Type systems for ICC analysis of imperative programs

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ICC's last stand

The aim of ICC is to find machine independent characterizations of complexity classes:

- Function algebra (Bellantoni, Cook, Leivant, Marion, ...)
- Lights logics (Girard, Lafont, Baillot, Gaboardi, Ronchi Della Rocca, ...)
- Interpretations of TRS (Bonfante, Marion, Moyen, Péchoux, ...)
- Non-size increasing principle (Hofmann, ...)
- Matrices calculus for imperative programs(Jones, Kristiansen, Wunderlich, Moyen,...)
- Imperative pointer graph languages for subpolynomial classes (Hofmann, Schoepp, ...)

Mixture

Marion's idea (Lics 2011) is to take advantage of two well-known lines of work:

- Safe (or tiered) recursion by Bellantoni and Cook [1992]
- Non-interference by Volpano et al [1996]

in order to obtain a polynomial time characterization on imperative languages.

Safe recursion

The class of functions that can be defined using:

- constants, projections, successor, predecessor, conditional,
- safe composition:

$$f(\overline{x};\overline{a}) = h(r(\overline{x};);t(\overline{x};\overline{a}))$$

- and safe recursion (on notation):

$$\begin{aligned} f(0,\overline{x};\overline{a}) &= g(\overline{x};\overline{a}) \\ f(i(x),\overline{y};\overline{a}) &= h_i(x,\overline{y};f(x,\overline{y};\overline{a})) \qquad i \in \{0,1\}, \end{aligned}$$

provided h, r, t, g, h_i are already defined in the class,

is exactly the set of functions computable in polynomial time (FPtime).

The tiered viewpoint

The class of functions that can be defined using:

- constants, projections, successor, predecessor, conditional,
- safe composition:

$$f(\overline{x}^{\mathbf{1}}; \overline{a}^{\mathbf{0}}) = h(r(\overline{x}^{\mathbf{1}};); t(\overline{x}^{\mathbf{1}}; \overline{a})^{\mathbf{0}})$$

- and safe recursion (on notation):

$$\begin{aligned} f(0, \overline{x}^{\mathbf{1}}; \overline{a}^{\mathbf{0}}) &= g(\overline{x}^{\mathbf{1}}; \overline{a}^{\mathbf{0}}) \\ f(i(x)^{\mathbf{1}}, \overline{y}^{\mathbf{1}}; \overline{a}) &= h_i(x^{\mathbf{1}}, \overline{y}^{\mathbf{1}}; f(x^{\mathbf{1}}, \overline{y}^{\mathbf{1}}; \overline{a})^{\mathbf{0}}) \qquad i \in \{0, 1\}, \end{aligned}$$

provided h, r, t, g, h_i are already defined in the class,

is exactly the set of functions computable in polynomial time (FPtime).

Non-interference

Two security levels:

- H for high
- L for low

and typing rules of the shape:

$$\frac{\Gamma \vdash E : \tau \quad \Gamma \vdash I : \tau\ Cmd}{\Gamma \vdash \mathtt{while}(E)\{I\} : \tau\ Cmd}\ (Wh)$$

$+$ command subtyping:

$$\frac{\Gamma \vdash I : \tau\ Cmd \quad \tau < \tau'}{\Gamma \vdash I : \tau'\ Cmd}\ (Sub)$$

Non-interference example

It prevent us from typing the following program:

```
while(x>0 : H) {
    x = x-- ; : H Cmd
    y = y++ ; : L Cmd
}
```

if x is High and y is Low (Indeed there is a flow from x to y) and provided that $H < L$.

Duality of non-interference and tiering

We would like to type following program:

```
while(x>0 : 1) {
    x = x-- ; : 1 Cmd
    y = y++ ; : 0 Cmd
}
```

if x is of tier $\mathbf{1}$ (High) and y is of tier $\mathbf{0}$ Low (preventing flows from y to x) and provided that $\mathbf{0} < \mathbf{1}$.

Small imperative language

Every data type is encoded by words over $\mathbf{W}$.
The size $|w|$ of a word $w \in \mathbf{W}$ is standard.

- Expressions :
  $E ::= \ \mathtt{x} \mid c \mid \mathtt{true} \mid \mathtt{false} \mid op(\overline{E})$
- Instructions :
  $I ::= \ ; \mid [\tau]\ \mathtt{x}{:=}E; \mid I_1\ I_2 \mid \mathtt{while}(E)\{I\}$
  $\quad \mid \mathtt{if}(E)\{I_1\}\mathtt{else}\{I_2\}$

The types τ will be tiers in $\{\mathbf{0}, \mathbf{1}\}$ such that $\mathbf{0} < \mathbf{1}$.

Typing rules : expressions

Variable

$$\frac{\Gamma(\mathtt{x}) = \tau}{\Gamma \vdash \mathtt{x} : \tau}$$

Constant

$$\frac{}{\Gamma \vdash n : \tau}$$

Destructor

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash op(e) : \tau}$$

Constructor

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash op(e) : \mathbf{0}}$$

Typing rules : commands

Assign

$$\frac{\Gamma \vdash \mathtt{x} : \tau \qquad \Gamma \vdash E : \tau'}{\Gamma \vdash \mathtt{x} := E : \tau} \; \tau \leq \tau'$$

Compose

$$\frac{\Gamma \vdash I_1 : \tau \qquad \Gamma \vdash I_2 : \tau'}{\Gamma \vdash I_1 \;\; I_2 : \tau \vee \tau'}$$

If

$$\frac{\Gamma \vdash e : \tau \qquad \Gamma \vdash I_i : \tau}{\Gamma \vdash \mathtt{if}(E)\{I_1\}\mathtt{else}\{I_2\} : \tau}$$

While

$$\frac{\Gamma \vdash E : \mathbf{1} \qquad \Gamma \vdash I : \tau}{\Gamma \vdash \mathtt{while}(E)\{I\} : \mathbf{1}}$$

Improvements

We can extend the type system to more general operators $op :: \tau_1 \times \ldots \times \tau_n \to \tau$ such that $\tau \leq \wedge_i \tau_i$.

- Neutral operators:
  - either a computable predicate
  - or a subword operator:

$$\forall \overline{w},\ \exists i \in \{1, \ldots, n\},\ [\![op]\!](\overline{w}) \trianglelefteq w_i$$

- Positive operators:

$$\forall \overline{w},\ |[\![op]\!](\overline{w})| \leq \max_{i \in [1,n]} |w_i| + c,\ \text{for } c \geq 0$$

  - In this case $\tau = \mathbf{0}$.

We can also add procedure calls.

Example: addition

```
int add(int x,int y)
{
while (x>0)
        {
        x--;
        y++;
        }
return y
}
```

- y is necessarily of tier $\mathbf{0}$
- x is necessarily of tier $\mathbf{1}$
- and, consequently, add $:: \mathbf{1} \times \mathbf{0} \rightarrow \mathbf{0}$

Example: multiplication

```
int mult(int x,int y)
{
int z=0;
while (x)
      {
      x--;
      z = add(y,z);
      }
return z;
}
```

- the output of add is $\mathbf{0}$. Consequently, z is of tier $\mathbf{0}$.
- both x and y are of tier $\mathbf{1}$
- and, consequently, `mult` :: $\mathbf{1} \times \mathbf{1} \to \mathbf{0}$

Example: exponential

```
int expo(int x)
{
int y=1;
while (x)
        {
        x--;
        y = add(y,y);
        }
return y;
}
```

- x is of tier **1**,
- the output of add is of tier **0**,
- but y has to be of tier **1** in the first argument of add !!!

Results

We have a (weak) subject reduction property:

Theorem [Marion and Péchoux (TAMC 2014)]

If $\sigma \vDash I \to \sigma' \vDash I'$ and $\Gamma \vdash I : \tau$ then $\Gamma \vdash I' : \tau'$ where $\tau' \leq \tau$.

We obtain a characterization of FPtime:

Theorem [Marion (Lics 2011)]

The set of functions computable by a typable and terminating program with FPtime computable operators is exactly FPtime.

Moreover, type inference is decidable:

Theorem [Hainry, Marion and Péchoux (Fossacs 2013)]

Type inference can be done in polynomial time.

Mechanism

FPtime soundness:

- No flow from **0** to **1**: tier **1** variables cannot increase
- Only tier **1** arguments in the guards
- At most n^k configurations under termination assumption

FPtime completeness:

- Any polynomial can be computed
- We simulate polynomial time TMs by an imperative typable (and terminating) program

Type inference:

- All the constraints are inequalities over 2 tiers
- That can be reduced to a 2-SAT formula

Multi-threaded

Now we consider multi-threads M to be a fixed collection of commands:

$$M(\alpha) = I,\ \alpha \in dom(M)$$

and non-deterministic reduction:

$$\frac{M(\alpha) = I \quad \sigma \vDash I \to \sigma_1 \vDash I_1}{\sigma \vDash M \ \to \ \sigma_1 \vDash M[\alpha := I_1]}\ (Step) \quad \frac{M(\alpha) = I \quad \sigma \vDash I \to \sigma_1}{\sigma \vDash M \ \to \ \sigma_1 \vDash M - \alpha}\ (Stop)$$

and we extend the typing rule by:

$$\frac{\forall \alpha \in dom(M),\ \exists \tau,\ \Gamma \vdash M(\alpha) : \tau}{\Gamma \vdash M : \diamond}\ (Multi)$$

Results

We obtain a polynomial time soundness criterion:

Theorem [Marion and Péchoux (TAMC 2014)]

A typable and strongly normalizing multi-thread terminates in a polynomially bounded number of transitions.

The strong normalization assumption can be weakened under a fair scheduling policy (depending only on M and tier **1** values):

Theorem [Marion and Péchoux (TAMC 2014)]

A typable a multi-thread terminating under a fair scheduling policy terminates in a polynomially bounded number of transitions.

Moreover, type inference remains decidable:

Theorem [Hainry, Marion and Péchoux (Fossacs 2013)]

Type inference can be done in polynomial time.

Forks: motivation

- May the analysis be generalized to more expressive languages ?
- Can we analyze parallelism ?
- Is it possible to jump from time (FPtime) to space (Pspace or FPspace) ?

In [Fossacs 2013], we have presented an extension to forks.
The syntax of the language is extended by two commands:

$$X = \texttt{fork}() \mid X = \texttt{wait}\{E\}$$

Forks informal semantics

On the execution of $X = \mathit{fork}(); I$ in a parent process:

- a new son of (fresh) pid n and instruction I is created (by default, $X := 0$)
- the father has instruction I and knows the pid of its new son ($X := n$)

On the execution of $X = \mathit{wait}(E); I$ in a parent process:

- if E evaluates to n and the process of pid n returns v then $X := v$ in the parent process
- otherwise the father has to wait.

Forks typing rules

We need to add an extra tier $-\mathbf{1}$ ($-\mathbf{1} < \mathbf{0} < \mathbf{1}$) in order to prevent accumulation.

$$\frac{\Gamma \vdash \mathtt{x} : \mathbf{0}}{\Gamma \vdash \mathtt{x{:=}fork()} : \mathbf{0}}\ (F) \qquad \frac{\Gamma \vdash E : \mathbf{0} \quad \Gamma \vdash \mathtt{x} : -\mathbf{1}}{\Gamma \vdash \mathtt{x{:=}wait}(E) : -\mathbf{1}}\ (W)$$

Operators $op :: \tau_1 \times \ldots \times \tau_n \to \tau$ are extended to max operations :

$$\forall \overline{w},\ |[\![op]\!](\overline{w})| \leq \max_{i \in [1,n]} |w_i|$$

provided that $\tau < \mathbf{1}$.

- It means that forks' pid cannot be used as guards
- The values returned by sons cannot be accumulated (at most max or neutral operators).

Example "rien que pour les yeux"

```
max_reduce(n^1, A^0) ::= r^0 := 0: 0; f^-1 := A[r]^0: -1;
  flag^0 := tt: 0;
  while (n^1 ≠ 1)^1 do {
      if flag^0 then {      // not finished
          pidl^0 := fork(): 0
          if (pidl>0)^0 then {      // father process
              r^0 := 2*r+2: 0;
              pidr^0 := fork(): 0}
              else { r^0 := 2*r+1: 0 }     // left son
          if (pidr==0)^0 or(pidl==0)^0 then { f^-1 := A[r]^0:
              else {
                  flag^0 := ff: 0;     // father
                  xl^-1 := wait(pidl): 0;
                  xr^-1 := wait(pidr): 0;
                  f^-1 := max(f^-1, max(xl,xr)): 0; } }
      n^1 := half(n)^1: 1 }   // end of while
    return f:-1
```

Results

We obtain a characterization of Pspace computable functions:

Theorem [Hainry, Marion and Péchoux (Fossacs 2013)]

The set of functions computed by typable, strongly normalizing and confluent processes is exactly the set of polynomial space computable functions *FPspace*.

Soundness:

- As for multi-threads, the computation tree has a polynomially bounded depth
- tier $-\mathbf{1}$ prevents accumulation
- the considered programs are confluent, consequently, we can perform a "in depth" evaluation

Completeness:

- Each FPspace function can be bitwise computed
- We show that QBF can be encoded and typed in our formalism.

OO State of the art

Some techniques and programs to bound resource consumptions

- Amortised analysis for linear heap (Hofmann & Jost)
- "Costa" for analyzing Java bytecode (Albert, Arenas, Genaim, Puebla & Zanardini)
- "Speed" for C++ (Gulwani *et al.*)
- "ResAna" analyzes Java programs (Shkaravska *et al.*)
- Non-interference and tiering for a graph based imperative language (Leivant & Marion)

OO: motivation

- Extend our results to a "daily-life" real programming language
- Analyze the complexity of the OO paradigm
- Obtain "practical" upper bound on both the heap and stack space usage
- Analyze OO features:
  - mixture of while loops and recursive method calls
  - objects in loop guards
  - inheritance
  - control flow statements such as `break` or `continue`

Core Java

In [FOPARA2013], we have considered the Java-like language:

- Expressions $E ::= \ \ldots \mid \mathtt{null} \mid \mathtt{this} \mid \mathtt{new}\ \mathsf{C}(\overline{E}) \mid E.m(\overline{E})$
- Instructions $I ::= \ \ldots \mid E.m(\overline{E});$
- Methods $M_{\mathsf{C}} ::= \tau\ m(\tau_1\ \mathtt{x}_1, \ldots, \tau_n\ \mathtt{x}_n)\{I[\mathtt{return\ x};]\}$
- Cons $K_{\mathsf{C}} ::= \mathsf{C}(\tau_1\ \mathtt{y}_1, \ldots, \tau_n\ \mathtt{y}_n)\{\mathtt{x}_1{:=}\mathtt{y}_1; \ldots \mathtt{x}_n{:=}\mathtt{y}_n;\}$
- Classes $\mathfrak{C} ::= \mathsf{C}\{\tau_1\ \mathtt{x}_1; \ldots; \tau_n\ \mathtt{x}_n;\ K_{\mathsf{C}}\ M^1_{\mathsf{C}} \ldots M^k_{\mathsf{C}}\}$

Core Java Programs

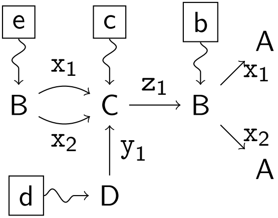
Definition [Core Java Program]

A Core Java Program is a collection of classes and exactly one executable:

$$\text{Exe}\{\texttt{main}()\{\underbrace{\tau_1\ \texttt{x}_1 := E_1; \ldots ; \tau_n\ \texttt{x}_n := E_n;}_{\text{Initialization}}\ \underbrace{\ I\ }_{Computation}\ \}\}.$$

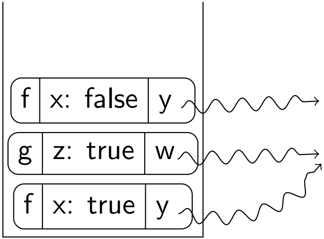
Heap

- Where objects are created and kept in memory.
- Maximal heap space is defined at the launch of the JVM.
- Pointers to the objects, arrows between objects and their attributes.



Stack

- Where arguments of a method call are put.
- Primitive types are put by value.
- Object types are put by reference, *i.e.* a pointer to the heap.
- May grow indefinitely because of recursive calls.



Tiered types

- Expressions, Instructions, Constructors and Methods are annotated by tiered types (i.e. a type and a tier (0 or 1)).
- For instructions, the type will always be `void`.
- For Constructors and methods the tiered type is functional:

$$\texttt{boolean}(1) \times \texttt{BList}(1) \to \texttt{BList}(0)$$

- For methods, the tiered type of the caller object is included: e.g. for `void setQueue(BList q) {...}`

$$\texttt{BList}(0) \times \texttt{BList}(1) \to \texttt{void}(0)$$

Typing Simple Expressions

$$\frac{}{\Gamma \vdash \texttt{true} : \texttt{boolean}(\mathbf{1})}\ \textit{(True)} \qquad \frac{}{\Gamma \vdash \texttt{false} : \texttt{boolean}(\mathbf{1})}\ \textit{(False)}$$

$$\frac{}{\Gamma \vdash \texttt{null} : \mathsf{C}(\mathbf{1})}\ \textit{(Null)}$$

$$\frac{\alpha \preceq \min\{\text{tiers of the attributes}\}}{(m^{\mathsf{C}}, \Delta) \vdash \texttt{this} : \mathsf{C}(\alpha)}\ \textit{(Self)} \qquad \frac{\Delta(m^{\mathsf{C}})(\texttt{x}) = \tau(\alpha)}{(m^{\mathsf{C}}, \Delta) \vdash \texttt{x} : \tau(\alpha)}\ \textit{(Var)}$$

$$\frac{\forall i\ \Gamma \vdash E_i : \tau_i(\alpha) \qquad op :: \tau_1 \times \cdots \times \tau_n \to \texttt{boolean}}{\Gamma \vdash op(E_1, \ldots, E_n) : \texttt{boolean}(\alpha)}\ \textit{(Op)}$$

Typing Instructions

$$\frac{}{\Gamma \vdash \ ; \ : \texttt{void}(\mathbf{0})}\ \textit{(Skip)} \qquad \frac{\Gamma \vdash \texttt{x} : \tau(\alpha) \qquad \Gamma \vdash E : \tau(\beta) \quad \alpha \preceq \beta}{\Gamma \vdash [\tau]\ \texttt{x}{:=}E; : \texttt{void}(\alpha)}\ \textit{(Ass)}$$

$$\frac{\Gamma \vdash I : \texttt{void}(\alpha) \quad \alpha \preceq \beta}{\Gamma \vdash I : \texttt{void}(\beta)}\ \textit{(Sub)} \qquad \frac{\forall i\ \Gamma \vdash I_i : \texttt{void}(\alpha_i)}{\Gamma \vdash I_1\ I_2 : \texttt{void}(\alpha_1 \vee \alpha_2)}\ \textit{(Seq)}$$

$$\frac{\Gamma \vdash E : \texttt{boolean}(\alpha) \quad \forall i\ \Gamma \vdash I_i : \texttt{void}(\alpha)}{\Gamma \vdash \texttt{if}(E)\{I_1\}\texttt{else}\{I_2\} : \texttt{void}(\alpha)}\ \textit{(If)}$$

$$\frac{\Gamma \vdash E : \texttt{boolean}(\mathbf{1}) \quad \Gamma \vdash I : \texttt{void}(\mathbf{1})}{\Gamma \vdash \texttt{while}(E)\{I\} : \texttt{void}(\mathbf{1})}\ \textit{(Wh)}$$

Typing Constructors

$$\frac{\forall i\ (m^{\mathsf{C}}, \Delta) \vdash E_i : \tau_i(\beta_i) \quad \alpha_i \preceq \beta_i \qquad (\epsilon, \Delta) \vdash \mathsf{C}(\ldots \tau_i\ \mathtt{y}_i \ldots)\{\ldots \mathtt{x}_i := \mathtt{y}_i; \ldots\} : \cdots \times \tau_i(\alpha_i) \times \cdots \to \mathsf{C}(\mathbf{0})}{(m^{\mathsf{C}}, \Delta) \vdash \mathtt{new}\ \mathsf{C}(E_1, \ldots, E_n) : \mathsf{C}(\mathbf{0})}\ \textit{(New)}$$

$$\frac{\forall i\ (\epsilon, \Delta) \vdash \mathtt{y}_i : \tau_i(\alpha_i)}{(\epsilon, \Delta) \vdash \mathsf{C}(\ldots, \tau_i\ \mathtt{y}_i, \ldots)\{\ldots \mathtt{x}_i := \mathtt{y}_i; \ldots\} : \cdots \times \tau_i(\alpha_i) \times \cdots \to \mathsf{C}(\mathbf{0})}\ (K_C)$$

Constructors make the heap increase, hence output something of tier $\mathbf{0}$.

Safety assumption

Definition [Safety]

A well-typed program with respect to a typing environment Δ is safe if for each recursive method $M_C = \tau\ m(\ldots)\{I\ [\texttt{return x;}]\}$:

- there is exactly one call (even nested) to m,
- there is no while loop inside I,
- and the following judgment can be derived:

$$(\epsilon, \Delta) \vdash M_C : \mathsf{C}(\mathbf{1}) \times \tau_1(\mathbf{1}) \times \cdots \times \tau_n(\mathbf{1}) \to \tau(\mathbf{1}).$$

Results

Theorem [Hainry and Péchoux]

In the execution of a safe Core Java program terminating on input $\mathcal{C}$, the size of the heap and of the stack are in $O(|\mathcal{C}|^{n_1((\nu+1)\lambda)})$.

- n_1 the number of variables and attributes of tier **1**,
- λ the maximum number of nested while and
- ν the maximum number of nested methods.

We are still complete wrt FPtime and type inference is decidable:

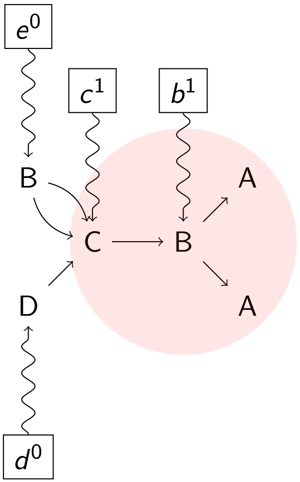
Proposition [Hainry and Péchoux]

The set of functions computable by typable, safe and terminating programs is exactly FPtime

Proposition [Type inference]

The type inference can be done in time linear in the size of the program.

Idea of the proof



- The subheap of tier **1** never grows.
- Only tier **1** variables control `while` and recursive functions.
- The number of tier **1** configurations is bounded by $|\mathcal{C}|^{2\times n_1}$.
- Hence a bound on the stack and heap.

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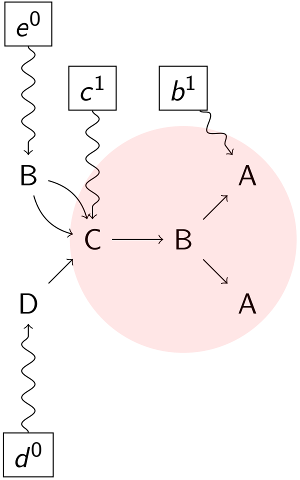

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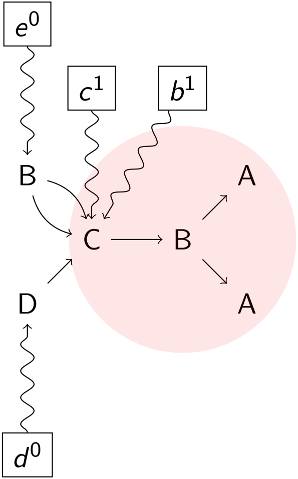

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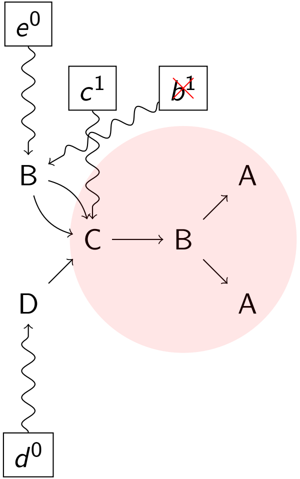

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Conclusion

Result

A static analysis for resource consumption dealing with:

- several languages (imperative, fork, multi-thread, OO, ...)
- several classes (FPtime, FPspace,...)
- both extensional and intensional (heap, stack) properties

Drawbacks and Open questions

- Not intentionnally complete: improve expressiveness by program transformation
- Capture Thread creation (work in progress)
- Do the implementation
- Extend the characterizations (PP, BPP, ...)