Introduction to the IEEE 1788-2015 Standard for Interval Arithmetic

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Agenda

Verified, guaranteed computations: Interval arithmetic

Principle

Numbers are replaced by intervals.

π is replaced by $[3.14159, 3.14160]$ or $[3.14, 3.15]$ ou $[3, 4]$.

Fundamental theorem (Thou Shalt Not Lie): the interval contains the exact value(s).

Interval arithmetic

How should $\sqrt{[-1, 2]}$ be defined?

By convention, $\sqrt{[-1, 2]} = \sqrt{[0, 2]} = [0, \sqrt{2}]$.
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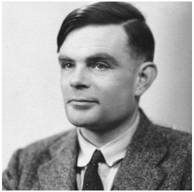
The computation should signal that something odd happened:

- how? by "decorating" intervals: 
- how can it be done without harm for the performances, in terms on computing time, memory usage?

Regarding the chosen definitions and conventions: this is the topic of this talk.

Historical remarks



Alan Turing (23-06-1912 - 07-06-1954)
according to Wilkinson in his Turing lecture (1970).

- **Foundations** in the late fifties and in the sixties.
- **Childhood** until the seventies.
- **Popularization** in the 1980, German school (U. Kulisch).
- **IEEE-754 standard for floating-point arithmetic** in 1985.
- **Since the nineties:** interval **algorithms**.
- **IEEE-1788 standard for interval arithmetic** in 2015.

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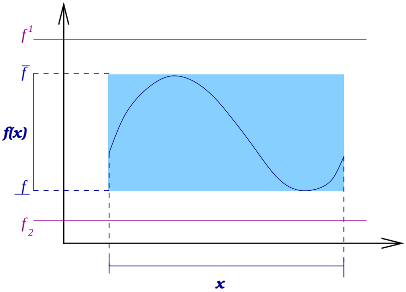
Fundamental Theorem of Interval Arithmetic

Interval computations are **guaranteed** computations.

FTIA: Fundamental Theorem of Interval Arithmetic
Let $\mathbf{x}_1, \ldots, \mathbf{x}_n$ be intervals,
let f be an arithmetic expression defined over $\mathbf{x}_1, \ldots, \mathbf{x}_n$,
the $\mathbf{y}$ obtained by interval evaluation of f over $\mathbf{x}_1, \ldots, \mathbf{x}_n$ encloses the range $f(\mathbf{x}_1, \ldots, \mathbf{x}_n)$.

Set computing

On $\boldsymbol{x}$, are the extrema of the function $f > f^1$, $< f_2$?



No if $f(\boldsymbol{x}) = [\underline{f}, \overline{f}] \subset [f_2, f^1]$.

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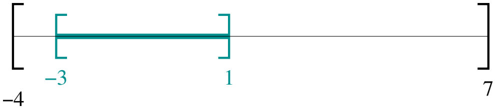
Constraint solving

Can we solve

$$[-3.1] + z = [-4, 7]?$$



$$z = [-4, 7] - [-3, 1] = [-5, 10]$$

is not the solution:

$$[-3, 1] + [-5, 10] = [-8, 11] \neq [-4, 7].$$

A (the) solution exists: $z = [-1, 6]$.

$$z = [-4 - (-3), 7 - 1].$$

Constraint solving

Can we solve

$$\boldsymbol{x} + \boldsymbol{z} = \boldsymbol{y} \quad : \quad [\underline{x}, \overline{x}] + [\underline{z}, \overline{z}] = [\underline{y}, \overline{y}]?$$

Yes if $\mathrm{width}(\boldsymbol{y}) \geq \mathrm{width}(\boldsymbol{x})$ and the solution is given by

$$\boldsymbol{z} = [\underline{y} - \underline{x}, \overline{y} - \overline{x}].$$

This is neither the addition nor the subtraction of intervals, it is a new operation so that the addition has a reciprocal.
It is very useful to solve constraints such as $x + z \geq -4$ and $x + z \leq 7$ with $x \in [-1, 3]$ for instance.

Agenda

Algorithm: solving a nonlinear system: Newton

Why a specific iteration for interval computations?

Usual formula:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Direct interval transposition:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \frac{f(\boldsymbol{x}_k)}{f'(\boldsymbol{x}_k)}$$

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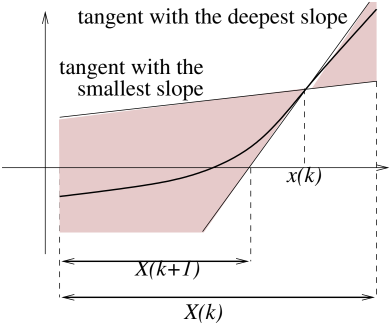
Width of the resulting interval:

$$w(\mathbf{x}_{k+1}) = w(\mathbf{x}_k) + w\left(\frac{f(\mathbf{x}_k)}{f'(\mathbf{x}_k)}\right) > w(\mathbf{x}_k)$$

divergence!

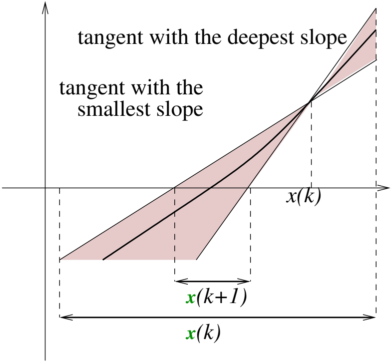
Algorithm: interval Newton (Hansen-Greenberg 83, Baker Kearfott 95-97, Mayer 95, van Hentenryck et al. 97)



$$\mathbf{x}_{k+1} := \left(x_k - \frac{\mathbf{f}(\{x_k\})}{\mathbf{f}'(\mathbf{x}_k)} \right) \bigcap \mathbf{x}_k$$

Interval Newton: Brouwer theorem

If the new iterate (before intersection) is a subset of the previous iterate, then f has a zero on it.
Furthermore, if it is included in its interior, then this zero is unique.



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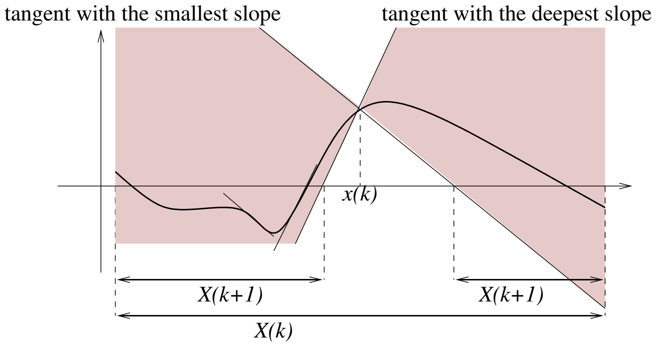
Interval Newton and the extended division



$$(\mathbf{x}_{k+1,1}, \mathbf{x}_{k+1,2}) := \left(x_k - \frac{\mathbf{f}(\{x_k\})}{\mathbf{f}'(\mathbf{x}_k)} \right) \bigcap \mathbf{x}_k$$

Agenda

Precious features of interval arithmetic

- **Fundamental theorem of interval arithmetic ("Thou shalt not lie"):** the returned result contains the sought result: abbreviated as FTIA;
- **constraint solving:** reverse operations are needed: abbreviated as CS;
- **Brouwer theorem:** proof of existence (and uniqueness) of a solution: abbreviated as Brouwer;
- **ad hoc, extended division:** gap between two semi-infinite intervals is preserved: abbreviated as extended division.

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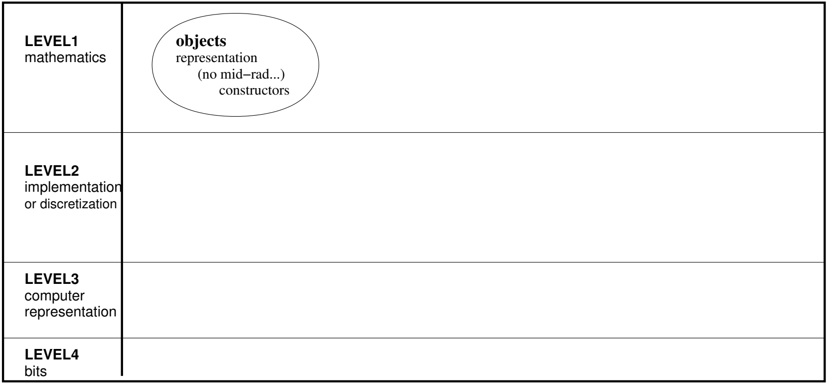
Goal of a standardization: keep the nice properties, have common definitions.

IEEE-1788 standard: the big picture

LEVEL1 mathematics	
LEVEL2 implementation or discretization	
LEVEL3 computer representation	
LEVEL4 bits	

IEEE-1788 standard: the big picture



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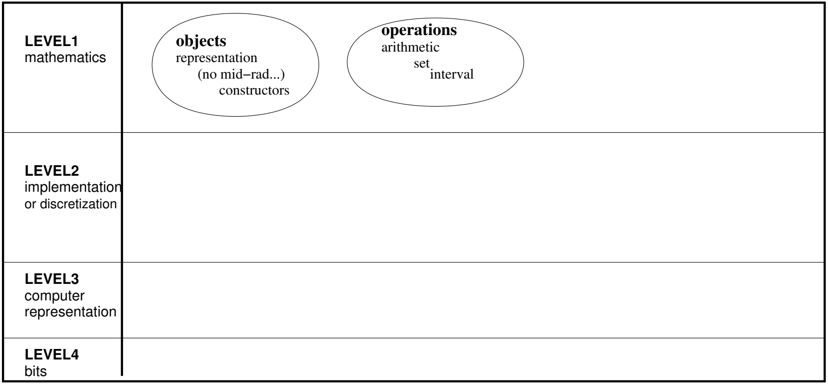

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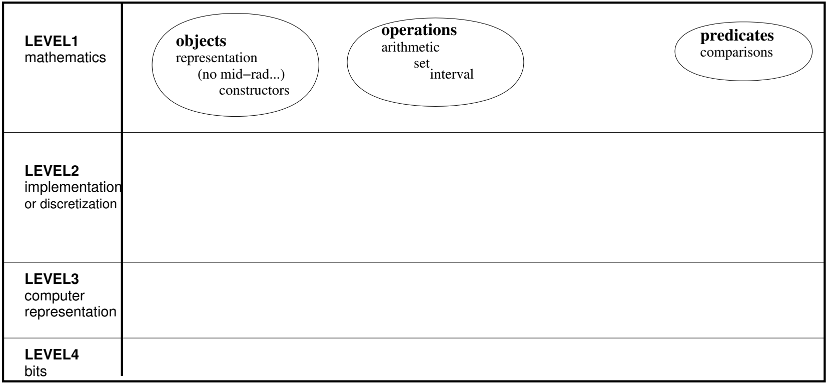

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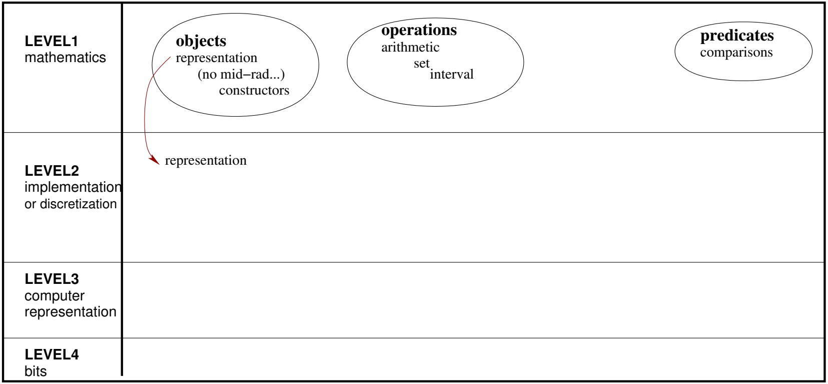

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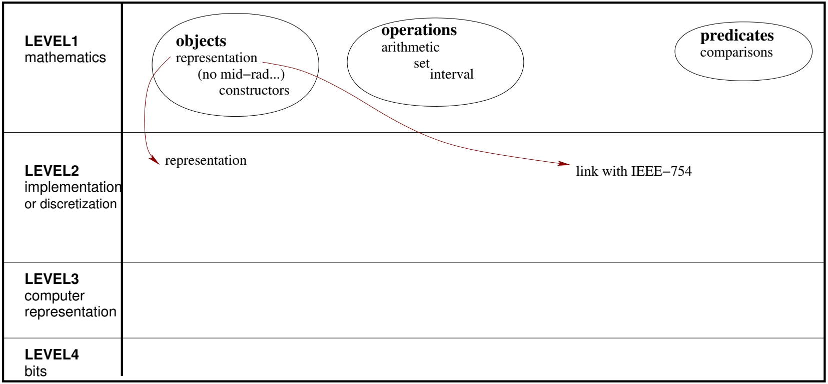

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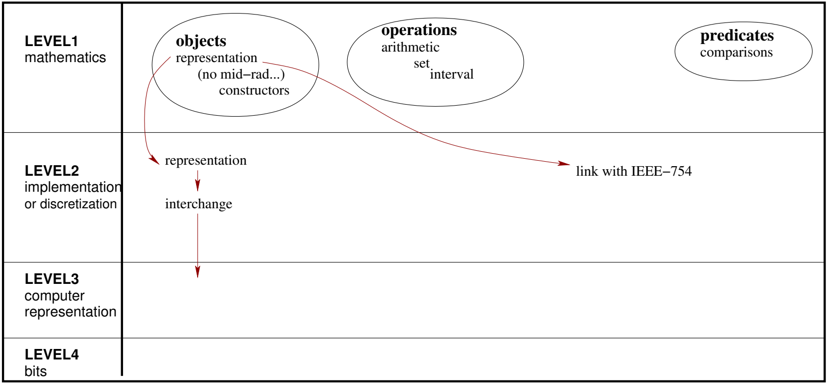

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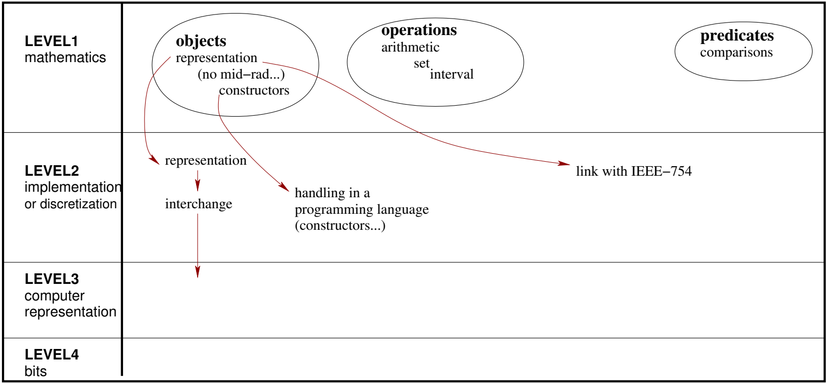

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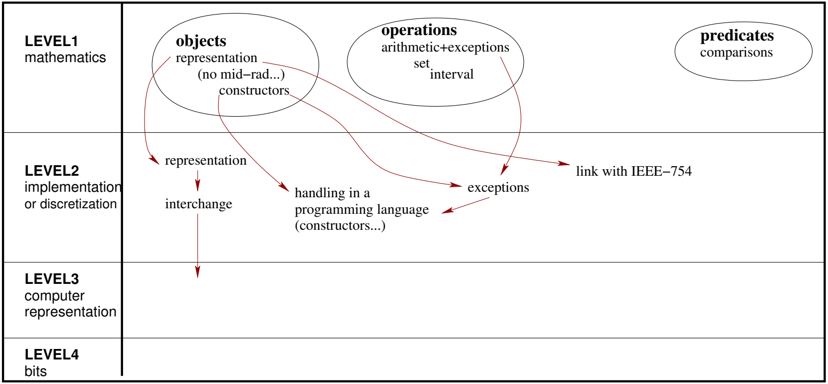

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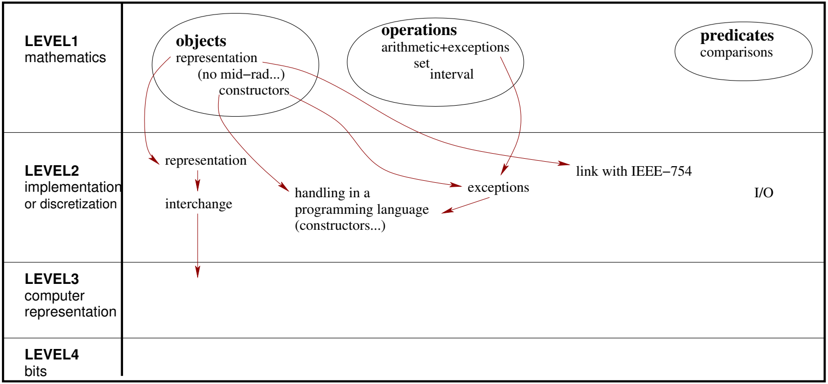

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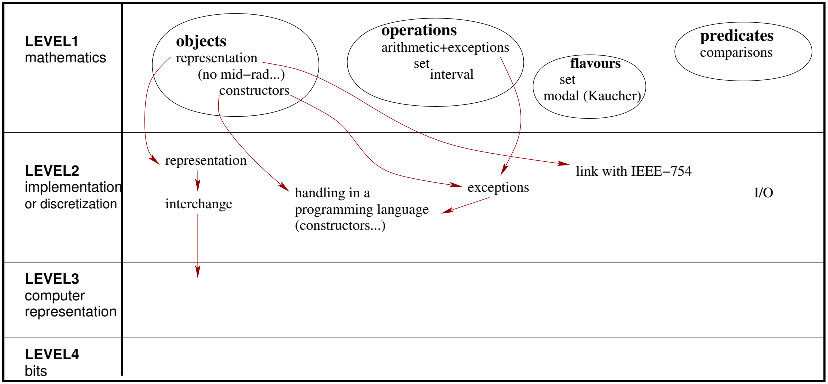
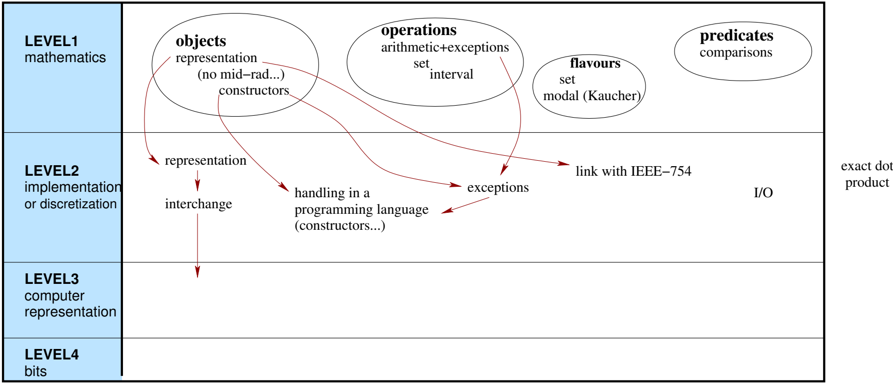


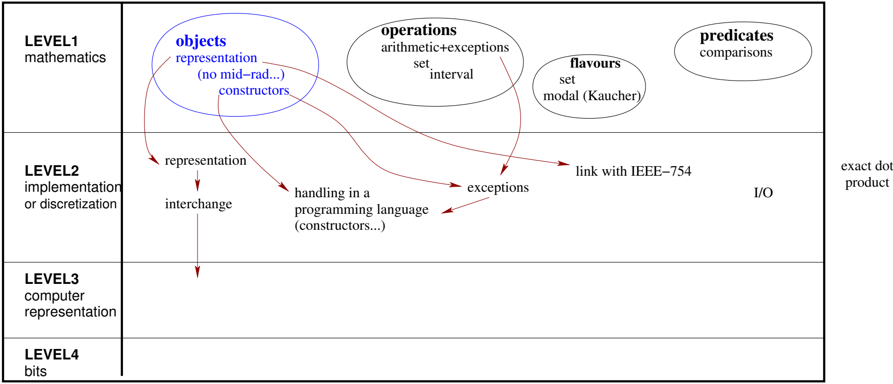
IEEE-1788 standard: levels



IEEE-1788 standard: levels

- **Level 1: mathematical level**
  $[0, \pi]$
- **Level 2: discretization level**
  $[\mathrm{rl}(0), \mathrm{ru}(\pi)]$ where rl and ru map a real number to a number from an abstract finite set; **issues: going from a continuous to a discrete, finite set**;
- **Level 3: computer representation level**
  $[\mathrm{zero}, \mathrm{pi}]$ where $\mathrm{zero} =$ RD (0), $\mathrm{pi} =$ RU (π) are two `binary64` numbers: **the discrete finite set is specified**;
- **Level 4: encoding level**
  `0x8000000000000000`
  `0x400921fb54442d19`

IEEE-1788 standard: intervals



IEEE-1788 standard: intervals

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What about $\emptyset$?

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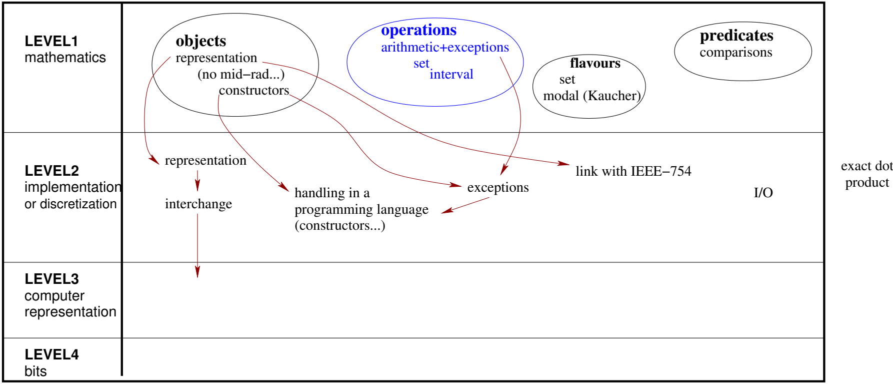
What about $\emptyset$? $[5, +\infty)$? $[3, 1]$?

Common basis: **an interval is a non-empty bounded closed connected subset of $\mathbb{R}$.**

IEEE-1788 standard: operations



IEEE-1788 standard: operations

Operations are defined so as to satisfy **FTIA**:

- $[-1, 3] + [4, 8] = [3, 11]$
- $[2, 5] - [1, 2] = [0, 4]$
- $[2, 3] * [-2, 1] = [-6, 3]$
- $[2, 3]/[1, 2] = [1, 3]$
- $\sqrt{[1, 2]} = [1, \sqrt{2}]$
- $\sin([3, 5]) \subset [-1, +0.14113]$
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Common basis: **an operation φ evaluated on interval arguments $\mathbf{x}_1, \ldots, \mathbf{x}_k$ within its domain returns its range on these arguments (or an enclosure of it)**.

IEEE-1788 standard: operations

Operations specific to sets:

- intersection: $[2, 4] \cap [3, 7] = [3, 4]$
- convex hull of the union: $[-2, -1] \cup [3, 7] = [-2, 7]$

and to intervals:

- infimum, supremum: $\inf([-1, 3]) = -1$, $\sup([-1, 3]) = 3$
- midpoint: $\mathrm{mid}([-1, 3]) = 1$
- width, radius: $\mathrm{wid}([-1, 3]) = 4$, $\mathrm{rad}([-1, 3]) = 2$
- magnitude, mignitude: $\mathrm{mag}([-1, 3]) = 3$, $\mathrm{mig}([-1, 3]) = 0$

IEEE-1788 standard: operations

`cancelPlus` and `cancelMinus` are defined as reverse operations for $+$ and $-$:

- $\texttt{cancelMinus}([2,5],[1,3]) = [1,2]$
- $\texttt{cancelPlus}([2,5],[1,3]) = \texttt{cancelMinus}([2,5],-[1,3]) = [5,6]$

so as to satisfy CS.

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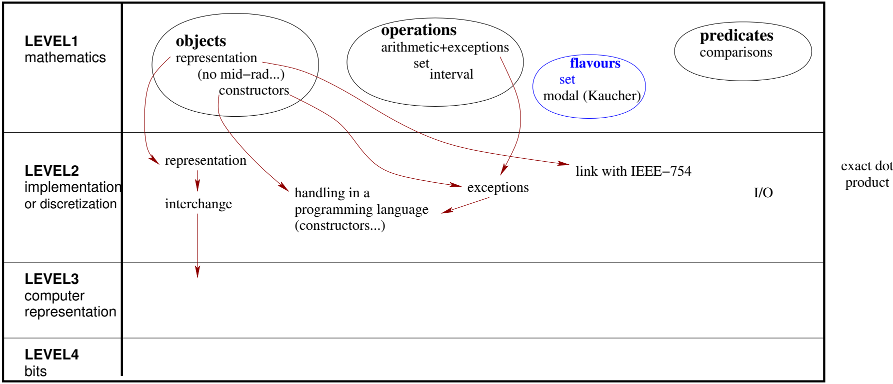
IEEE-1788 standard: flavors



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IEEE-1788 standard: flavors

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IEEE 1788-2015 defines a common basis (seen so far) and provides "hooks" to accommodate a theory, called a **flavor**.

The only available flavor in IEEE 1788-2015 is the **set-based flavor**. Any implementation is IEEE 1788-2015 compliant if it provides at least one flavor.

IEEE-1788 standard: set-based flavor

Set-based interval: a ~~non-empty bounded~~ closed connected subset of $\mathbb{R}$. Ex.: $\emptyset$ $[5, +\infty)$ ~~$[3, 1]$~~

IEEE-1788 standard: set-based flavor

Set-based interval: a ~~non-empty bounded~~ closed connected subset of $\mathbb{R}$. Ex.: $\emptyset$ $[5, +\infty)$ ~~$[3, 1]$~~

Set-based operation:

$$\varphi(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k) = \mathrm{Hull}\{\varphi(x_1, \ldots, x_k) : (x_1, \ldots, x_k) \in (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k) \cap \mathrm{Dom}(\varphi)\}.$$

$$[2, 3]/[0, 2] = [1, +\infty), \quad [2, 3]/[-1, 2] = (-\infty, +\infty), \quad \sqrt{[-1, 2]} = [0, \sqrt{2}].$$

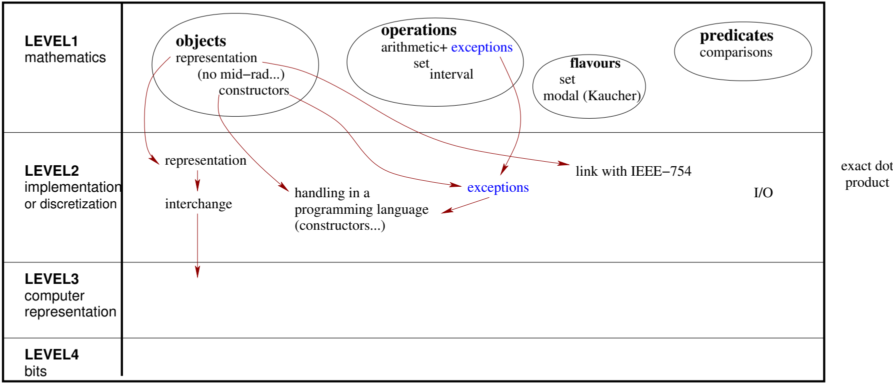
IEEE-1788 standard: set-based flavor

Some more operations, specific to the set-based flavor:

- more reverse operations: for $|.|$, sqr, sin. . . .
  Ex.: $\mathrm{sqrRev}([1, 4]) = \mathrm{Hull}([-2, -1] \cup [1, 2]) = [-2, 2]$,
  $\mathrm{sinRev}([-0.3, 0.5], [3\pi, 5\pi]) \subset [9.4247, 15.708]$.

- `mulRevToPair` corresponds to the extended division:
  $[2, 3]/[-1, 2] = ((-\infty, -2], [1, +\infty))$.

IEEE-1788 standard: decorations



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Mechanism for handling exception: **decoration**.

IEEE-1788 standard: decorations

Best way to handle exceptions?

To avoid global flags, "tags" attached to each interval: decorations.

Meaning (in the set-based flavor): piece of information regarding the history, the computation that led to the interval.

Discussions about what should be in the decorations.
For set-based flavor:

- `com` for common,
- `dac` for defined and continuous,
- `def` for defined,
- `trv` for no information (trivial),
- `ill` for nowhere defined, or ill-formed.

IEEE-1788 standard: decorations

Propagation rule (for the set-based flavor):

$$(\boldsymbol{y}, d_y) = \varphi((\boldsymbol{x}_1, d_1), \ldots, (\boldsymbol{x}_k, d_k))$$

where

- $\boldsymbol{y} = \varphi(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k)$;
- d is a decoration corresponding to the application of φ to $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k$;
- $d_y = \min(d, d_1, \ldots, d_k)$ where the order is `com` $>$ `dac` $>$ `def` $>$ `trv` $>$ `ill`.

IEEE-1788 standard: decorations

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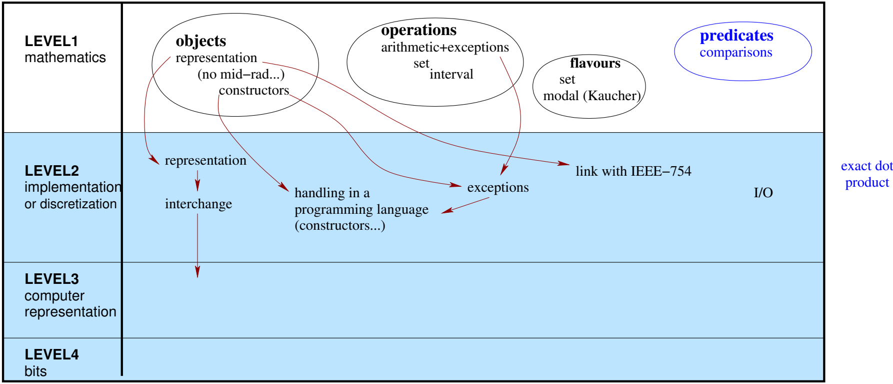
FTDIA for the set-based flavor: the FTIA holds and the decorations mean what they are meant to mean.

IEEE-1788 standard: misc



IEEE-1788 standard: comparisons

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- **7 relations:** equal ($=$), subset ($\subset$), less than or equal to ($\leq$), precedes or touches ($\preceq$), interior to, less than ($<$), precedes ($\prec$).
- **predicates:** before, meets, overlaps, starts, containedBy, finishes, equal, finishedBy, contains, startedBy, overlappedBy, metBy, after.

IEEE-1788 standard: exact dot product

Recommended operation: exact dot product or **edp**.
This operation concerns vectors of floating-point numbers, not vectors of intervals.

IEEE-1788 standard: levels

Level 2 issues:

- rounding: at Level 1, $\boldsymbol{x} = [\underline{x}, \overline{x}]$ – at Level 2, $\boldsymbol{x}$ is represented as $[\,\mathrm{RD}\,(\underline{x}),\ \mathrm{RU}\,(\overline{x})]$;
- similar issue for the result of each operation;
- cornercases: $\mathrm{wid}(\emptyset)$? $\mathrm{mid}(\mathbb{R})$? By convention: NaN, or 0...
- representation: by endpoints, by midpoint-radius
- constructors.

Level 3 issues:

- issues mostly related to IEEE 754-2008.

Level 4 issues:

- issues mostly related to IEEE 754-2008;
- encoding of decorations is specified.

Agenda

IEEE 1788-2015 compliant libraries

The IEEE 1788-2015 compliant libraries have been developed after the standard, as proof-of-concept mostly:

- **Octave interval** by Oliver Heimlich
- **libieee1788** by Marco Nehmeier
- jInterval : Java interface for several libraries + compliance-tests: Dmitry Nadezhin, Sergei Zhilin

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Both developers/maintainers have left academia: what is the future of these libraries?

New flavors?

Flavors mentioned during the development of IEEE 1788-2015:

- cset
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Any person proposing a new flavor should submit it as a revision of the standard.

Anyway, the standard will incur a revision for 2025.

Agenda

Why do I like working on interval arithmetic?

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because there is plenty of research to do. . .

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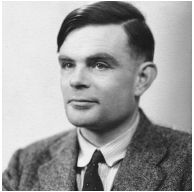
because there is plenty of research to do. . .

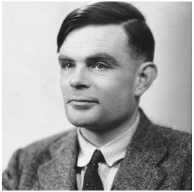
because interval arithmetic is magic!

Magic interval arithmetic

Alan Turing (23-06-1912 - 07-06-1954)



Magic interval arithmetic



Alan Turing (23-06-1912 - 07-06-1954)

- number between 1 and 100 $\in [1,\ 100]$

Magic interval arithmetic

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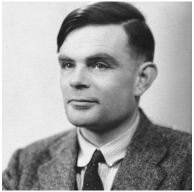
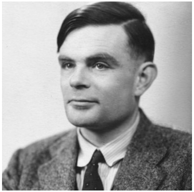


- number between 1 and 100 $\in [1, 100]$
- multiply by birth day: $\in [1, 31]$ $\in [1, 3100]$

Magic interval arithmetic



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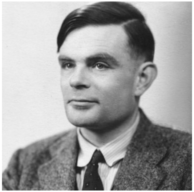

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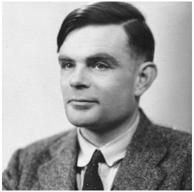

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- divide by 8xxx: $\in [8000, 8999]$ $\in (0, 0.4)$
- subtract 241 $\in (-241, -240.6)$
- divide by 4 $\in (-60.25, -60.15)$

Magic interval arithmetic

5	10	21	24
22	23	6	9
22	19	12	7
11	8	19	20