Internship proposal

Contact

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Title

Smoothed analysis of walking strategies in Delaunay triangulation

Context

Smoothed analysis

On the one hand, classical analysis of geometric algorithms is done in the worst case. The worst case is often very pessimistic and need to designed carefully pathological data set to be reached. On the other hand, analysis of geometric algorithms can also be done under probabilistic hypotheses on the data distribution. Unfortunately, common hypotheses, such as Poisson distribution or uniform distribution, are often too optimistic compared to real world data.

In 2004, Spielman and Teng [8] introduced a new technique called *smoothed analysis* to interpolate between this two situations. A particular data set is analyzed under the hypothesis of some level of noise δ, then the worst case on all data set of size n is taken. The complexity obtained depends on n and δ. Spielman and Teng proved that the simplex algorithm for linear programming, whose worst case is exponential in n, decreases quickly with δ removing the exponential behavior if δ is a reasonable function of n.

Recently, we proposed a smoothed analysis of the size of the convex hull [4]. As an example of result, in a lemma, we prove that if each vertex of the regular n-gon is perturbed by a uniform noise in a disk of radius $\delta \in [\frac{1}{n^2}, 1]$ the complexity of the convex hull is $O(n^{\frac{1}{4}}\delta^{-\frac{3}{8}})$ and thus decreasing with δ.

Walking algorithms

Point location in a planar subdivision is a classical problem of computational geometry for which several data structures have been designed with good complexities in the worst case [7]. These intricate solutions are often unused in favor of simpler algorithms based on traversal of the planar subdivisions using neighborhood relations between faces, also known as *walking algorithms* [1, 2, 5]. These walking algorithms can also be used as a building block in randomized data structures for point location [6, 3].

Amongst planar subdivisions, Delaunay triangulations received a lot of attention because of their practical importance. And amongst walking strategies, the simplest to analyze is the straight walk. The straight walk visits all faces crossed by a line segment between a known face and the query point (see Figure 1).

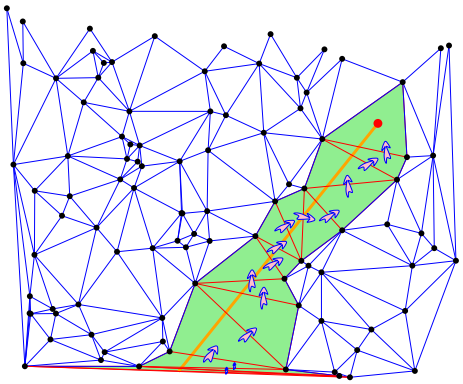

Figure 1: Straight walk in a Delaunay triangulation.

Subject

The topic of the internship is to obtained a smoothed analysis of the straight walk. A first objective is to start with some particular configurations where the worst case for the walk is obtained and analyze how the complexity changes when some noise is introduced (see Figure2).

We will look at two different kind of noise: uniform noise in a disk of radius δ and Gaussian noise.

The aim is to obtained precise bounds through theoretical analysis, but to better understand the problem, the intern will also code simulations using the CGAL library [9].

Skills

Some knowledge in probablity is a good thing,

To conduct simulations, knowledge of C++ is welcome.

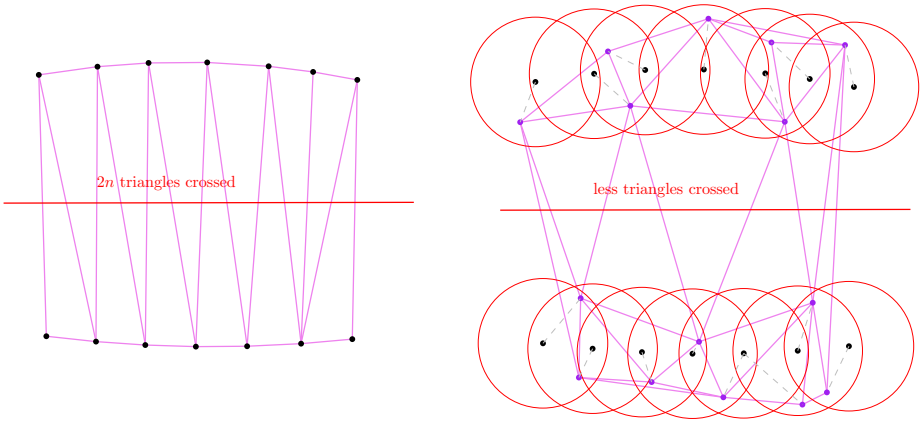

Figure 2: Worst case (left) and perturbed worst case (right) for the straight walk in a Delaunay triangulation.

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