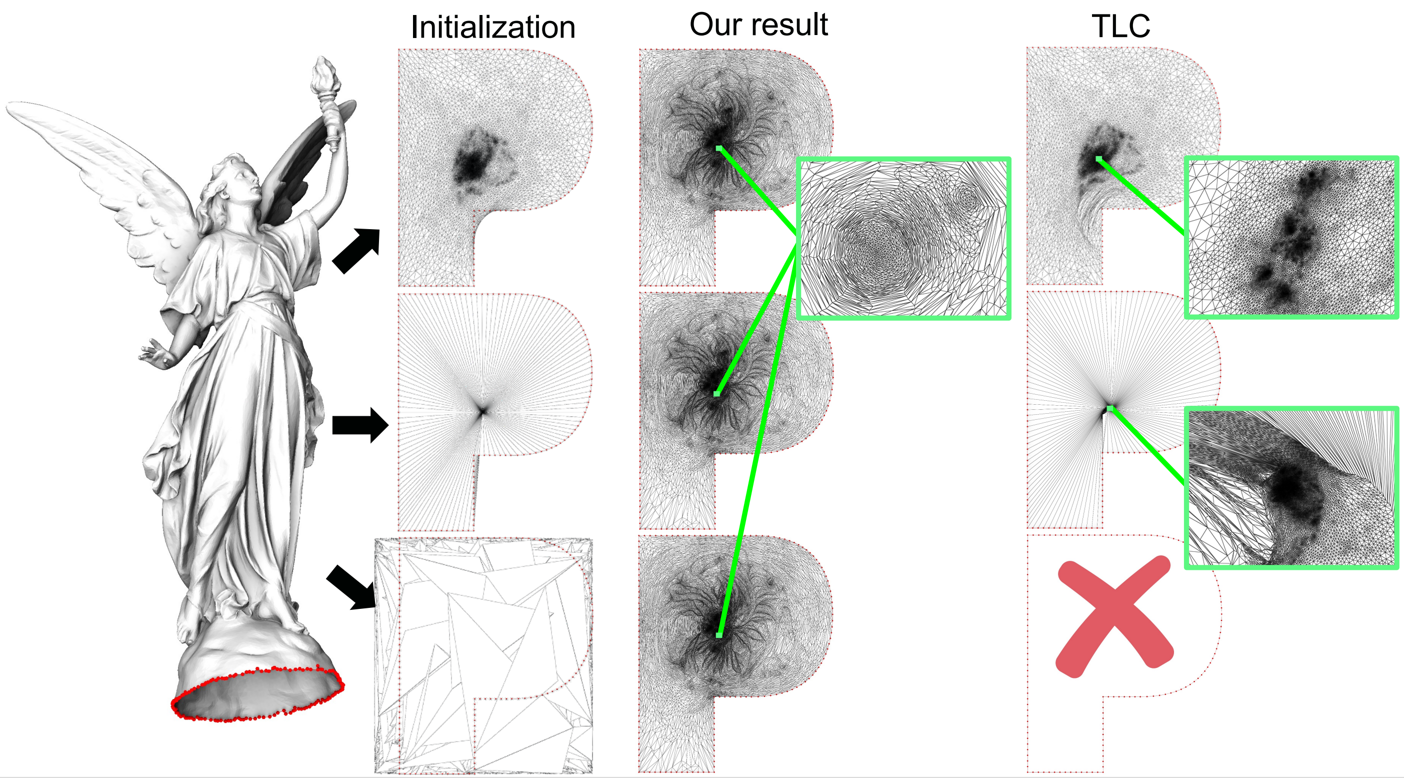






COMPARISON - TLC

- TOTAL LIFTED CONTENT [Du *et al* 2020]

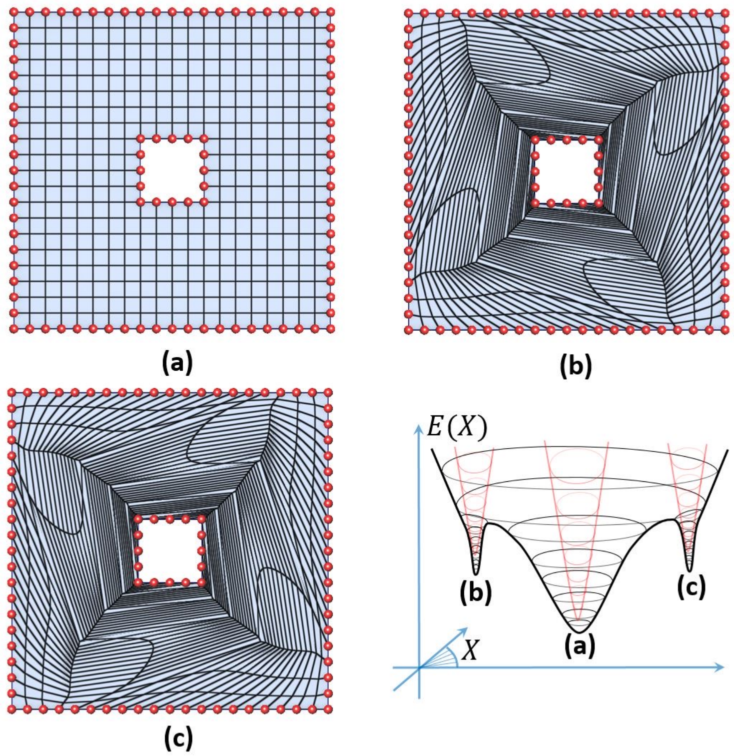


Our approach:

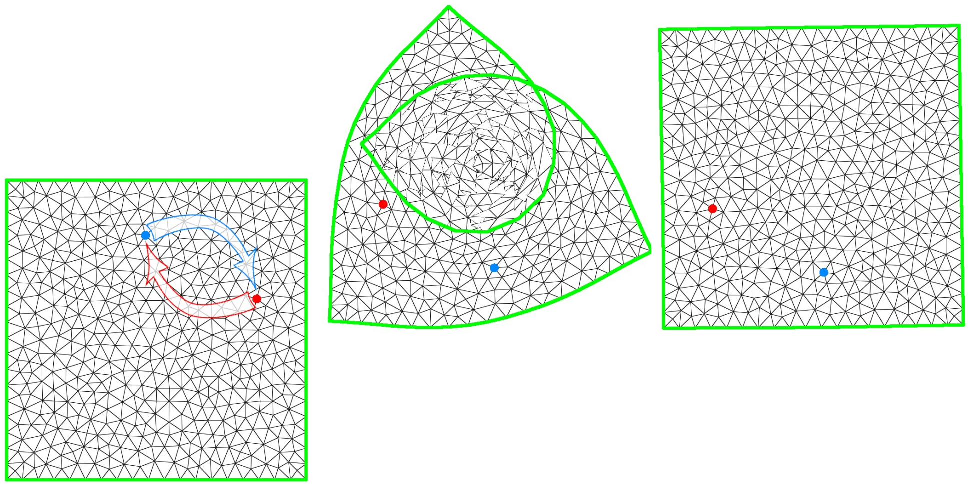
- Initialization invariant
- Control of distortion
- Robust to very bad initializations

LIMITATIONS - FUTURE WORK

- Local minima



- k-coverings



CONCLUSION

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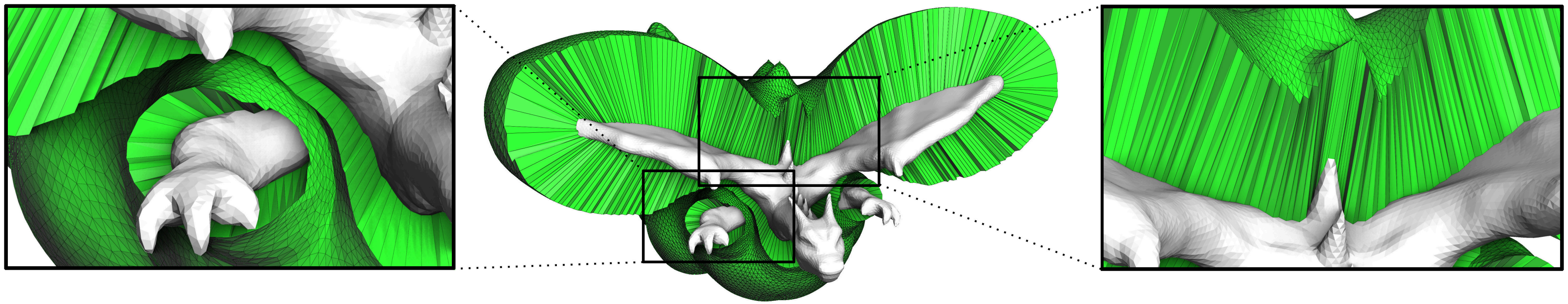

https://github.com/ssloy/invertible-maps

Thank you for your attention!