



## Associate Team proposal 2020-2022 Submission Form

**Title:** Termination and Complexity Properties of Probabilistic Programs.

**Associate Team acronym:** TC(Pro)<sup>3</sup>

**Principal investigator (Inria):** Romain Péchoux, Inria project team Mocqua, Inria Nancy Grand Est

**Principal investigator (Main team):** Georg Moser, University of Innsbruck, Austria

**Other participants:** Inria project team Focus, Inria Sophia

**Key Words:**

- A2.4. Méthodes formelles pour vérification, sûreté, certification
- A7. Informatique théorique
- A8. Mathématiques pour l'informatique

## 1 Partnership

### 1.1 Detailed list of participants

- Inria project team Mocqua:
  - **Romain Péchoux** (PI), Lorraine University (délégation CNRS 2019-2020), associate professor, permanent contract, <https://members.loria.fr/RPechoux/>. Romain Péchoux has worked on techniques for studying the implicit computational complexity of programming languages. He has an expertise in interpretation methods, tier-based type systems and linear logic-based type systems.
  - **Titouan Carette**, PhD student, non permanent.
  - **Alexandre Clément**, PhD student, non permanent.
  - **Emmanuel Hainry**, Lorraine University, associate professor, permanent contract, <https://members.loria.fr/EHainry/>. Emmanuel Hainry has a strong expertise in tier-based type systems for implicit computational complexity.
  - **Emmanuel Jeandel**, Lorraine University, professor, permanent contract, <https://members.loria.fr/EJeandel/>. Emmanuel Jeandel has a strong expertise in quantum programming. He is team leader of the Inria project team Mocqua.
  - **Simon Perdrix**, CNRS, researcher HDR, permanent contract, <https://members.loria.fr/SPerdrix/>. Simon Perdrix has a strong expertise in quantum programming. He is principal investigator of the ANR SoftQPro.

- University of Innsbruck, Austria:
  - **Georg Moser** (PI), associate professor with habilitation, permanent contract, <http://cl-informatik.uibk.ac.at/users/georg/>. Georg Moser has a strong expertise on automated termination analysis and automated complexity analysis of term rewrite systems. He is head of the research group *Computation with Bounded Resources* and he is a contributor of the Tyrolean Complexity Tool (TCT), an open source software licensed under GNU LGPL, allowing to infer automatically upper bounds on the derivational complexity of term rewrite systems, <http://cl-informatik.uibk.ac.at/software/tct/>.
  - **Gemma De las Cuevas**, assistant professor, permanent contract, <https://www.gemmadelascuevas.com/>. Gemma De las Cuevas is a theoretical physicist working mainly at the intersection of mathematical physics and quantum many-body physics. Recently she has been in particular interested in the connection between the universality of spin models and the universality of computability theory.
  - **David Obwaller**, PhD student, non permanent.
  - **Florian Zuleger**, associate professor, permanent contract, <https://informatiks.tuwien.ac.at/people/florian-zuleger>. Florian Zuleger has an expertise on automated methods for termination and resource-bound analysis and program verification. He has also worked on average case complexity analysis.
- Inria project team Focus:
  - **Martin Avanzini**, Inria, junior researcher, permanent contract, <https://www-sop.inria.fr/members/Martin.Avanzini/>. Martin Avanzini has expertise on automated termination analysis and automated complexity analysis of term rewrite systems and of probabilistic programs. He is also a main contributor of the Tyrolean Complexity Tool.
  - **Ugo Dal Lago**, Università di Bologna, associate professor, permanent contract, <https://www.cs.unibo.it/~dallago/>. Ugo Dal Lago has an expertise in the study of semantics and complexity properties of programming languages, including probabilistic and quantum programming languages.

## 1.2 Nature and history of the collaboration

The major goal of the associate team TC(Pro)<sup>3</sup> is to develop automatic tools for termination and complexity static analysis of probabilistic program. To accomplish this goal, a synergy between the different expertises of the partners will be achieved. Indeed, while the Inria project team Mocqua has a strong expertise in complexity analysis (Hainry, Péchoux) and in quantum computing (Jeandel, Perdrix), the Austrian partner is specialized in termination and complexity analysis of term rewrite systems (Moser, Zuleger) and the Inria project team Focus has obtained several results on the termination of probabilistic programs (Avanzini, Dal Lago). These three kinds of skills and expertises are complementary and will be combined and merged to achieve the major goal.

This collaboration will be made effective and fruitful through several means. First, the three entities are all belonging to the implicit computational complexity community and are involved in the regular scientific events of this community (workshops, conferences, special issues, ...). Consequently, their members (Avanzini, Dal Lago, Hainry, Moser, Péchoux, Zuleger) all have a solid common background on term rewrite systems and programming languages and they share common scientific knowledge, culture, and expertise. Second, it includes researchers from the

three partners who have a strong expertise in quantum programming (De las Cuevas, Dal Lago, Jeandel, Perdrix), which makes the collaboration realistic and natural.

A scientific cooperation is already existing between Inria project team Focus and the Austrian partner (Martin Avanzini was PhD there and contributed to the theoretical research and software development).

A scientific cooperation is already existing between Inria project team Mocqua and Inria project team Focus. The two teams participate to the ANR project ELICA (ending in 2019) and Martin Avanzini and Romain Péchoux are coeditors a Theoretical Computer Science special issue on developments in implicit computational complexity DICE 2016, 2017, and 2018 (edition in progress).

The collaborations between the partners is summarized in Figure 1.

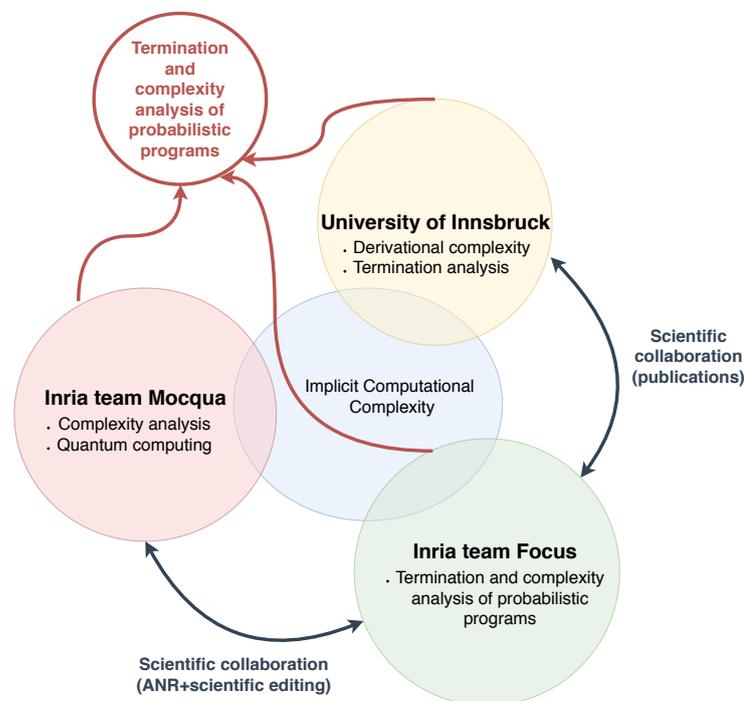


Figure 1: Collaboration between partners

## 2 Scientific program

### 2.1 Context

Probabilistic languages consist in higher-order functional, imperative languages, and reduction systems with sampling and conditioning primitive instructions. Starting from the seminal work of Kozen [19], semantics of probabilistic languages have been intensively studied (e.g. with respect to denotational [12, 33] or small step [11, 7] semantics). It is only recently that we have seen an increased interest on the verification of their computational and algorithmic properties, most notably *termination*, e.g. [8, 9, 3, 10, 1], and *complexity*, e.g. [18, 25, 22, 2]. This renewed popularity is due to the strong need of applications of such probabilistic languages in algorithmics [21], robotics [32], and cryptography [28].

While deep theoretical results have been established, applications of termination and complexity analysis are restricted to academic examples so far. Indeed, the situation is reminiscent of that in termination analysis of non-probabilistic systems in the nineties, where termination was predominantly proven by embeddings into well-ordered sets, an inherently non-modular method that does not scale up well in an automated setting. In practice, *modular* methods are required, i.e., techniques that break down into the analysis of individual programs components. While various modular techniques exist for reasoning about non-probabilistic systems, a vast majority of these results cannot be directly transposed from classical models to probabilistic models. For example, studies on termination of classical systems that mostly enforce strong normalization were not straightforwardly adaptable to the probabilistic setting where a program having a small probability to diverge can be considered to be terminating.

### 2.2 Objectives (for the three years)

The associate team TC(Pro)<sup>3</sup> has the aim to contribute to the field by developing methods for reasoning on quantitative properties of probabilistic programs and models. Such tools have applications in quantum computing as quantum programs can be considered as particular cases of probabilistic transition systems where measurement plays the role of probabilistic choice. Consequently, they could be applied to quantum programming languages such as **QPL** [29] or **quantum lambda-calculus** [30]. We establish our objective along the following axes:

- Our first goal is to develop novel methods for ensuring **termination** properties (e.g. almost sure termination) and **complexity** properties (e.g. average case polynomial time) of **probabilistic programs**, with a particular emphasis on **automated techniques**. This study encompasses imperative as well as higher-order functional programs. While the consortium has achieved here already notable results, so far the methods are either relatively weak on industrial scale examples [3, 10] or expressive (even complete) but lacking any form of automation [2, 18, 13]. To overcome these limitations, on the one hand, we investigate how these methods can be coupled with modularity techniques, and how inherently modular methods from ICC, such as data-tiering [16, 15], can be lifted. On the other hand, we are interested in delineating a subsystem of complete systems [2, 18] that admits tractable inference, paving the way towards automation.
- While with the previous goal we will foremost study prototypical languages, our second goal is to lift these techniques to the new languages emerging in statistical reasoning (e.g. **Anglican** [34] or **Church** [14]) and quantum computing (e.g. **QPL**). On the side of probabilistic languages, it is foremost necessary to broaden the language to standard programming constructs such as arbitrary data-types and to integrate *conditioning*. On the side of quantum languages, *quantum primitives* have to be integrated into the analysis.

- Besides the previous two theoretical goals, we then pursue the **integration** of the developed methods in our tool  $\mathsf{TCT}$ . To date,  $\mathsf{TCT}$  is among the most powerful tools to reason about the complexity (and termination) of various forms of programs (e.g. rewrite systems, imperative and functional programs). However, probabilistic and quantum programs are currently not supported.

### 2.3 Work-program (for the first year)

**Planned activities:** Thematically, the first year will be devoted to approaching the central theoretical challenges posed by the problem and to devising prototypical implementations thereof. The three main research tasks pursued in the first year will be:

- We will study a minimal probabilistic model of computation, such as first-order reduction systems or a core imperative programming language, and investigate to what extent existing termination and complexity techniques can be lifted from the non-probabilistic to the probabilistic setting. This study will encompass in particular the highly-influential *dependency pair framework* and its complexity aware variations [23, 4, 17, 20, 26], which enables a form of compositional call-graph analysis. On the other hand, we are interested in adapting promising ICC systems based on the notion of data-tiering [15, 5] to probabilistic programs. This tasks involve Avanzini, Dal Lago, Hainry, Moser, Obwaller, Péchoux, Zuleger.
- In recent work [6], we have developed a fully automated inference procedure to reason about the termination behavior and expected runtime of a prototypical imperative probabilistic language, based on the weakest precondition calculus of Kaminski et al. [18]. The main objective of this task is to extend this procedure to stochastic languages. New results of Olmedo et al. [24], which integrate conditioning into this calculus will give a first lead. Then we will extend the calculus to a non-recursive fragment of **QPL** [29]. While the calculus is well-equipped to deal with probabilistic choices stemming from measurements, the main concerns here will be to reflect quantum primitives into the calculus and to extend the analysis tools as, for example, [27] extended abstract interpretations to perform quantum entanglement analysis. Another main concern in quantum computing is to control space, i.e. the number of qubits. Indeed, space is very restricted in current systems<sup>1</sup> and, consequently, lifting space complexity techniques to the quantum paradigm is of particular interest. These tasks involve Avanzini, Carette, Clément, Dal lago, De las Cuevas, Hainry, Jeandel, Moser, Péchoux, Perdrix.
- As usual, we provide **prototypical implementations** along these developments, that is, we turn the theoretical results from the previous to tasks into concrete algorithms that scale well while giving informative results (e.g.[31]). This task will allow us to give experimental evidence highlighting the usefulness of the theoretical results. It will also provide a starting point towards our endeavor of extending  $\mathsf{TCT}$  to a versatile tool capable of reasoning about stochastic and quantum languages. This task involves Avanzini, Hainry, Moser, Obwaller, Péchoux, Perdrix, Zuleger.

**Kickoff Meeting:** A three-day kickoff meeting is planned in the first months of the project, which will take place in Nancy. This is of paramount importance as it will give the right opportunity for participants to meet and exchange ideas right at the beginning of the project. We expect all the participants of the project to attend the meeting.

<sup>1</sup><https://www.technologyreview.com/f/614416/google-researchers-have-reportedly-achieved-quantum-supremacy/>

**Visits (2020):** Some of the participants will visit their partner during one or two weeks.

- Avanzini (permanent, Focus) will visit Innsbruck for one week, during autumn.
- Dal Lago (permanent, Focus) will visit Innsbruck for one week, during autumn.
- Hainry (permanent, Mocqua) will visit Innsbruck for one week, during autumn.
- Péchoux (permanent, Mocqua) will visit Innsbruck for two weeks, during autumn.
- Moser (permanent, Innsbruck) will visit Mocqua for one week, during autumn.
- Zuleger (permanent, Innsbruck) will visit Mocqua for one week, during autumn.

### 3 Data Management Plan

Data management plan is not relevant with respect to this proposal.

## 4 Budget

### 4.1 Budget (for the first year)

The budget for 2020 (see Figure 2) is of 15 080€, of which 9 000€ will be provided by this associate team, and 2 750€ consists in co-funding from the ANR Project SoftQPro, (“Solutions logicielles pour l’optimisation des programmes et ressources quantiques”), 2018-2022, led by Simon Perdrix, and to which Mocqua participates (secured). Co-funding on the Austrian side will be applied by the University of Innsbruck for a total amount of 3 330€ (secured).

The budget will finance journeys of participants. More specifically:

- coffee breaks of the kickoff meeting (2,7€ per person, 4 coffee breaks), summing to 130€.
- Avanzini and Dal Lago (Sophia) and De las Cuevas, Moser, Obwaller, Zuleger (Innsbruck) will participate to the kickoff meeting. The expected cost for each participant from Sophia is 800€ (indemnité nuit 70€, avion AR 300€, taxi AR 180€, indemnités/nuit 70€, indemnités/repas 15,25€) and the expected cost for each participant from Innsbruck is 570€ (train AR 300€, navette Gare TGV 20€, indemnité journalière 175€), summing to 3 880€. Hence the total cost of the kickoff meeting will be 4 010€.
- There will also be four trips, each of one week, of French members to Austrian sites and two trips from Austrian members to Nancy. Each trip Nancy-Innsbruck has an expected cost of 1 630€ for one week (3) and 2 860€ for two weeks (1) and each trip Sophia-Innsbruck has an expected cost of 1 660€ for one week (2), summing to 11 070€.

Income		Expenses	
TC(Pro) <sup>3</sup>	9 000€	Kickoff meeting	4 010€
ANR SoftQPro	2 750€	Visits	11 070€
University of Innsbruck	3 330€		
<b>Total Income</b>	<b>15 080€</b>	<b>Total Expenses</b>	<b>15 080€</b>

Figure 2: Budget for 2020

## 4.2 Strategy to get additional funding

The participants of the associate team will submit a proposal to the H2020 FET FLAGSHIPS – Tackling grand interdisciplinary science and technology challenges | Call ID: H2020-FETFLAG-2018-2020: Complementary call on Quantum Computing, ID: FETFLAG-05-2020. The aim of this proposal will be to target the development of quantum applications and libraries that facilitate the link from a high-level description of algorithms to a low-level implementation, for solving concrete problems, and the current associate team achievements will be complementary objectives towards this goal.

In a long-term strategy, we also anticipate PhD students involved in this associate team (Carette, Clément, Obwaller) to apply for a Marie Curie Fellowship: Individual fellowship after their PhD defense (planned in 2021, 2022, and 2021, respectively).

## 5 Added value

The following is a non-exhaustive list of strengths which will benefit the research of the TC(Pro)<sup>3</sup> participants:

- The major benefit of this associate team will be to obtain new tools for the termination and complexity analysis of probabilistic programs. Such kind of tools are highly expected and will be completely new results in this area.
- The obtained results on probabilistic program termination and complexity analysis will be transferred to quantum programs, which is a hot topic in verification and programming language research.
- The integration of the results obtained on termination and complexity analysis of probabilistic programs into the **TCT** tool developed by the Austrian partner.
- A transfer of knowledge between the three partners (see Figure 1). The aim of this associate team is to reinforce the links between Inria project team Mocqua and the University of Innsbruck, taking advantage of the expertise of each site - implicit complexity (Mocqua), automatic termination (Innsbruck), and verification of probabilistic programs (Focus) - to develop automatic techniques for termination and complexity analysis of probabilistic programs.
- On both the Austrian and French sides, graduate and PhD students (Carette, Clément, Obwaller) will play central roles. We plan to hire new students in the coming years and this international collaboration will make it possible to plan visits, exchanges between partners, and to present joint results in meetings, which will constitute invaluable experience for the students future academic career.

## 6 Previous Associate Teams

Romain Péchoux was principal investigator of the associate team CRISTAL (Contrôle des Ressources par Interprétation et Systèmes de Types Affines et Linéaires) between the Inria project team CARTE, Inria Nancy Grand Est, and the University of Torino, Italy, from 2009 to 2012. He was not involved in any associate team in the last five years.

## 7 Impact

The results produced by this associate team are of two kinds. Some software component, implementing the obtained probabilistic results, will be written by the participants of the three partners (Avanzini, Carette, Clément, Hainry, Moser, Obwaller, Péchoux, Zuleger) and will be integrated into the TCT tool developed by the University of Innsbruck. The results will also consist in publications to international journals (targeted: Journal of Automated Reasoning, Computer Science Programming) and high ranking conferences (targeted: POPL, PLDI, ESOP, ...).

## 8 Intellectual Property Right Management

### 8.1 Background

Tyrolean Complexity Tool (TCT) is an open source software developed by the University of Innsbruck and licensed under GNU LGPL, allowing to infer automatically upper bounds on the derivational complexity of term rewrite systems, <http://cl-informatik.uibk.ac.at/software/tct/>.

### 8.2 Protective measures

All the software components (joint results) produced in this collaboration will be open source software licensed under GNU LGPL. The results will be published following the editorial and publishing policies of the corresponding conference or journal. In accordance to these policies, they will also be made available on [hal.inria.fr](http://hal.inria.fr).

## 9 References

### 9.1 Joint publications of the partners

- [4] M. Avanzini and G. Moser. “A Combination Framework for Complexity”. In: *IC* 248 (2016), pp. 22–55.
- [5] M. Avanzini and G. Moser. “Polynomial Path Orders”. In: *LMCS* 9.4 (2013).
- [6] M. Avanzini, M. Schaper, and G. Moser. “Modular Runtime Complexity Analysis of Probabilistic While Programs.” In: *Proc. of 10th DICE and 6th FOPARA*. 2019.

### 9.2 Main publications of the participants relevant to the project

#### Focus:

- [2] M. Avanzini, U. Dal Lago, and A. Ghyselen. “Type-Based Complexity Analysis of Probabilistic Functional Programs”. In: *Proc. of 34<sup>th</sup> LICS*. IEEE, 2019, pp. 1–13.
- [3] M. Avanzini, U. Dal Lago, and A. Yamada. “On Probabilistic Term Rewriting”. In: *Proc. of 14<sup>th</sup> FLOPS*. Vol. 10818. LNCS. Springer, 2018, pp. 132–148.
- [10] U. Dal Lago and C. Grellois. “Probabilistic Termination by Monadic Affine Sized Typing”. In: *Proc. of 26th ESOP*. 2017, pp. 393–419.
- [11] U. Dal Lago and M. Zorzi. “Probabilistic Operational Semantics for the Lambda Calculus”. In: *RAIRO - TIA* 46.3 (2012), pp. 413–450.

#### Innsbruck:

- [17] N. Hirokawa and G. Moser. “Automated Complexity Analysis Based on the Dependency Pair Method”. In: *Proc. of 4<sup>th</sup> IJCAR*. Vol. 5195. LNAI. Springer, 2008, pp. 364–380.
- [20] G. Moser and A. Schnabl. “The Derivational Complexity Induced by the Dependency Pair Method”. In: *LMCS* 7.3 (2011).
- [31] M. Sinn, F. Zuleger, and H. Veith. “A Simple and Scalable Static Analysis for Bound Analysis and Amortized Complexity Analysis”. In: *Proc. of 26th CAV*. Vol. 8559. LNCS. Heidelberg, DE: Springer, 2014, pp. 745–761.

#### Mocqua:

- [15] E. Hainry and R. Péchoux. “A Type-based Complexity Analysis of Object Oriented programs”. In: *IC* 261.Part (2018), pp. 78–115.
- [16] E. Hainry and R. Péchoux. “Objects in Polynomial Time”. In: *Proc. of 13<sup>th</sup> APLAS*. 2015, pp. 387–404.
- [26] R. Péchoux. “Synthesis of Sup-interpretations: A Survey”. In: *TCS* 467 (2013), pp. 30–52.
- [27] S. Perdrix. “Quantum Entanglement Analysis Based on Abstract Interpretation”. In: *Proc. of 15<sup>th</sup> SAS*. 2008, pp. 270–282.

### 9.3 Other publications

- [1] S. Agrawal, K. Chatterjee, and P. Novotný. “Lexicographic Ranking Supermartingales: An Efficient Approach to Termination of Probabilistic Programs”. In: *PACMPL* 2.POPL (2018), 34:1–34:32.

- [7] J. Borgström et al. “A Lambda-Calculus Foundation for Universal Probabilistic Programming”. In: *Proc. of 21st ICFP*. ACM, 2016, pp. 33–46.
- [8] O. Bournez and F. Garnier. “Proving Positive Almost-Sure Termination”. In: *Proc. of 16th RTA*. Vol. 3467. LNCS. Springer, 2005, pp. 323–337.
- [9] A. Chakarov and S. Sankaranarayanan. “Probabilistic Program Analysis with Martin-gales”. In: *Proc. of 25th CAV*. Vol. 8044. LNCS. Springer, 2013, pp. 511–526.
- [12] T. Ehrhard, M. Pagani, and C. Tasson. “Measurable Cones and Stable, Measurable Functions: a Model for Probabilistic Higher-order Programming”. In: *PACMPL 2.POPL (2018)*, 59:1–59:28.
- [13] L. María F. Fioriti and H. Holger. “Probabilistic Termination: Soundness, Completeness, and Compositionality”. In: *Proc. of 42<sup>nd</sup> POPL*. ACM, 2015, pp. 489–501.
- [14] N. D. Goodman et al. “Church: A Language for Generative Models”. In: *Proc. of 24th UAI*. AUAI Press, 2008, pp. 220–229.
- [18] B. L. Kaminski et al. “Weakest Precondition Reasoning for Expected Runtimes of Randomized Algorithms”. In: *J. ACM* 65.5 (2018), 30:1–30:68.
- [19] D. Kozen. “Semantics of Probabilistic Programs”. In: *JCSS* 22.3 (1981), pp. 328–350.
- [21] R. Motwani and P. Raghavan. *Randomized Algorithms*. Cambridge University Press, 1995.
- [22] N. C. Ngo, Q. Carbonneaux, and J. Hoffmann. “Bounded Expectations: Resource Analysis for Probabilistic Programs”. In: *Proc. of 39th PLDI*. 2018, pp. 496–512.
- [23] L. Noschinski, F. Emmes, and J. Giesl. “Analyzing Innermost Runtime Complexity of Term Rewriting by Dependency Pairs”. In: *JAR* 51.1 (2013), pp. 27–56.
- [24] F. Olmedo et al. “Conditioning in Probabilistic Programming”. In: *ACM Trans. Program. Lang. Syst.* 40.1 (2018), 4:1–4:50.
- [25] F. Olmedo et al. “Reasoning about Recursive Probabilistic Programs”. In: *Proc. of 31<sup>st</sup> LICS*. 2016, pp. 672–681.
- [28] T. Sato et al. “Formal Verification of Higher-order Probabilistic Programs: Reasoning about Approximation, Convergence, Bayesian Inference, and Optimization”. In: *PACMPL 3.POPL (2019)*, 38:1–38:30.
- [29] P. Selinger. “Towards a quantum programming language”. In: *Mathematical Structures in Computer Science* 14.4 (2004), pp. 527–586.
- [30] P. Selinger and B. Valiron. “A lambda calculus for quantum computation with classical control”. In: *Mathematical Structures in Computer Science* 16.3 (2006), pp. 527–552.
- [32] S. Thrun, W. Burgard, and D. Fox. *Probabilistic Robotics (Intelligent Robotics and Autonomous Agents)*. MIT Press, 2005.
- [33] M. Vákár, O. Kammar, and S. Staton. “A Domain Theory for Statistical Probabilistic Programming”. In: *PACMPL 3.POPL (2019)*, 36:1–36:29.
- [34] F. D. Wood, J.-W. van de Meent, and V. Mansinghka. “A New Approach to Probabilistic Programming Inference”. In: *Proc. of 17<sup>th</sup> AISTATS*. Vol. 33. JMLR. 2014, pp. 1024–1032.



## Letter of Intent

### Commitment of the corresponding partner institution

**Associate Team acronym: TC(Pro)<sup>3</sup>**

**Principal investigator (Inria):**

Romain Péchoux, Inria project team Mocqua, Inria Nancy Grand Est

**Principal investigator (Partner team):**

Georg Moser, University of Innsbruck

The University of Innsbruck, which is the partner team's institution legal entity, confirms its intention to participate in the Associate Team entitled TC(Pro)<sup>3</sup> and has been informed that it will be requested to sign an Associated Team agreement with Inria, in case the Associate Team entitled TC(Pro)<sup>3</sup> should be retained.

Date: 3.10.2019

Name and function: Head projekt.service.büro

Signature:

Stamp of the partner institution



projekt.service.büro

Technikerstraße 21a  
6020 Innsbruck  
Österreich / Austria