Structures Informatiques et Logiques pour la Modélisation Linguistique
(MPRI 2.27.1 - second part)

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A man entered the room. He switched on the light.

I am the only one in this group who dares to say that I am wrong.

Every farmer who owns a donkey beats it.

A wolf might come in. It would eat you first.

John does not have a car. He would not know where to park it.

Either there is no bathroom in this apartment or it is in a funny place.

The man who gives his paycheck to his wife is wiser than the man who gives it to his mistress.
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Definition

Terms

\[ t ::= v \mid c \]

Conditions

\[ C ::= \top \mid Pt_1 \ldots t_n \mid v = t \mid v \neq t \mid \neg D \]

Structures

\[ D ::= (\{v_1, \ldots, v_n\}, \{C_1, \ldots, C_m\}) \]
Box notation

\[
\begin{array}{|c|}
\hline
v_1 \cdots v_n \\
C_1 \\
\vdots \\
C_m \\
\hline
\end{array}
\]
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$v_1 \cdots v_n$

\begin{array}{|c|}
\hline
C_1 \\
\vdots \\
C_m \\
\hline
\end{array}

is interpreted as \ \exists v_1 \ldots v_n. C_1 \land \ldots \land C_m
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Merging

A man entered the room. He switched on the light

Every man loves a woman. He smiles at her
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Knowledge-poor approach

*Peter loves Mary. But she does not love him. It is John whom she loves. He is a nicer guy.*

Based on:

- morphological features,
- grammatical roles,
- discourse functions: theme (topic), rheme, ...

Signaling the topic:

- stating it as the subject,
- using the passive voice — to turn an object into the subject,
- clefting (it is from Mary that I learned the news),
- periphrastic constructions (“as for”, “concerning”, “speaking of”, …),
- dislocation, a.k.a. topicalization (Mary, I love her).
The utility (CDVU) shows you a LIST4250, LIST38PP, or LIST3820 file on your terminal for a format similar to that in which it will be printed.
Anaphora resolution

Knowledge based approach

John hid Bill’s keys. He was drunk.

John hid Bill’s keys. He was playing a joke on him.
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Typing the left and right contexts

Montague semantics is based on Church’s simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

- $\iota$, the type of individuals (a.k.a. entities).
- $o$, the type of propositions (a.k.a. truth values).

We add a third atomic type, $\gamma$, which stands for the type of the left contexts.

What about the type of the right contexts?

```
  left context  \downarrow  right context
  \gamma  \quad  \gamma \rightarrow o
  o
```
Revisiting DRT
Left and right contexts

Updating and accessing the context

\[
\text{nil} : \gamma \\
_::_ : \iota \rightarrow \gamma \rightarrow \gamma \\
\text{sel} : \gamma \rightarrow \iota
\]
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Let $s$ be the syntactic category of sentences. Remember that we intend to abstract our notions of left and right contexts over the meaning of the sentences.

$$[[s]] = \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$$
Composition of two sentence interpretations

$$[[S_1 \cdot S_2]] = \lambda e \phi. [[S_1]] e (\lambda e'. [[S_2]] e' \phi)$$
Consider a DRS:

\[
\begin{array}{c}
\vdots \\
C_m \\
C_1
\end{array}
\]

\(x_1 \ldots x_n\)

To such a structure, corresponds the following \(\lambda\)-term of type \(\gamma \rightarrow \gamma \rightarrow o \rightarrow o\):

\[
\lambda e \phi. \exists x_1 \ldots x_n . C_1 \land \ldots \land C_m \land \phi e'
\]

where \(e'\) is a context made of \(e\) and of the variables \(x_1, \ldots, x_n\).
Revisiting DRT

Semantic interpretation of the sentences

Example

*John loves Mary. He smiles at her.*

\[
\begin{align*}
\lbrack John\ loves\ Mary \rbrack &= \lambda e \phi.\ love j m \land \phi (m::j::e) \\
\lbrack He\ smiles\ at\ her \rbrack &= \lambda e \phi.\ smile (sel_{he} e) (sel_{her} e) \land \phi e
\end{align*}
\]
\[ \lambda e\phi. \; \llbracket John \; loves \; Mary\rrbracket \; e \; (\lambda e'. \; \llbracket He \; smiles \; at \; her\rrbracket \; e' \; \phi) \]
\[
= \; \lambda e\phi. \; (\lambda e\phi. \; \text{love} \; j \; m \; \land \; \phi \; (m::j::e)) \; e \; (\lambda e'. \; \llbracket He \; smiles \; at \; her\rrbracket \; e' \; \phi) \\
\rightarrow \beta \; \lambda e\phi. \; (\lambda \phi. \; \text{love} \; j \; m \; \land \; \phi \; (m::j::e)) \; (\lambda e'. \; \llbracket He \; smiles \; at \; her\rrbracket \; e' \; \phi) \; (m::j::e) \\
\rightarrow \beta \; \lambda e\phi. \; \text{love} \; j \; m \; \land \; \llbracket He \; smiles \; at \; her\rrbracket \; (m::j::e) \; \phi \\
= \; \lambda e\phi. \; \text{love} \; j \; m \; \land \; (\lambda e\phi. \; \text{smile} \; (sel \; he \; e) \; (sel \; her \; e) \; \land \; e \; (m::j::e) \; \phi) \\
\rightarrow \beta \; \lambda e\phi. \; \text{love} \; j \; m \; \land \\
\quad (\lambda \phi. \; \text{smile} \; (sel \; he \; (m::j::e)) \; (sel \; her \; (m::j::e)) \; \land \; \phi \; (m::j::e)) \; \phi \\
\rightarrow \beta \; \lambda e\phi. \; \text{love} \; j \; m \; \land \; \text{smile} \; (sel \; he \; (m::j::e)) \; (sel \; her \; (m::j::e)) \; \land \; \phi \; (m::j::e) \\
= \; \lambda e\phi. \; \text{love} \; j \; m \; \land \; \text{smile} \; j \; (sel \; her \; (m::j::e)) \; \land \; \phi \; (m::j::e) \\
= \; \lambda e\phi. \; \text{love} \; j \; m \; \land \; \text{smile} \; j \; m \; \land \; \phi \; (m::j::e) \]
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Montague’s interpretation

\[
\begin{align*}
[s] & = o \\
[n] & = \iota \rightarrow o \\
[np] & = (\iota \rightarrow o) \rightarrow o
\end{align*}
\]

may be rephrased as follows:

\[
\begin{align*}
[s] & = o & (1) \\
[n] & = \iota \rightarrow [s] & (2) \\
[np] & = (\iota \rightarrow [s]) \rightarrow [s] & (3)
\end{align*}
\]

Replacing (1) with:

\[
[s] = \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o
\]

we obtain:

\[
\begin{align*}
[n] & = \iota \rightarrow \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o \\
[np] & = (\iota \rightarrow \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o) \rightarrow \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o
\end{align*}
\]
This interpretation results in handcrafted lexical semantics such as the following:

\[
\begin{align*}
[farmer] & = \lambda x e \phi. \textbf{farmer} \ x \ \land \ \phi \ e \\
[donkey] & = \lambda x e \phi. \textbf{donkey} \ x \ \land \ \phi \ e \\
[owns] & = \lambda o s. (\lambda x. o (\lambda y e \phi. \textbf{own} \ x \ y \ \land \ \phi \ e)) \\
[beats] & = \lambda o s. (\lambda x. o (\lambda y e \phi. \textbf{beat} \ x \ y \ \land \ \phi \ e)) \\
[who] & = \lambda r n x e \phi. n \ x \ e \ (\lambda e. r (\lambda \psi. \psi \ x) e \ \phi) \\
[a] & = \lambda n \psi e \phi. \exists x. n \ x \ e \ (\lambda e. \psi \ x (x::e) \ \phi) \\
[every] & = \lambda n \psi e \phi. (\forall x. \neg (n \ x \ e \ (\lambda e. \neg (\psi \ x (x::e) (\lambda e. T)))))) \ \land \ \phi \ e \\
[it] & = \lambda \psi e \phi. \psi (\textbf{sel} \ e) e \ \phi
\end{align*}
\]

...which might seem a little bit involved.
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Questions

- is there a systematic way of obtaining the new lexical semantics from Montague’s?
- can we find any “modular” presentation of the approach?
- is there some dynamic logic hidden in the approach?
Let $\Omega \triangleq \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$. We intend to design a logic acting on propositions of type $\Omega$.

We share with DRT the two following assumptions:

- discourse composition is mainly conjunctive (roughly speaking, a discourse consists in the conjunction of its sentences);
- the main form of quantification is existential (it introduces referential markers).

Consequently, our logic will be based on conjunction and existential quantification (defined as primitives). The other connectives will be obtained using negation (a third primitive) and de Morgan’s laws.
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We consider a simply-typed $\lambda$-calculus, the terms of which are built upon a signature including the following constants:

**FIRST-ORDER LOGIC**

\[ \top : o \quad \text{(truth)} \]
\[ \neg : o \to o \quad \text{(negation)} \]
\[ \land : o \to o \to o \quad \text{(conjunction)} \]
\[ \exists : (\iota \to o) \to o \quad \text{(existential quantification)} \]

**DYNAMIC PRIMITIVES**

\[ :: : \iota \to \gamma \to \gamma \quad \text{(context updating)} \]
\[ \text{sel} : \gamma \to \iota \quad \text{(choice operator)} \]
Conjunction is nothing but sentence composition. We therefore define:

$$A \land B \triangleq \lambda e \phi. A e (\lambda e. B e \phi)$$
Existential quantification introduces “reference markers”. It is therefore responsible for context updating:

$$\exists x. P x \triangleq \lambda e\phi. \exists x. P x (x::e) \phi$$
Negation

We do not want the continuation of the discourse to fall into the scope of the negation. Consequently, negation must be defined as follows:

\[ \neg A \triangleq \lambda e. \neg (A e (\lambda e. \top)) \land \phi e \]
These are defined using de Morgan’s laws:

\[ A \Rightarrow B \triangleq \neg (A \land \neg B) \]
\[ \forall x. P x \triangleq \neg \exists x. \neg (P x) \]
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Embedding and conservativity

\[
\overline{R t_1 \ldots t_n} = \lambda e \phi. R t_1 \ldots t_n \land \phi e
\]

\[
\overline{\neg A} = \neg \overline{A}
\]

\[
\overline{A \land B} = \overline{A} \land \overline{B}
\]

\[
\overline{\exists x. A} = \exists \overline{x}. \overline{A}
\]

This embedding is such that, for every term \( e \) of type \( \gamma \):

\[
A \equiv \overline{A} e (\lambda e. \top)
\]
Montague-like semantic interpretation:

- $\llbracket \text{farmer} \rrbracket = \text{farmer}$
- $\llbracket \text{donkey} \rrbracket = \text{donkey}$
- $\llbracket \text{owns} \rrbracket = \lambda OS. S (\lambda x. O (\lambda y. \text{own } xy))$
- $\llbracket \text{beats} \rrbracket = \lambda OS. S (\lambda x. O (\lambda y. \text{beat } xy))$
- $\llbracket \text{who} \rrbracket = \lambda RQx. Qx \land R (\lambda P. P x)$
- $\llbracket \text{a} \rrbracket = \lambda PQ. \exists x. P x \land Q x$
- $\llbracket \text{every} \rrbracket = \lambda PQ. \forall x. P x \supset Q x$
- $\llbracket \text{it} \rrbracket = ???$
Dynamic interpretation:

\[ [\text{farmer}] = \lambda x. \texttt{farmer } x \]
\[ [\text{donkey}] = \lambda x. \texttt{donkey } x \]
\[ [\text{owns}] = \lambda OS. S (\lambda x. O (\lambda y. \texttt{own } x y)) \]
\[ [\text{beats}] = \lambda OS. S (\lambda x. O (\lambda y. \texttt{beat } x y)) \]
\[ [\text{who}] = \lambda RQx. Q x \land R (\lambda P. P x) \]
\[ [\text{a}] = \lambda PQ. \exists x. P x \land Q x \]
\[ [\text{every}] = \lambda PQ. \forall x. P x \Rightarrow Q x \]
\[ [\text{it}] = \lambda Pe\phi. P (\texttt{sel } e) e \phi \]
With the dynamic interpretation we have that:

\[
[\text{beats}] [\text{it}] ([\text{every}] ([\text{who}] ([\text{owns}] ([a] [\text{donkey}])))) [\text{farmer}])
\]

\(\beta\)-reduces to the following term (modulo de Morgan’s laws):

\[
\lambda e. (\forall x. \textbf{farmer} \ x \supset \ (\forall y. \textbf{donkey} \ y \supset (\textbf{own} \ x \ y \supset \textbf{beat} \ x \ (\text{sel} (x::y::e)))))) \land \phi_e
\]

that is, assuming that \text{sel} is a “perfect” anaphora resolution operator:

\[
\lambda e. (\forall x. \textbf{farmer} \ x \supset (\forall y. \textbf{donkey} \ y \supset (\textbf{own} \ x \ y \supset \textbf{beat} \ x \ y))) \land \phi_e
\]