Formal Languages

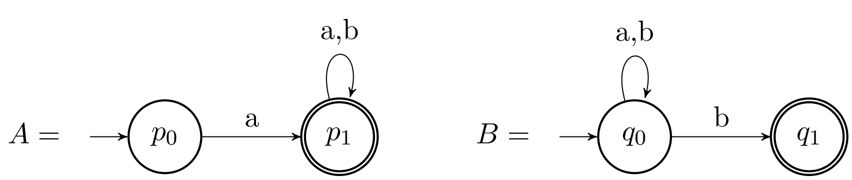
I. Give a finite state automaton that accepts the language specified by the following regular expression:

$$a \cdot (a+b)^* \cdot b$$

II. Let $A = \langle Q_A, \Sigma, \delta_A, q_A, F_A \rangle$, and $B = \langle Q_B, \Sigma, \delta_B, q_B, F_B \rangle$ be two NFSAs. Define an NFSA, $C = \langle Q_C, \Sigma, \delta_C, q_C, F_C \rangle$, as follows:

- $Q_C = \{(p,q) : p \in Q_A \text{ and } q \in Q_B\}$
- $\delta_C((p,q),a) = \{(p',q') : p' \in \delta_A(p,a) \text{ and } q' \in \delta_B(q,a)\}$
- $q_C = (q_A, q_B)$
- $F_C = \{(p,q) : p \in F_A \text{ and } q \in F_B\}$

II.A. Apply the above construction to the two following automata.



II.B. Show, by induction on the length of α, that:

$$\text{if } (p',q') \in \hat{\delta}_C((p,q),\alpha) \text{ then } p' \in \hat{\delta}_A(p,\alpha) \text{ and } q' \in \hat{\delta}_B(q,\alpha).$$

II.C. Conclude that $L(C) \subset L(A) \cap L(B)$.

III. Let $G_1 = \langle N_1, \Sigma_1, P_1, S_1 \rangle$ and $G_2 = \langle N_2, \Sigma_2, P_2, S_2 \rangle$ be two context-free grammars such that $N_1 \cap N_2 = \varnothing$. Define a context-free grammar G_3 such that $L(G_3) = L(G_1) \cup L(G_2)$.