Formal Semantics of Natural Language

Philippe de Groote and Yoad Winter

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Additional Topics:

Intensionality
Sinn und bedeutung

Gottlob Frege (1848-1925)
Sinn und bedeutung

► *Sinn* (sense)/*Bedeutung* (reference)
— Frege

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Sinn und bedeutung

► **Sinn** (sense)/**Bedeutung** (reference) — Frege

► **Intension/Extension** — Carnap

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According to Frege, the sense of an expression is its “mode of presentation”, while the reference or denotation of an expression is the object it refers to.

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Sinn und bedeutung

► **Sinn (sense)/Bedeutung (reference)** — Frege

► **Intension/Extension** — Carnap

► According to Frege, the sense of an expression is its “mode of presentation”, while the reference or denotation of an expression is the object it refers to.

► For instance, both expressions “1 + 1” and “2” have the same denotation but not the same sense.

Gottlob Frege (1848-1925)
Intensional propositions
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Intensional propositions

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- Frege gives the example of “the morning star” and “the evening star” which both refer to the planet Venus.

- Compare “the morning star is the evening star” with “the ancients did not know that the morning star is the evening star”.
Possible world semantics

G.W. von Leibniz
(1646–1716)
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Pangloss enseignait la métaphysico-théologo-cosmolo-nigologie.
Il prouvait admirablement qu’il n’y a point d’effet sans cause, et que,
dans ce meilleur des mondes possibles, le château de monseigneur le baron
était le plus beau des châteaux et madame la meilleure des baronnes possibles.

G.W. von Leibniz (1646–1716)

Voltaire (Candide)
Modals
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  ► Deontic logic: *It is mandatory that... It is allowed that...*

  ► Epistemic logic: *Bob knows that... Bob ignores that...*

  ► Temporal logic: *It will always be the case that... It will eventually be the case that...*
Modal logic
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Syntax:

\[ F ::= a | \neg F | F \lor F | \Box F \]

Define the other connectives in the usual way. Define \( \Diamond A \) as \( \neg \Box \neg A \).

\( \Box A \) stands for “necessarily A”. \( \Diamond A \) stands for “possibly A”.

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Validity:

let \( M = \langle W, P \rangle \), where \( W \) is a set of “possible worlds”, and \( P \) is a function that assigns to each atomic proposition a subset of \( W \).

\[ \begin{align*}
\forall M, s \models a & \iff s \in P(a). \\
\forall M, s \models \neg A & \iff \text{not } M, s \models A. \\
\forall M, s \models A \lor B & \iff \text{either } M, s \models A \text{ or } M, s \models B, \text{ or both.} \\
\forall M, s \models \Box A & \iff \text{for every } t \in W, \ M, t \models A.
\end{align*} \]
Modal logic and type theory
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- Use three atomic types: \( e \), \( s \), and \( t \).
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\text{all}_m := \lambda P \ w. \ \forall x. \ P \ x \ w
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\[
\text{all}_m := \lambda P w. \forall x. P x w
\]

\[
\text{necessarily} := \lambda A w. \forall v. (A v)
\]
Intension and extension
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This red car is a Ferrari
Intension and extension

This red car is a Ferrari

This skillful surgeon is Dr Johnson
Intension and extension

This red car is a Ferrari

This skillful surgeon is Dr Johnson

$$(\forall x. \text{surgeon } x) \leftrightarrow (\text{driver } x)$$

$$(\forall x. ((\text{skillful surgeon} ) x) ((\text{skillful driver} ) x))$$
Intension and extension

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(\forall x. (\text{surgeon } x) \leftrightarrow (\text{driver } x)) \\
(\forall x. ((\text{skillful surgeon}) x) ((\text{skillful driver}) x))
\]

Solution:

\[
\begin{align*}
\text{surgeon} & : e(s \ t) \\
\text{driver} & : e(s \ t) \\
\text{skillful} & : (e(s \ t)) e(s \ t)
\end{align*}
\]