Structures Informatiques et Logiques pour la Modélisation Linguistique (MPRI 2.27.1 - second part)

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Inria

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Discourse Analysis

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Examples

A man entered the room. He switched on the light.

I am the only one in this group who dares to say that I am wrong.

Every farmer who owns a donkey beats it.

A wolf might come in. It would eat you first.

John does not have a car. He would not know where to park it.

Either there is no bathroom in this apartment or it is in a funny place.

The man who gives his paycheck to his wife is wiser than the man who gives it to his mistress.

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Definition

Terms

$$t ::= v \mid c$$

Conditions

$$C ::= \top \mid P t_1 \ldots t_n \mid v \doteq t \mid v \neq t \mid \neg D$$

Strucutures

$$D ::= (\{v_1, \ldots, v_n\}, \{C_1, \ldots, C_m\})$$

Box notation

$v_1 \cdots v_n$
C_1 $\vdots$ C_m

Discourse Analysis

Interpretation

$v_1 \cdots v_n$
C_1
$\vdots$
C_m

is interpreted as $\exists v_1 \ldots v_n . C_1 \wedge \ldots \wedge C_m$

Discourse Analysis

Merging

A man entered the room. He switched on the light

x
man x entered_the_room x

$\bullet$

y
switched_on_the_light y

$\rightarrow$

$x\ y$
man x entered_the_room x $y \doteq x$ switched_on_the_light y

Every man loves a woman. $^?$*He smiles at her*

$\neg$ [x | man x, $\neg$ [y | woman y, love x y]]

$\bullet$

$u\ v$
smile_at u v

$\rightarrow$

$u\ v$
$\neg$ [x | man x, $\neg$ [y | woman y, love x y]] smile_at u v

Merging

A man entered the room. He switched on the light

$$\begin{array}{|c|}\hline x \\ \hline \text{man}\ x \\ \text{entered\_the\_room}\ x \\ \hline \end{array} \bullet \begin{array}{|c|}\hline y \\ \hline \text{switched\_on\_the\_light}\ y \\ \hline \end{array} \rightarrow \begin{array}{|c|}\hline x\ y \\ \hline \text{man}\ x \\ \text{entered\_the\_room}\ x \\ y \doteq x \\ \text{switched\_on\_the\_light}\ y \\ \hline \end{array}$$

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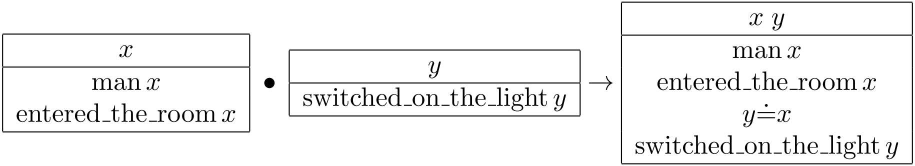

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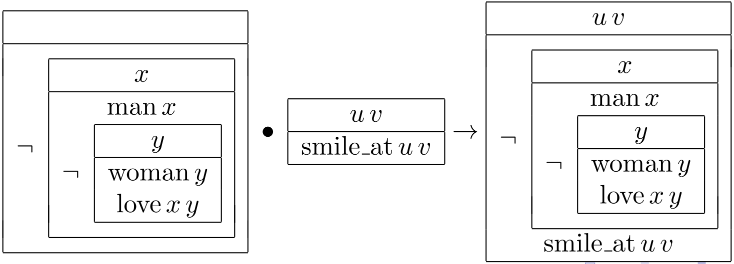

Discourse Analysis

Knowledge-poor approach

Peter loves Mary. But she does not love him. It is John whom she loves. He is a nicer guy.

Based on:

- morphological features,
- grammatical roles,
- discourse functions: theme (topic), rheme, ...

Signaling the topic:

- stating it as the subject,
- using the passive voice — to turn an object into the subject,
- clefting (it is from Mary that I learned the news),
- periphrastic constructions ("as for", "concerning", "speaking of", ...),
- dislocation, a.k.a. topicalization (Mary, I love her).

Knowledge-poor approach + statistical training

The utility (CDVU) shows you a LIST4250, LIST38PP, or LIST3820 file on your terminal for a format similar to that in which it will be printed

Knowledge based approach

John hid Bill's keys. He was drunk.

John hid Bill's keys. He was playing a joke on him.

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Typing the left and right contexts

Montague semantics is based on Church's simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

- $\mathbf{e}$, the type of entities (a.k.a. individuals).
- $\mathbf{t}$, the type of truth values (a.k.a. propositions).

We add a third atomic type, $\mathbf{c}$, which stands for the type of the left contexts.

What about the type of the right contexts?

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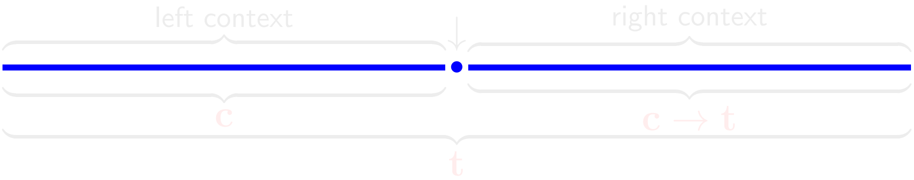

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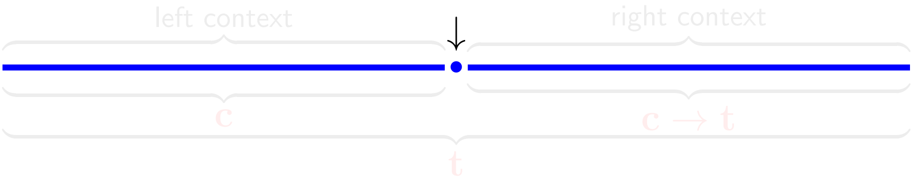

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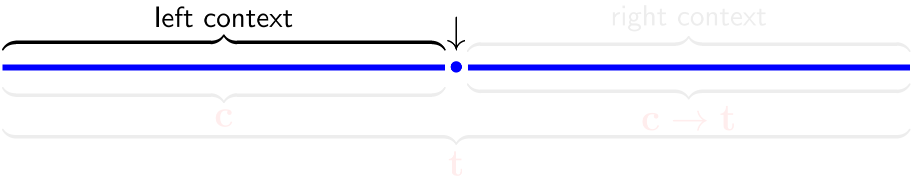

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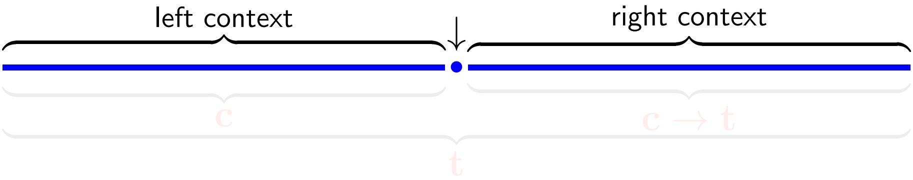

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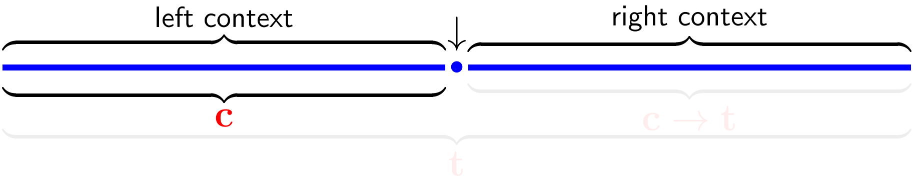

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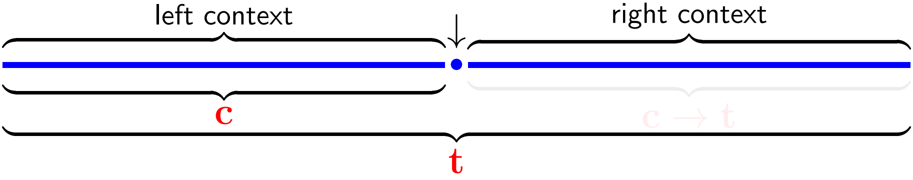

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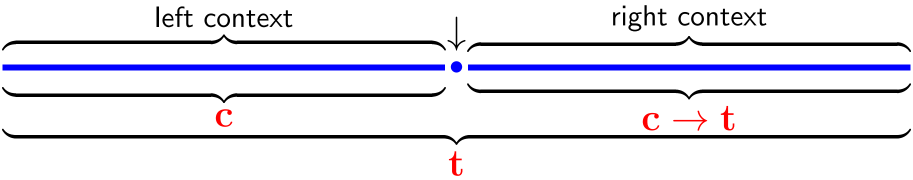

Updating and accessing the context

$$
\begin{array}{lcl}
\texttt{nil} & : & \mathbf{c} \\
\_ :: \_ & : & \mathbf{e} \rightarrow \mathbf{c} \rightarrow \mathbf{c} \\
\texttt{sel} & : & \mathbf{c} \rightarrow \mathbf{e}
\end{array}
$$

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Semantic interpretation of the sentences

Let s be the syntactic category of sentences. Remember that we intend to abstract our notions of left and right contexts over the meaning of the sentences.

$$[\![s]\!] = \mathbf{c} \rightarrow (\mathbf{c} \rightarrow \mathbf{t}) \rightarrow \mathbf{t}$$

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Composition of two sentence interpretations

$$[\![S_1 . S_2]\!] = \lambda e \phi . [\![S_1]\!]\, e\, (\lambda e' . [\![S_2]\!]\, e'\, \phi)$$

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Back to DRT and DRSs

Consider a DRS:

$$\begin{array}{|c|}\hline x_1 \ldots x_n \\ \hline C_1 \\ \vdots \\ C_m \\ \hline\end{array}$$

To such a structure, corresponds the following λ-term of type $\mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}$:

$$\lambda e \phi . \exists x_1 \ldots x_n . C_1 \wedge \cdots \wedge C_m \wedge \phi\, e'$$

where e' is a context made of e and of the variables $x_1, \ldots, x_n$.

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Example

John loves Mary. He smiles at her.

$$[\![\textit{John loves Mary}]\!] = \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \phi\,(\mathbf{m}{::}\mathbf{j}{::}e)$$

$$[\![\textit{He smiles at her}]\!] = \lambda e\phi.\, \mathbf{smile}\,(\mathtt{sel}_{he}\,e)\,(\mathtt{sel}_{her}\,e) \wedge \phi\,e$$

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$$\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\begin{aligned}
&= \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\\
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$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$= \ \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$= \ \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$= \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}::\mathbf{j}::e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}::\mathbf{j}::e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}::\mathbf{j}::e)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}::\mathbf{j}::e)\, \phi$$

$$= \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}::\mathbf{j}::e)\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}::\mathbf{j}::e))\, (\mathtt{sel}_{her}\, (\mathbf{m}::\mathbf{j}::e)) \wedge \phi\, (\mathbf{m}::\mathbf{j}::e))\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}::\mathbf{j}::e))\, (\mathtt{sel}_{her}\, (\mathbf{m}::\mathbf{j}::e)) \wedge \phi\, (\mathbf{m}::\mathbf{j}::e)$$

$$= \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}::\mathbf{j}::e)) \wedge \phi\, (\mathbf{m}::\mathbf{j}::e)$$

$$= \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}::\mathbf{j}::e)$$

$$
\begin{aligned}
&\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi) \\
&= \ \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi) \\
&\to_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi) \\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}{::}\mathbf{j}{::}e) \\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi \\
&= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi \\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \\
&\qquad\qquad (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi \\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e) \\
&= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e) \\
&= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)
\end{aligned}
$$

$$\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$= \ \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi$$

$$\rightarrow_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$
\begin{aligned}
&\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\\
&= \ \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\\
&\to_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}{::}\mathbf{j}{::}e)\\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi\\
&= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi\\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge\\
&\qquad\qquad (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi\\
&\to_\beta \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)\\
&= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)\\
&= \ \lambda e\phi.\, \mathbf{love}\, \mathbf{j}\, \mathbf{m} \wedge \mathbf{smile}\, \mathbf{j}\, \mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)
\end{aligned}
$$

$$\lambda e\phi.\, [\![\textit{John loves Mary}]\!]\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$= \ \lambda e\phi.\, (\lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, e\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\to_\beta \lambda e\phi.\, (\lambda \phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)$$

$$\to_\beta \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge (\lambda e'.\, [\![\textit{He smiles at her}]\!]\, e'\, \phi)\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$\to_\beta \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge [\![\textit{He smiles at her}]\!]\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$= \ \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge (\lambda e\phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, e)\, (\mathtt{sel}_{her}\, e) \wedge \phi\, e)\, (\mathbf{m}{::}\mathbf{j}{::}e)\, \phi$$

$$\to_\beta \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \\ (\lambda \phi.\, \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e))\, \phi$$

$$\to_\beta \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \mathbf{smile}\, (\mathtt{sel}_{he}\, (\mathbf{m}{::}\mathbf{j}{::}e))\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \mathbf{smile}\,\mathbf{j}\, (\mathtt{sel}_{her}\, (\mathbf{m}{::}\mathbf{j}{::}e)) \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

$$= \ \lambda e\phi.\, \mathbf{love}\,\mathbf{j}\,\mathbf{m} \wedge \mathbf{smile}\,\mathbf{j}\,\mathbf{m} \wedge \phi\, (\mathbf{m}{::}\mathbf{j}{::}e)$$

Discourse Analysis

Montague's interpretation

$$\begin{array}{rcl} [\![s]\!] & = & \mathbf{t} \\ [\![n]\!] & = & \mathbf{e} \to \mathbf{t} \\ [\![np]\!] & = & (\mathbf{e} \to \mathbf{t}) \to \mathbf{t} \end{array}$$

may be rephrased as follows:

$$\begin{array}{rcll} [\![s]\!] & = & \mathbf{t} & (1) \\ [\![n]\!] & = & \mathbf{e} \to [\![s]\!] & (2) \\ [\![np]\!] & = & (\mathbf{e} \to [\![s]\!]) \to [\![s]\!] & (3) \end{array}$$

Replacing (1) with:

$$[\![s]\!] = \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}$$

we obtain:

$$\begin{array}{rcl} [\![n]\!] & = & \mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \\ [\![np]\!] & = & (\mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}) \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \end{array}$$

Montague's interpretation

$$\begin{aligned} [\![s]\!] &= \mathbf{t} \\ [\![n]\!] &= \mathbf{e} \to \mathbf{t} \\ [\![np]\!] &= (\mathbf{e} \to \mathbf{t}) \to \mathbf{t} \end{aligned}$$

may be rephrased as follows:

$$\begin{aligned} [\![s]\!] &= \mathbf{t} && (1) \\ [\![n]\!] &= \mathbf{e} \to [\![s]\!] && (2) \\ [\![np]\!] &= (\mathbf{e} \to [\![s]\!]) \to [\![s]\!] && (3) \end{aligned}$$

Replacing (1) with:

$$[\![s]\!] = \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}$$

we obtain:

$$\begin{aligned} [\![n]\!] &= \mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \\ [\![np]\!] &= (\mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}) \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \end{aligned}$$

Montague's interpretation

$$
\begin{aligned}
[\![s]\!] &= \mathbf{t} \\
[\![n]\!] &= \mathbf{e} \to \mathbf{t} \\
[\![np]\!] &= (\mathbf{e} \to \mathbf{t}) \to \mathbf{t}
\end{aligned}
$$

may be rephrased as follows:

$$
\begin{aligned}
[\![s]\!] &= \mathbf{t} && (1) \\
[\![n]\!] &= \mathbf{e} \to [\![s]\!] && (2) \\
[\![np]\!] &= (\mathbf{e} \to [\![s]\!]) \to [\![s]\!] && (3)
\end{aligned}
$$

Replacing (1) with:

$$[\![s]\!] = \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}$$

we obtain:

$$
\begin{aligned}
[\![n]\!] &= \mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \\
[\![np]\!] &= (\mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}) \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}
\end{aligned}
$$

Montague's interpretation

$$
\begin{array}{rcl}
\llbracket s \rrbracket & = & \mathbf{t} \\
\llbracket n \rrbracket & = & \mathbf{e} \to \mathbf{t} \\
\llbracket np \rrbracket & = & (\mathbf{e} \to \mathbf{t}) \to \mathbf{t}
\end{array}
$$

may be rephrased as follows:

$$
\begin{array}{rcll}
\llbracket s \rrbracket & = & \mathbf{t} & (1) \\
\llbracket n \rrbracket & = & \mathbf{e} \to \llbracket s \rrbracket & (2) \\
\llbracket np \rrbracket & = & (\mathbf{e} \to \llbracket s \rrbracket) \to \llbracket s \rrbracket & (3)
\end{array}
$$

Replacing (1) with:

$$
\llbracket s \rrbracket = \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}
$$

we obtain:

$$
\begin{array}{rcl}
\llbracket n \rrbracket & = & \mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \\
\llbracket np \rrbracket & = & (\mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}) \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}
\end{array}
$$

Montague's interpretation

$$\begin{array}{rcl} [\![s]\!] & = & \mathbf{t} \\ [\![n]\!] & = & \mathbf{e} \to \mathbf{t} \\ [\![np]\!] & = & (\mathbf{e} \to \mathbf{t}) \to \mathbf{t} \end{array}$$

may be rephrased as follows:

$$\begin{array}{rcll} [\![s]\!] & = & \mathbf{t} & (1) \\ [\![n]\!] & = & \mathbf{e} \to [\![s]\!] & (2) \\ [\![np]\!] & = & (\mathbf{e} \to [\![s]\!]) \to [\![s]\!] & (3) \end{array}$$

Replacing (1) with:

$$[\![s]\!] = \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}$$

we obtain:

$$\begin{array}{rcl} [\![n]\!] & = & \mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \\ [\![np]\!] & = & (\mathbf{e} \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t}) \to \mathbf{c} \to (\mathbf{c} \to \mathbf{t}) \to \mathbf{t} \end{array}$$

This interpretation results in handcrafted lexical semantics such as the following:

$$
\begin{aligned}
[\![\text{farmer}]\!] &= \lambda x e \phi.\, \mathbf{farmer}\, x \wedge \phi\, e \\
[\![\text{donkey}]\!] &= \lambda x e \phi.\, \mathbf{donkey}\, x \wedge \phi\, e \\
[\![\text{owns}]\!] &= \lambda o s.\, s\, (\lambda x.\, o\, (\lambda y e \phi.\, \mathbf{own}\, x\, y \wedge \phi\, e)) \\
[\![\text{beats}]\!] &= \lambda o s.\, s\, (\lambda x.\, o\, (\lambda y e \phi.\, \mathbf{beat}\, x\, y \wedge \phi\, e)) \\
[\![\text{who}]\!] &= \lambda r n x e \phi.\, n\, x\, e\, (\lambda e.\, r\, (\lambda \psi.\, \psi\, x)\, e\, \phi) \\
[\![\text{a}]\!] &= \lambda n \psi e \phi.\, \exists x.\, n\, x\, e\, (\lambda e.\, \psi\, x\, (x{::}e)\, \phi) \\
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Principles

Let $\Omega \triangleq \mathbf{c} \rightarrow (\mathbf{c} \rightarrow \mathbf{t}) \rightarrow \mathbf{t}$. We intend to design a logic acting on propositions of type Ω

We share with DRT the two following assumptions:

- discourse composition is mainly conjunctive (roughly speaking, a discourse consists in the conjunction of its sentences);
- the main form of quantification is existential (it introduces referential markers).

Consequently, our logic will be based on conjunction and existential quantification (defined as primitives). The other connectives will be obtained using negation (a third primitive) and de Morgan's laws.

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Discourse Analysis

Formal Framework

We consider a simply-typed λ-calculus, the terms of which are built upon asignature including the following constants:

FIRST-ORDER LOGIC

$$\top \; : \; \mathbf{t} \qquad (\textit{truth})$$

$$\neg \; : \; \mathbf{t} \to \mathbf{t} \qquad (\textit{negation})$$

$$\wedge \; : \; \mathbf{t} \to \mathbf{t} \to \mathbf{t} \qquad (\textit{conjunction})$$

$$\exists \; : \; (\mathbf{e} \to \mathbf{t}) \to \mathbf{t} \qquad (\textit{existential quantification})$$

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Conjunction is nothing but sentence composition. We therefore define:

$$A \mathbin{/\!\!\!\wedge} B \triangleq \lambda e\phi.\, A\, e\, (\lambda e.\, B\, e\, \phi)$$

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Existential quantification introduces "reference markers". It is therefore responsible for context updating:

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We do not want the continuation of the discourse to fall into the scope of the negation. Consequently, negation must be defined as follows:

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These are defined using de Morgan's laws:

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Discourse Analysis

Embedding and conservativity

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\begin{aligned}
\overline{R\,t_1\,\ldots\,t_n} &= \lambda e\phi.\, R\,t_1\,\ldots\,t_n \wedge \phi\,e \\
\overline{\neg A} &= \neg\!\!\!\cdot\, \overline{A} \\
\overline{A \wedge B} &= \overline{A} \mathbin{/\!\!\!\wedge} \overline{B} \\
\overline{\exists x.\, A} &= \exists\!\!\!\mid x.\, \overline{A}
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This embedding is such that, for every term e of type $\mathbf{c}$:

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Donkey sentence revisited

Montague-like semantic interpretation:

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