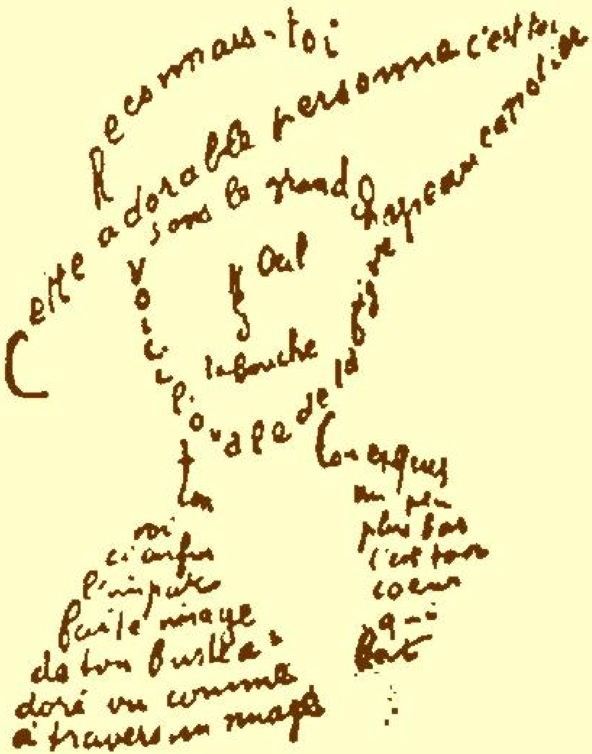
Reconnais-toi
Cette adorable personne c'est toi
sous le grand chapeau canotier
la bouche
voici l'ovale de ta figure
ton cou exquis
un peu plus bas c'est ton cœur qui bat
voici enfin l'imparfaite image de ton buste adoré vu comme à travers un nuage

A Type-Theoretic View of Dynamic Logic

Philippe de Groote

LORIA & Inria-Lorraine

A Type-Theoretic Reconstruction of DRT

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Motivation:

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Proposed solution:

- to interpret a sentence according to both its left and right contexts;
- to abstract these two kinds of contexts over the meaning of the sentences.

Typing the left and the right contexts

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Montague semantics is based on Church's simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

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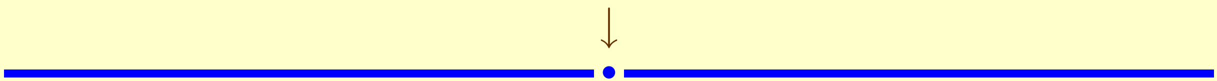

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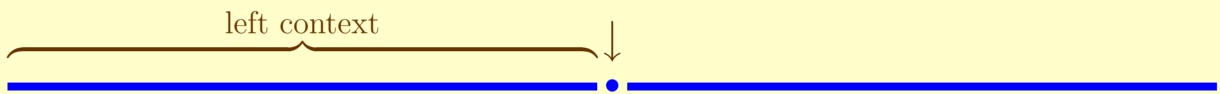

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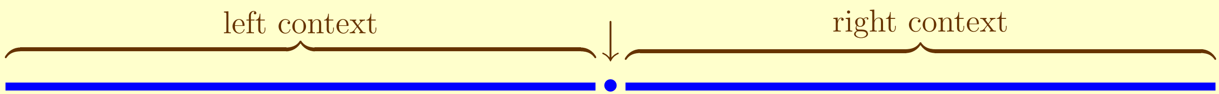

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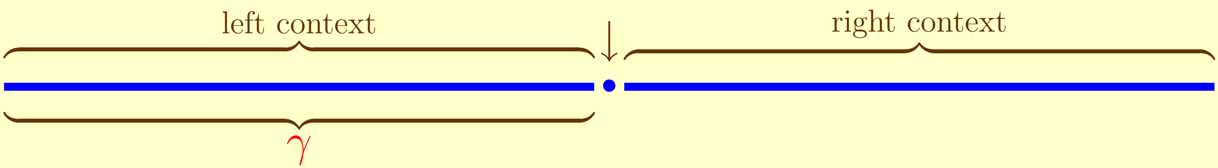

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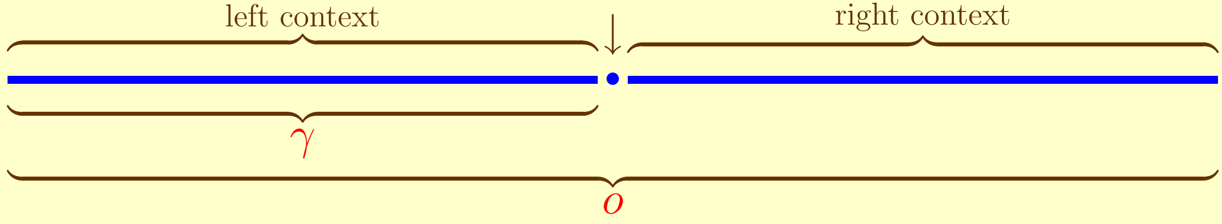

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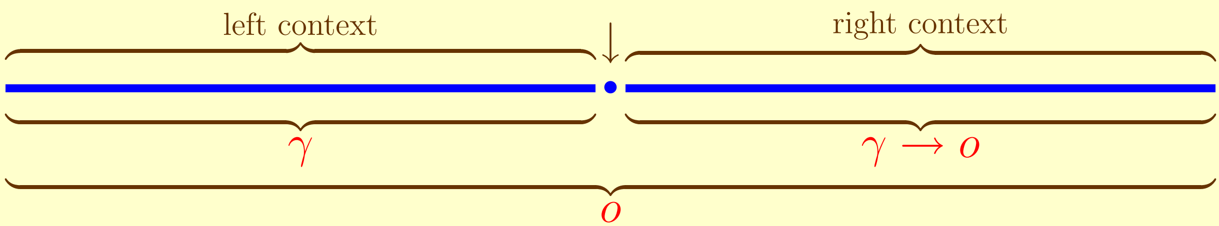

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Composition of two sentence interpretations

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Composition of two sentence interpretations

$$[\![S_1. \, S_2]\!] = \lambda e \phi. \, [\![S_1]\!] \, e \, (\lambda e'. \, [\![S_2]\!] \, e' \, \phi)$$

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Montague's interpretation

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$$\begin{aligned} [\![n]\!] &= \iota \to \gamma \to (\gamma \to o) \to o \\ [\![np]\!] &= (\iota \to \gamma \to (\gamma \to o) \to o) \to \gamma \to (\gamma \to o) \to o \end{aligned}$$

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...which might seem a little bit involved.

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- can we find any "modular" presentation of the approach?
- is there some dynamic logic hidden in the approach?

A Dynamic Logic

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We share with DRT the two following assumptions:

- discourse composition is mainly conjunctive (roughly speaking, a discourse consists in the conjunction of its sentences);
- the main form of quantification is existential (it introduces referential markers).

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We share with DRT the two following assumptions:

- discourse composition is mainly conjunctive (roughly speaking, a discourse consists in the conjunction of its sentences);
- the main form of quantification is existential (it introduces referential markers).

Consequently, our logic will be based on conjunction and existential quantification (defined as primitives). The other connectives will be obtained using negation (a third primitive) and de Morgan's laws.

Formal Framework

We consider a simply-typed λ-calculus, the terms of which are built upon asignature including the following constants:

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\begin{array}{lcll}
\top & : & o & (\textit{truth}) \\
\neg & : & o \rightarrow o & (\textit{negation}) \\
\wedge & : & o \rightarrow o \rightarrow o & (\textit{conjunction}) \\
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DYNAMIC PRIMITIVES

$$\begin{array}{rcll} :: & : & \iota \to \gamma \to \gamma & (\textit{context updating}) \\ \mathsf{sel} & : & \gamma \to \iota & (\textit{choice operator}) \end{array}$$

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Negation

We do not want the continuation of the discourse to fall into the scope of the negation. Consequently, negation must be defined as follows:

$$\sim A \triangleq \lambda e\phi.\, \neg\,(A\, e\,(\lambda e.\, \top)) \wedge \phi\, e$$

Implication and Universal Quantification

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These are defined using de Morgan's laws:

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\begin{aligned}
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$$\begin{aligned} \overline{R\,t_1\,\ldots\,t_n} &= \lambda e\phi.\, R\,t_1\,\ldots\,t_n \wedge \phi\,e \\ \overline{\neg A} &= {\sim}\overline{A} \\ \overline{A \wedge B} &= \overline{A} \sqcap \overline{B} \\ \overline{\exists x.\, A} &= \Sigma x.\, \overline{A} \end{aligned}$$

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This embedding is such that, for every term e of type γ:

$$A \equiv \overline{A}\, e\, (\lambda e.\, \top)$$

Donkey Sentence Revisited

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Montague-like semantic interpretation:

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With the dynamic interpretation we have that:

$$[\![\text{beats}]\!]\ [\![\text{it}]\!]\ ([\![\text{every}]\!]\ ([\![\text{who}]\!]\ ([\![\text{owns}]\!]\ ([\![\text{a}]\!]\ [\![\text{donkey}]\!]))\ [\![\text{farmer}]\!]))$$

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β-reduces to the following term (modulo de Morgan's laws):

$$\lambda e\phi.\,(\forall x.\,\mathbf{farmer}\,x \supset (\forall y.\,\mathbf{donkey}\,y \supset (\mathbf{own}\,x\,y \supset \mathbf{beat}\,x\,(\mathsf{sel}\,(x::y::e))))) \wedge \phi\,e$$

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that is, assuming that sel is a "perfect" anaphora resolution operator:

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Define type "dynamization" as follows:

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Then, define term "dynamization" as follows:

$$\begin{array}{rcll} \mathsf{D}t &=& \lambda x_1 \ldots x_n.\, t\,(\mathsf{R}_{\mathsf{nil}} x_1)\,\ldots\,(\mathsf{R}_{\mathsf{nil}} x_n) & \text{at type } \alpha_1 \to \ldots \alpha_n \to \iota \\ \mathsf{D}t &=& \lambda x_1 \ldots x_n e \phi.\, t\,(\mathsf{R}_e x_1)\,\ldots\,(\mathsf{R}_e x_n) \wedge (\phi\, e) & \text{at type } \alpha_1 \to \ldots \alpha_n \to o \\ \\ \mathsf{R}_e t &=& \lambda x_1 \ldots x_n.\, t\,(\mathsf{D}x_1)\,\ldots\,(\mathsf{D}x_n) & \text{at type } \mathsf{D}(\alpha_1 \to \ldots \alpha_n \to \iota) \\ \mathsf{R}_e t &=& \lambda x_1 \ldots x_n.\, t\,(\mathsf{D}x_1)\,\ldots\,(\mathsf{D}x_n)\, e\,(\lambda e.\, \top) & \text{at type } \mathsf{D}(\alpha_1 \to \ldots \alpha_n \to o) \end{array}$$

Finally, define λ-term translation as follows:

$$
\begin{array}{rcl}
\overline{x} & = & x \\
\overline{\wedge} & = & \sqcap \\
\overline{\exists} & = & \Sigma \\
\overline{\neg} & = & \sim \\
\overline{k} & = & \mathsf{D}k \quad \text{for the other constants} \\
\overline{\lambda x.\, t} & = & \lambda x.\, \overline{t} \\
\overline{t\, u} & = & \overline{t}\, \overline{u}
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$$
\begin{array}{rcll}
\overline{x} & = & x & \\
\overline{\wedge} & = & \sqcap & \\
\overline{\exists} & = & \Sigma & \\
\overline{\neg} & = & \sim & \\
\overline{k} & = & \mathsf{D}k & \text{for the other constants} \\
\overline{\lambda x.\, t} & = & \lambda x.\, \overline{t} & \\
\overline{t\, u} & = & \overline{t}\, \overline{u} &
\end{array}
$$

Then, for every closed term t of type o, and every context e, we have that:

$$\overline{t}\, e\, (\lambda e.\, \top) \equiv t$$

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and the semantics of Groenendijk & Stokhof's DPL would be rephrased as follows:

$$\begin{array}{rcll}
A_{\mathrm{d}} & \triangleq & \lambda gh.\, h{=}g \wedge A & \text{(atomic proposition)} \\
(\neg P)_{\mathrm{d}} & \triangleq & \lambda gh.\, h{=}g \wedge \neg(\exists k.\, P_{\mathrm{d}}\, h\, k) & \text{(negation)} \\
(P \wedge Q)_{\mathrm{d}} & \triangleq & \lambda gh.\, \exists k.\, P_{\mathrm{d}}\, g\, k \wedge Q_{\mathrm{d}}\, k\, h & \text{(conjunction)} \\
(\exists x.\, P)_{\mathrm{d}} & \triangleq & \lambda gh.\, \exists k.\, k[x]g \wedge P_{\mathrm{d}}\, k\, h & \text{(existential)}
\end{array}$$

There exists a canonical embedding $\lceil \cdot \rceil$ from $\gamma \to \gamma \to o$ into $\gamma \to (\gamma \to o) \to o$:

$$\lceil R \rceil \triangleq \lambda e \phi.\, \exists e'.\, \phi\, e' \wedge R\, e\, e'$$

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As for the existential quantifier:

$$\begin{aligned} \lceil (\exists x.\, P)_{\mathrm{d}} \rceil &= \lambda e\phi.\, \exists e'.\, \phi\, e' \wedge \exists k.\, k[x]e \wedge \lceil P_{\mathrm{d}} \rceil\, k\, e' \\ \Sigma x.\, \lceil P_{\mathrm{d}} \rceil &= \lambda e\phi.\, \exists e'.\, \phi\, e' \wedge \exists x.\, \lceil P_{\mathrm{d}} \rceil\, (x :: e)\, e' \end{aligned}$$

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- Deduction is replaced by computation.