1997 Report on Integrated Action Procope 97036 *Design and implementation of efficient algorithms for computer algebra*

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1 Travels and events

January. To start the Integrated Action and to make the participants know each other, a meeting was organized on January 10 in Oberwolfach. Participants from the german side were: Benno Fuchssteiner, Klaus Drescher, Ralf Kraume, Christian Heckler, Torsten Metzner, Ralf Hillebrand, Andreas Sorgatz, Oliver Kluge, Frank Postel. From the french side: Fabrice Rouillier and Paul Zimmermann. Some interested german colleagues from Karlsruhe took also part to this meeting: Werner Seiler, Winfried Fakler and Eckhard Pflügel.

February. The Internet address `www.mupad.de` was created on February 21st.

March. Paul Zimmermann visited the MuPAD group in Paderborn 2 days on March 13 and 14, during the CeBIT in Hannover, where MuPAD was exhibited. In March too, MuPAD was reviewed by Barry Simon in the *Desktop Engineering* journal [9].

April. Andreas Sorgatz visited INRIA-Lorraine 2 days on April 7 and 8.

May. Oliver Kluge visited INRIA-Lorraine 2 days on May 29 and 30.

June. Christian Heckler and Torsten Metzner visited INRIA-Lorraine 4 days from June 16 to June 19.

July. Andreas Sorgatz participated to the joint ISSAC/PASCO/IMACS conferences in Maui, where he was awarded for the best poster [10]. He also presented there the MuPAD solutions to the ISSAC System Challenge, which were prepared by Stefan Wehmeier, Walter Oevel and Paul Zimmermann.

August. Paul Zimmermann visited the MuPAD group 3 days from August 4 to 6.

Other visits planned for 1997: Fabrice Rouillier, Vladimir Grebinski and Paul Zimmermann plan to visit the MuPAD group during the MuPAD workshop in September (22 and 23).

Frank Postel plans to visit INRIA-Lorraine in October or November.

All those informations are available on `http://www.loria.fr/~zimmerma/mupad/procope.html` on the french side and `http://www.mupad.de/MuPAD/PROJECTS/procope.html` on the german side.

2 Scientific work

Firstly, to enable common distributed development, Frank Postel installed in January a new version of the CVS system in Paderborn so that people from Karlsruhe and Nancy can easily access to the development version of MuPAD and add new algorithms.

2.1 Ordinary Differential Equations (ODEs)

During the meeting in Oberwolfach, it was discussed about the current state of the ODE package and the future developments. A new strategy was decided to optimally use the existing algorithms to find rational, exponential (designed and implemented by Eckhard Pflügel [7]) and liouvillian [4] solutions of linear ODEs with polynomial coefficients. A new and very efficient algorithm from Barkatou [1] to find polynomial solutions of linear equations was implemented by Eckhard Pflügel and optimized for MuPAD by Paul Zimmermann.

```
>> Ln:=n*x*diff(y(x),x,x)-x*diff(y(x),x)+n^2*y(x):
>> time(ode::polynomialSolutions(subs(Ln,n=20),y(x)));

                                        3550
```

Winfried Fakler implemented a test to recognize linear ordinary differential equations, and many useful auxiliary functions. He also implemented d'Alembert method for the reduction of homogeneous ODEs (integrated to the ODE package by P. Zimmermann), the computation of unimodular equations, functions for dealing with equations of Fuchsian type, as well as for finding factors of order 1 or 2 in linear ODEs.

Werner Seiler made a proposal for new categories and domains to deal with differential functions and equations, which was already partially implemented in the domains package of MuPAD.

2.2 Recurrence Solver

Paul Zimmermann implemented algorithms to find polynomial solutions of linear recurrences with constant and polynomial coefficients, and some hypergeometric solutions of linear recurrences:

```
>> solve(rec(n*u(n+2)-5*u(n+1)-(n+1)*u(n)=0, u(n)));

                              {    /         2      \ }
                              {    | n   3 n      3 | }
                              { a3 | - - ---- + n   | }
                              {    \ 2    2         / }
```

Some auxiliary functions were also implemented, like the Borel transform which gives the recurrence for $u_n/n!$ from that for u_n.

2.3 Numerical evaluation of special functions

A first step towards the certified numerical evaluation of special functions was done, namely an efficient arbitrary precision interval arithmetic was implemented by Manfred Radimersky (student in Paderborn, advised by Oliver Kluge, Frank Postel and Paul Zimmermann) on top of the Pari system [2] which is used for MuPAD's arithmetic. This represents about 4000 lines of C code, and allows all kinds of directed rounding specified by the IEEE 754 standard [6]. This extension will provide firstly certified computations for the four basic operations ($+$, $-$, $\times$, $/$); it is planned to extend it to the elementary functions ($\sin, \cos$) and to the special functions (ζ, ψ) in the second year of the Integrated Action.

2.4 Algebraic systems

During Andreas Sorgatz' visit in Nancy in April, Fabrice Rouillier and he designed an interface between MuPAD and the systems GB and RealSolving developped by Jean-Charles Faugère and Fabrice Rouillier inside the European Frisco project. This interface enables one to use the efficient implementations of GB and RealSolving to symbolically solve algebraic systems (either with Groebner bases or Rational Univariate Representations), then to use the efficient factorization functions from MuPAD to further simplify the solution, and the MuPAD implementation of Schönhage's algorithm to find numerical approximations of the solutions.

2.5 Pattern Matching

During Oliver Kluge's visit in Nancy, the efficient associative-commutative matching program from Steven Eker [3] was integrated into MuPAD by Oliver Kluge.

```
>> ac_match(a+b, c+2+x, {}, {hold(_plus)}, {});

                                   {b = 2, a = x + c}
```

It is now planned to improve the user-interface for this pattern matcher, with the possibility to use it to program with rewriting rules.

2.6 Parallel version

Torsten Metzner and Christian Heckler made the instructions `break` and `return` work as described in the MuPAD manual, i.e. other tasks in loops are now killed. A report has been written, which describes the current state of the parallel version of MuPAD, current problems and future developments [5].

3 Workplan for 1998

3.1 Numerical Evaluation of Special functions

It is planned to integrate the work of Manfred Radimersky (extension of Pari to interval arithmetic) into MuPAD. This will provide an efficient arbitrary precision interval arithmetic, on top of which MuPAD procedures will be developed for the certified numerical evaluation of elementary and special functions.

3.2 Distributed version

It is planned to make a first prototype of the distributed version using either the PVM or MPI tools for distributed computing, and the machines of the Centre Charles Hermite at INRIA Lorraine.

3.3 Development of the library

Fabrice Rouillier, Jean-Charles Faugère and Stefan Wehmeier will work together on algebraic systems and polynomials ideals, developing news domains to deal with them and efficient procedures through the GB/RealSolving interface, with support from Andreas Sorgatz for the module part.

Manfred Radimersky and Paul Zimmermann will continue working on the interval arithmetic package, its integration into MuPAD and the development of algorithms using it.

Guillaume Hanrot will work on arithmetics and algorithms requiring a high mathematical knowledge.

Eckhard Pfluegel, Paul Zimmermann and Frank Postel will continue working on the ODE package, mainly improving the current strategies. A new version of the paper [8] will be written describing the algorithms used to solve ODEs, with several examples. This should appear as chapter in a book edited by Michael Wester.

Isabelle Dutour will work together with Benno Fuchssteiner on interactive books, and with Paul Zimmermann on combinatorics and random generation.

Vladimir Grebinski will work with Torsten Metzner and Christian Heckler on the parallel implementation of algorithms in the field of combinatorial research, with many applications to biology.

Name	Fonction	Travels	Weeks	Place	Subject
Paul Zimmermann	Chargé de recherche	3	2	Nancy	Project Leader
Fabrice Rouillier	Chargé de recherche	1	1	Nancy	algebraic systems
Jean-Charles Faugère	Chargé de recherche	1	1	Paris	algebraic systems
Guillaume Hanrot	Chargé de recherche	1	1	Paris	arithmetics
Eckhard Pfluegel	Doctor	1	1	Grenoble	ODEs
Isabelle Dutour	Doctor	1	1	Nancy	combinatorics
Vladimir Grebinski	PhD student	1	1	Nancy	biology
Benno Fuchssteiner	Prof. Dr	2	1	Paderborn	Project Leader
Christian Heckler	Doctor	2	2	Paderborn	Parallelism
Torsten Metzner	PhD student	2	2	Paderborn	Parallelism
Stefan Wehmeier	PhD student	1	1	Paderborn	algebraic systems
Manfred Radimersky	MS student	1	1	Paderborn	interval arithmetic
Frank Postel	PhD student	1	1	Paderborn	ODEs
Andreas Sorgatz	PhD student	1	1	Paderborn	modules

References

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[2] Batut, C., Bernardi, D., Cohen, H., and Olivier, M. *User's Guide to PARI-GP*, Jan. 1995. URL `ftp://megrez.math.u-bordeaux.fr/pub/pari`.

[3] Eker, S. M. Improving the efficiency of AC matching and unification. Research Report 2104, Institut National de Recherche en Informatique et en Automatique, 1993.

[4] Fakler, W. On second order homogeneous linear differential equations with liouvillian solutions. *Theoretical Comput. Sci.*. To appear.

[5] Heckler, C., Metzner, T., and Zimmermann, P. Progress report on parallelism in MuPAD. Research Report 3154, Institut National de Recherche en Informatique et en Automatique, Apr. 1997. 14 pages.

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[8] Postel, F., and Zimmermann, P. A review of the ODE solvers of Axiom, Derive, Maple, Mathematica, Macsyma, MuPAD and Reduce. In *Proceedings of the 5th RHINE workshop on computer algebra* (Saint-Louis (France), Apr. 1996), A. Carriére and L. R. Oudin, Eds., pp. 2.1–2.10.

[9] Simon, B. Symbolic math powerhouses revisited. *Desktop Engineering 2*, 7 (Mar. 1997), 12–53.

[10] Sorgatz, A. Dynamic modules – software integration in mupad. Poster Abstracts, ISSAC'97, Maui, Hawaii, USA, July 1997.