

Traceable Encryption for Verifiable Receipt-free Online Voting

Quentin Yang

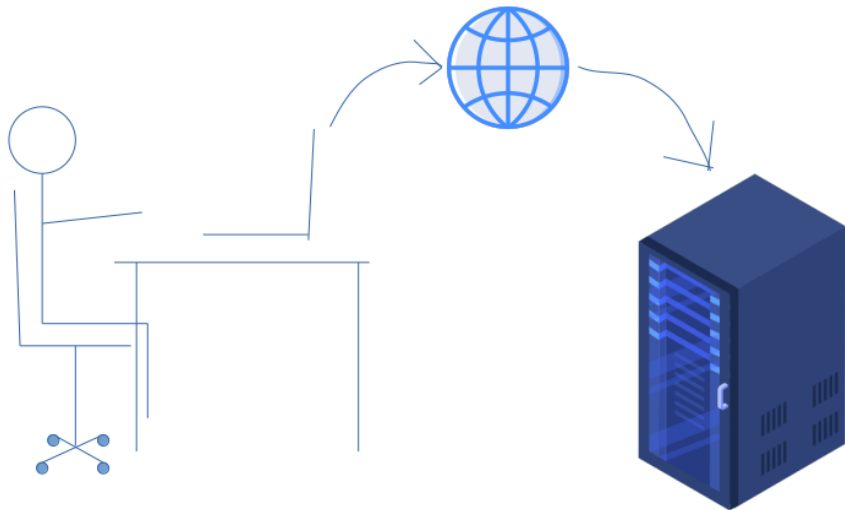
in collaboration with

Henri Devillez, Thomas Peters and Olivier Pereira

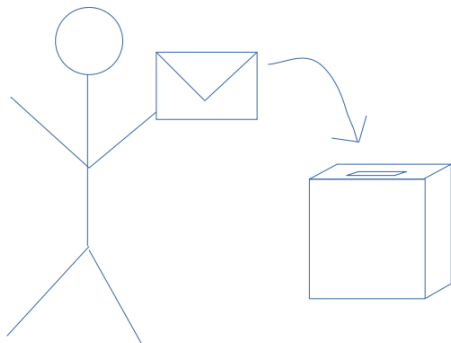
Université Catholique de Louvain

Journées au Vert, September 2022

What is electronic voting?



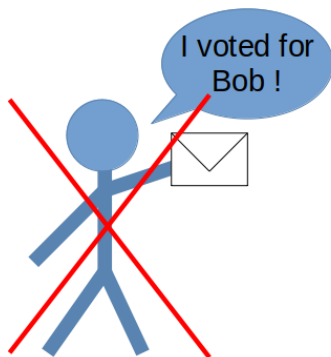
The security goals in electronic voting



Security properties

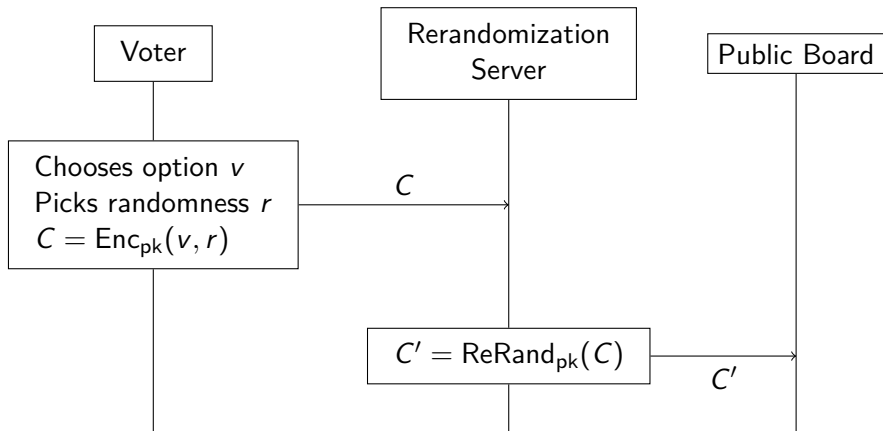
- ✓ Eligibility
- ✓ Cast-as-intended
- ✓ Vote secrecy
- ✓ Confidence in the result





Receipt-freeness: a voter cannot prove how they voted.

The rerandomization paradigm



An example: **BeleniosRF**, Chaidos, Cortier, Fuchsbauer and Galindo (CCS'16)

Outline

- 1 Introduction
- 2 Formalization**
- 3 Application
- 4 Construction
- 5 Security
- 6 Conclusion

Definition

A traceable encryption (TREnc) is a public key encryption scheme $(\text{Gen}, \text{Enc}, \text{Dec})$, augmented with:

- $\text{LGen}(\text{pk})$: Generates a (random) link key lk ;
- $\text{LEnc}_{\text{pk}}(\text{lk}, m, r)$: Encrypts a message m using the link key lk and the randomness r . We have that $\text{Enc}_{\text{pk}}(m, r) = \text{LEnc}(\text{pk}, \text{LGen}(\text{pk}), m, r)$;
- $\text{Trace}_{\text{pk}}(C)$: Outputs a trace from a ciphertext;
- **ReRand** $_{\text{pk}}(C, r)$: Rerandomizes a ciphertext C . We require that $\text{Dec}_{\text{sk}}(C) = \text{Dec}_{\text{sk}}(\text{ReRand}_{\text{pk}}(C))$;
- $\text{Ver}_{\text{pk}}(C)$: Verifies the validity of a ciphertext.

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- **Trace**_{pk}(C): Outputs a trace from a ciphertext;

⋮

Definition (Link traceability)

A TREnc has link traceability if, for all (pk, lk, m, m', r, r') , we have

$$\text{Trace}_{pk}(\text{LEnc}_{pk}(lk, m, r)) = \text{Trace}_{pk}(\text{LEnc}_{pk}(lk, m', r')).$$

Traceable encryption

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- $\text{Ver}_{\text{pk}}(C)$: Verifies the validity of a ciphertext.

Definition (Verifiability)

A TREnc is verifiable if no PPT adversary can produce some C such that $\text{Ver}_{\text{pk}}(C) = 1$ while C is not in the range of Enc .

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We also require the link traceability and verifiability properties.

Definition (Traceability)

A TREnc is traceable if, for every PPT adversary \mathbb{A} , the following experiment returns 1 with negligible probability.

$$\text{Exp}_{\mathbb{A}}^{\text{trace}}(\lambda)$$

$(pk, sk) \leftarrow \$ \text{Gen}(\lambda)$

$m \leftarrow \$ \mathbb{A}(pk, sk)$

$C \leftarrow \$ \text{Enc}_{pk}(m)$

$C^* \leftarrow \$ \mathbb{A}(C)$

if $\text{Trace}_{pk}(C) = \text{Trace}_{pk}(C^*)$ and
 $\text{Ver}_{pk}(C^*) = 1$ and $\text{Dec}_{sk}(C^*) \neq m$

then return 1

else return 0

Definition (IND-TCCA)

A TREnc is IND-TCCA secure if for every PPT adversary \mathbb{A} , the following experiment returns 1 with a probability negligibly close to $1/2$.

$$\text{Exp}_{\mathbb{A}}^{\text{tcca}}(\lambda)$$

$$(pk, sk) \leftarrow \text{Gen}(\lambda)$$
$$(C_0, C_1) \leftarrow \mathbb{A}^{\text{Dec}(\cdot)}(pk)$$
$$b \leftarrow \{0, 1\}$$

if $\text{Trace}_{pk}(C_0) \neq \text{Trace}_{pk}(C_1)$ or

$\text{Ver}_{pk}(C_0) = 0$ or $\text{Ver}_{pk}(C_1) = 0$

then return b

$$C^* \leftarrow \text{ReRand}_{pk}(C_b)$$
$$b' \leftarrow \mathbb{A}^{\text{Dec}^*(\cdot)}(C^*)$$

return $b' == b$

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BeleniosRF uses an encryption which is:

- ✓ Rerandomizable: ElGamal encryption.
- ✓ Verifiable: Groth-Sahai proofs.
- ✓ Traceable: Asymmetric Waters Signature.

Application to BeleniosRF

BeleniosRF uses an encryption which is:

- ✓ Rerandomizable: ElGamal encryption.
- ✓ Verifiable: Groth-Sahai proofs.
- ✓ Traceable: Asymmetric Waters Signature.

However, it comes with a few limitations:

- Restricted to small bitstrings.
- Low modularity.

Asymmetric Waters Signatures

Setting:

- Bilinear groups $(\mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_T)$ of prime order p with generators $g \in \mathbb{G}$ and $\hat{g} \in \hat{\mathbb{G}}$ and a pairing e .
- Random elements $z, u_0, \dots, u_k \in \mathbb{G}$.
- A function $\mathcal{F} : \{0, 1\}^k \rightarrow \mathbb{G}$ such that $\mathcal{F}(m) = u_0 \prod_{i=1}^k u_i^{m_i}$.

Secret key: $\alpha \in \mathbb{Z}_p$ and $y = z^\alpha$.

Public key: $x = g^\alpha$ and $\hat{x} = \hat{g}^\alpha$.

Sign(m, s): $\sigma_1 = y\mathcal{F}(m)^s$, $\sigma_2 = g^s$ and $\hat{\sigma} = \hat{g}^s$.

Verify(m, σ): Check that

$$e(\sigma_1, \hat{g}) = e(z, \hat{x})e(\mathcal{F}(m), \hat{\sigma}) \text{ and } e(\sigma_2, \hat{g}) = e(\sigma_1, \hat{\sigma}).$$

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Groth-Sahai proofs

Setting:

- Bilinear groups $\mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_{\mathcal{T}}$ of prime order p with a pairing e .
- We rely on the SXDH assumption.

Groth-Sahai proofs are Non-Interactive Witness Indistinguishable proofs for bilinear equations, such as

$$\prod_{i=1}^n e(A_i, Y_i) \prod_{j=1}^n e(X_j, B_j) \prod_{i=1}^n \prod_{j=1}^n e(X_j, Y_i)^{\alpha_{i,j}} = t_{\mathcal{T}},$$

where **red** symbols represent the (secret) witnesses.

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Property: They are perfectly rerandomizable.

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Linearly Homomorphic Structure-Preserving Signature

Setting: Bilinear groups $(\mathbb{G}, \hat{\mathbb{G}}, \mathbb{G}_T)$ of prime order p with independent generators $\hat{g}, \hat{h} \in \hat{\mathbb{G}}$ and a pairing e .

Secret key: $(\alpha_i, \beta_i)_{i=1}^n \in \mathbb{Z}_p$

Public key: $\hat{k}_i = \hat{g}^{\alpha_i} \hat{h}^{\beta_i}$ for $i = 1$ to n .

Sign $(\mathbf{M}_1, \dots, \mathbf{M}_n)$: $\left(Z = \prod_{i=1}^n M_i^{\alpha_i}, R = \prod_{i=1}^n M_i^{\beta_i} \right)$

Verify (\mathbf{Z}, \mathbf{R}) : Check that $e(Z, \hat{g})e(R, \hat{h}) = \prod_{i=1}^n e(M_i, \hat{k}_i)$

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- ✓ Traceability
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An example: unforgeability of the LHSP signature under SXDH.

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An example: unforgeability of the LHSP signature under SXDH.

The adversary is given:

- $\hat{g}, \hat{h}, \hat{k}_1, \dots, \hat{k}_n$ such that $\hat{k}_i = \hat{g}^{\alpha_i} \hat{h}^{\beta_i}$.
- Some valid signatures on messages $\vec{M}_1, \dots, \vec{M}_k$: (Z_i, R_i) such that $Z_i = \prod_{j=1}^n M_{i,j}^{\alpha_j}$ and $R_i = \prod_{j=1}^n M_{i,j}^{\beta_j}$.
- A message $\vec{M} \notin \langle \vec{M}_1, \dots, \vec{M}_k \rangle$ to sign.

The adversary outputs Z', R' such that $e(Z', \hat{g})e(R', \hat{h}) = \prod_{i=1}^n e(M_i, \hat{k}_i)$.

By computing $Z = \prod_{i=1}^n M_i^{\alpha_i}$ and $R = \prod_{i=1}^n M_i^{\beta_i}$, we have

$$e(Z/Z', \hat{g})e(R/R', \hat{h}) = 1.$$

An example: unforgeability of the LHSP signature under SXDH.

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If I have a DDH challenge $(\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4)$, I use (\hat{g}_1, \hat{g}_2) as (\hat{g}, \hat{h}) .

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DDH tuple: there exists α such that

$$\begin{aligned}(\hat{g}_3, \hat{g}_4) &= (\hat{g}_1, \hat{g}_2)^\alpha \\ e(Z/Z', \hat{g}_3)e(R/R', \hat{g}_4) &= (e(Z/Z', \hat{g}_1)e(R/R', \hat{g}_2))^\alpha \\ &= 1.\end{aligned}$$

Random tuple: the pairing product is random.

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Second step: Conclude.

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The adversary outputs $Z = \prod_{i=1}^n M_i^{\alpha_i}$ and $R = \prod_{i=1}^n M_i^{\beta_i}$.

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- ✓ IND-TCCA + Verifiable + “nice” tally \implies receipt-freeness

The control flow in receipt-freeness

- ① The voter votes.
- ② They receive or produce a receipt.
- ③ They try to convince the adversary.

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The adversary could instruct to use a specific trace!

The definition of Receipt-freeness

$$\text{Exp}_{\mathcal{A}, \mathcal{V}, \mathcal{D}}^{\beta}(\lambda)$$

$\text{pk} \leftarrow \mathcal{O}\text{init}(\lambda); \mathcal{A}^{\mathcal{O}\text{cast}, \mathcal{O}\text{board}, \mathcal{O}\text{voteLR}, \mathcal{O}\text{receiptLR}}(\text{pk}); (r, \Pi) \leftarrow \mathcal{O}\text{tally}();$
 $g \leftarrow \mathcal{A}(r, \Pi); \text{return } (g == \beta)$

$$\mathcal{O}\text{init}(\lambda)$$

if $\beta = 0$
 then $(\text{pk}, \text{sk}) \leftarrow \text{Setup}(\lambda)$
else $(\text{pk}, \text{sk}, \tau) \leftarrow \text{SimSetup}(1^\lambda)$
 $\text{PB}_0 \leftarrow \emptyset; \text{PB}_1 \leftarrow \emptyset$
return pk

$$\mathcal{O}\text{receiptLR}(l, v)$$

$s_0, b_0 \leftarrow \text{Vote}(l, \text{RS}, \text{PB}_0)$
 $s_1, b_1 \leftarrow \text{Vote}(\mathcal{D}_1(v), \text{RS}, \text{PB}_1)$
if $\text{Valid}(\text{PB}_0, b_0) = 0$ **or** $v \notin \mathcal{C}$
 or $\text{Valid}(\text{PB}_1, b_0) = 0$ **then return** \perp
else $\text{Append}(\text{PB}_0, b_0); \text{Append}(\text{PB}_1, b_1)$
return s_β

$$\mathcal{O}\text{cast}(b)$$

if $\text{Valid}(\text{PB}_0, b) = 0$
 or $\text{Valid}(\text{PB}_1, b) = 0$
 then return \perp
else $\text{Append}(\text{PB}_0, b);$
 $\text{Append}(\text{PB}_1, b)$

$$\mathcal{O}\text{voteLR}(v_0, v_1)$$

if $v_0 \notin \mathcal{C}$ **or** $v_1 \notin \mathcal{C}$ **then return** \perp
 $b_0 \leftarrow \text{Vote}(v_0, \text{RS}, \text{PB}_0)$
 $b_1 \leftarrow \text{Vote}(v_1, \text{RS}, \text{PB}_1)$
 $\text{Append}(\text{PB}_0, b_0); \text{Append}(\text{PB}_1, b_1)$

$$\mathcal{O}\text{board}()$$

return PB_β

$$\mathcal{O}\text{tally}()$$

$(r, \Pi) \leftarrow \text{Tally}(\text{PB}_0, \text{sk})$
if $\beta = 1$ **then** $\Pi \leftarrow \text{SimProof}(\text{PB}_1, r, \tau)$
return (r, Π)

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- Overcame the limitations of BeleniosRF
- Took the notion of receipt-freeness to the next step
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- Overcame the limitations of BeleniosRF
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- Constructed a voting scheme and proved its security
- Adapted the scheme for cast-as-intended verification

Our work was submitted at S&P in August.

Bonus: the voting protocol

