Structures Informatiques et Logiques pour la Modélisation Linguistique (MPRI 2.27.1 - second part)

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Formal semantics

A formal point of view

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory. On this point I differ from a number of philosophers (...).

R. Montague,
Universal Grammar,
Theoria 36:373–398 (1970)

Semantic translations

- Interpret directly natural language utterances into a model (in the Tarskian tradition).
- Give the semantic interpretation of some logic (intensional logic, in Montague's case). Translate natural language utterances as formulas of this logic.

Montague's legacy

- The notion of fragment.
- Semantics as an homomorphic image of syntax.
- Semantic interpretation through a translation into an intermediate logical form.

1. Montague semantics
   - Introduction
   - **A direct naive interpretation**
   - Quantified noun phrases
   - Noun and determiners
   - Relative clauses
   - Adjectives
   - Scope ambiguities
   - De re and de dicto readings

Syntax/semantics interface:

$$\begin{aligned} \text{JOHN} &: \text{NP} \\ \text{MARY} &: \text{NP} \\ \text{LOVES} &: \text{NP} \to \text{NP} \to \text{S} \end{aligned}$$

$$\begin{aligned} [\![\text{NP}]\!] &= \iota \\ [\![\text{S}]\!] &= o \end{aligned}$$

Semantic interpretation:

$$\begin{aligned} [\![\text{JOHN}]\!] &= \mathbf{j} \\ [\![\text{MARY}]\!] &= \mathbf{m} \\ [\![\text{LOVES}]\!] &= \lambda y.\, \lambda x.\, \mathbf{love}\, x\, y \end{aligned}$$

where:

$$\begin{aligned} \mathbf{j}, \mathbf{m} &: \iota \\ \mathbf{love} &: \iota \to \iota \to o \end{aligned}$$

Syntax/semantics interface:

$$\begin{array}{lcl} \text{JOHN} & : & \text{NP} \\ \text{MARY} & : & \text{NP} \\ \text{EVERYBODY} & : & \text{NP} \\ \text{SOMEBODY} & : & \text{NP} \\ \text{LOVES} & : & \text{NP} \to \text{NP} \to \text{S} \end{array}$$

$$\begin{array}{ll} [\![\text{NP}]\!] & = (\iota \to o) \to o \\ [\![\text{S}]\!] & = o \end{array}$$

Semantic interpretation:

$$\begin{array}{ll} [\![\text{JOHN}]\!] & = \lambda k.\, k\, \mathbf{j} \\ [\![\text{MARY}]\!] & = \lambda k.\, k\, \mathbf{m} \\ [\![\text{EVERYBODY}]\!] & = \lambda k.\, \forall x.\, k\, x \\ [\![\text{SOMEBODY}]\!] & = \lambda k.\, \exists x.\, k\, x \\ [\![\text{LOVES}]\!] & = \lambda o.\, \lambda s.\, s\, (\lambda x.\, o\, (\lambda y.\, \mathbf{love}\, x\, y)) \end{array}$$

Syntax/semantics interface:

JOHN	:	NP
MARY	:	NP
EVERYBODY	:	NP
SOMEBODY	:	NP
MAN	:	N
WOMAN	:	N
EVERY	:	$N \rightarrow NP$
A	:	$N \rightarrow NP$
LOVES	:	$NP \rightarrow NP \rightarrow S$

$$[\![N]\!] = \iota \rightarrow o$$
$$[\![NP]\!] = (\iota \rightarrow o) \rightarrow o$$
$$[\![S]\!] = o$$

Semantic interpretation:

$$
\begin{aligned}
[\![\text{JOHN}]\!] &= \lambda k.\, k\,\mathbf{j} \\
[\![\text{MARY}]\!] &= \lambda k.\, k\,\mathbf{m} \\
[\![\text{EVERYBODY}]\!] &= \lambda k.\, \forall x.\, k\,x \\
[\![\text{SOMEBODY}]\!] &= \lambda k.\, \exists x.\, k\,x \\
[\![\text{MAN}]\!] &= \lambda x.\, \mathbf{man}\,x \\
[\![\text{WOMAN}]\!] &= \lambda x.\, \mathbf{woman}\,x \\
[\![\text{EVERY}]\!] &= \lambda n.\, \lambda m.\, \forall x.\, n\,x \supset m\,x \\
[\![\text{A}]\!] &= \lambda n.\, \lambda m.\, \exists x.\, n\,x \wedge m\,x \\
[\![\text{LOVES}]\!] &= \lambda o.\, \lambda s.\, s\,(\lambda x.\, o\,(\lambda y.\, \mathbf{love}\,x\,y))
\end{aligned}
$$

where:

$$\mathbf{woman}, \mathbf{man} : \iota \to o$$

1. Montague semantics
   - Introduction
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   - **Relative clauses**
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   - De re and de dicto readings

Syntax/semantics interface:

$$
\begin{array}{ll}
\text{JOHN} & : \text{NP} \\
\text{MARY} & : \text{NP} \\
\text{EVERYBODY} & : \text{NP} \\
\text{SOMEBODY} & : \text{NP} \\
\text{MAN} & : \text{N} \\
\text{WOMAN} & : \text{N} \\
\text{EVERY} & : \text{N} \rightarrow \text{NP} \\
\text{A} & : \text{N} \rightarrow \text{NP} \\
\text{LOVES} & : \text{NP} \rightarrow \text{NP} \rightarrow \text{S} \\
\text{WHO} & : (\text{NP} \rightarrow \text{S}) \rightarrow \text{N} \rightarrow \text{N}
\end{array}
$$

$$
\begin{aligned}
[\![\text{N}]\!] &= \iota \rightarrow o \\
[\![\text{NP}]\!] &= (\iota \rightarrow o) \rightarrow o \\
[\![\text{S}]\!] &= o
\end{aligned}
$$

Semantic interpretation:

$$
\begin{aligned}
[\![\text{JOHN}]\!] &= \lambda k.\, k\, \mathbf{j}\\
[\![\text{MARY}]\!] &= \lambda k.\, k\, \mathbf{m}\\
[\![\text{EVERYBODY}]\!] &= \lambda k.\, \forall x.\, k\, x\\
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[\![\text{MAN}]\!] &= \lambda x.\, \mathbf{man}\, x\\
[\![\text{WOMAN}]\!] &= \lambda x.\, \mathbf{woman}\, x\\
[\![\text{EVERY}]\!] &= \lambda n.\, \lambda m.\, \forall x.\, n\, x \supset m\, x\\
[\![\text{A}]\!] &= \lambda n.\, \lambda m.\, \exists x.\, n\, x \wedge m\, x\\
[\![\text{LOVES}]\!] &= \lambda o.\, \lambda s.\, s\, (\lambda x.\, o\, (\lambda y.\, \mathbf{love}\, x\, y))\\
[\![\text{WHO}]\!] &= \lambda r.\, \lambda n.\, \lambda x.\, n\, x \wedge r\, (\lambda k.\, k\, x)
\end{aligned}
$$

Syntax/semantics interface:

JOHN	:	NP
MARY	:	NP
EVERYBODY	:	NP
SOMEBODY	:	NP
MAN	:	N
WOMAN	:	N
EVERY	:	$\mathrm{N} \to \mathrm{NP}$
A	:	$\mathrm{N} \to \mathrm{NP}$
FRENCH	:	$\mathrm{N} \to \mathrm{N}$
LOVES	:	$\mathrm{NP} \to \mathrm{NP} \to \mathrm{S}$
WHO	:	$(\mathrm{NP} \to \mathrm{S}) \to \mathrm{N} \to \mathrm{N}$

$$[\![\mathrm{N}]\!] = \iota \to o$$
$$[\![\mathrm{NP}]\!] = (\iota \to o) \to o$$
$$[\![\mathrm{S}]\!] = o$$

Semantic interpretation:

$$
\begin{aligned}
[\![\text{JOHN}]\!] &= \lambda k.\, k\,\mathbf{j}\\
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[\![\text{EVERY}]\!] &= \lambda n.\, \lambda m.\, \forall x.\, n\,x \supset m\,x\\
[\![\text{A}]\!] &= \lambda n.\, \lambda m.\, \exists x.\, n\,x \wedge m\,x\\
[\![\text{FRENCH}]\!] &= \lambda n.\, \lambda x.\, n\,x \wedge \mathbf{french}\,x\\
[\![\text{LOVES}]\!] &= \lambda o.\, \lambda s.\, s\,(\lambda x.\, o\,(\lambda y.\, \mathbf{love}\,x\,y))\\
[\![\text{WHO}]\!] &= \lambda r.\, \lambda n.\, \lambda x.\, n\,x \wedge r\,(\lambda k.\, k\,x)
\end{aligned}
$$

where:

$$\mathbf{french} : \iota \to o$$

Adjective classification:

- **Intersective**:

  French, sick, carnivorous, red, ...
- **Subsective** but non intersective:

  typical, recent, skillful, ...
- **Privative**:

  fake, former, spurious, ...
- **Plain nonsubsective**:

  alleged, arguable, putative, ...

Meaning postulates:

$$\mathrm{INT}(A) = \exists P. \forall Q\, x.\, A\, Q\, x \equiv (P\, x \wedge Q\, x)$$

$$\mathrm{SUB}(A) = \forall Q\, x.\, A\, Q\, x \supset Q\, x$$

$$\mathrm{PRIV}(A) = \forall Q\, x.\, A\, Q\, x \supset \neg(Q\, x)$$

Beware!!!! Some intensionality involved!

Scope ambiguities

Every man loves a woman

$$\forall x.\mathbf{man}\,x \supset (\exists y.\mathbf{woman}\,y \wedge \mathbf{love}\,x\,y)$$
$$\exists y.\mathbf{woman}\,y \wedge (\forall x.\mathbf{man}\,x \wedge \mathbf{love}\,x\,y)$$

Subject wide scope:

$$\lambda o.\,\lambda s.\,s\,(\lambda x.\,o\,(\lambda y.\,\mathbf{love}\,x\,y))$$

Object wide scope:

$$\lambda o.\,\lambda s.\,o\,(\lambda y.\,s\,(\lambda x.\,\mathbf{love}\,x\,y))$$

Another solution:

$$\begin{array}{rcl} \text{every} & : & \mathrm{N} \rightarrow (\mathrm{NP} \rightarrow \mathrm{S}) \rightarrow \mathrm{S} \\ \text{a} & : & \mathrm{N} \rightarrow (\mathrm{NP} \rightarrow \mathrm{S}) \rightarrow \mathrm{S} \end{array}$$

with

$$\begin{array}{rcl} [\![\mathrm{S}]\!] & = & o \\ [\![\mathrm{NP}]\!] & = & \iota \end{array}$$

De re and de dicto readings as scope ambiguities

John seeks a unicorn

$$\exists x.\mathbf{unicorn}\,x \wedge \mathbf{try}\,\mathbf{j}\,(\lambda z.\,\mathbf{find}\,z\,x)$$
$$\mathbf{try}\,\mathbf{j}\,(\lambda z.\,\exists x.\mathbf{unicorn}\,x \wedge \mathbf{find}\,z\,x)$$

De re reading:

$$\lambda o.\,\lambda s.\,o\,(\lambda x.\,s\,(\lambda y.\,\mathbf{try}\,y\,(\lambda z.\,\mathbf{find}\,z\,x)))$$

De dicto reading:

$$\lambda o.\,\lambda s.\,s\,(\lambda y.\,\mathbf{try}\,y\,(\lambda z.\,o\,(\lambda x.\,\mathbf{find}\,z\,x)))$$

Beware!!!! Some intensionality involved!