Maintenance of the Visibility of a Moving Viewpoint, and Applications

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Soutenance de thèse
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1. Introduction

2. Connexity in the 3D Visibility Complex

3. Visibility Maintenance among Convex Polytopes in Space

4. Conclusions
Visibility in Image Synthesis and Computational Geometry

Computer Graphics oldest goal: create images of virtual worlds.

[Wonka et al. 2006]
Let’s look at some examples of visibility problems...
Example in the plane: given polygonal scene description and viewpoint $V$
Compute **objects** visible from **V**

Ordered around the viewpoint: table, bottle, lamp
Compute \textit{segments} visible from $V$

Possibly ordering the segments circularly
Compute parts visible from $V$. E.g. lit/shadowed parts if $V$ is a light source.

Adding discontinuity positions ($\circ$), to clip the invisible parts.
Further, the observer might vary (from-cell visibility)

Precomputation of visible sets, or area-lights
[Durand et al. 00, Haumont et al. 05]
V (or the objects) might move: update visibility during motion

E.g., if no preprocessing is available
Kinetic Data Structures (KDS)

Framework for design & analysis of algo. for maintaining an attribute of continuously moving items [Basch et al. 97].
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Event queue
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In the first part, we consider three problems on:

- Exact and continuous object-visibility maintenance.
- With moving point observer (viewpoint).
- And possibly continuously moving objects.

... and a property of the visibility complex.
Goal: maintain the ordering of $n$ points around $V$, as $V$ moves along line segment trajectories given on-line.

Optimal algorithm.
Tangents through $V$ describe the visibility polygon (in **green**).

Goal: maintain the visibility polygon as $V$ moves along algebraic trajectory given on-line.

Under mild assumptions, the Visible Zone algorithm is optimal.

We explain the algorithm and give a new and simpler proof of a crucial property used for the algorithm.
Contrib #3: Vis. Maintenance Among Polytopes in Space

- The visibility polyhedra encodes the set of objects visible from $V$.
- Goal: maintain the visibility polyhedra as $V$ moves along arbitrary pseudo-algebraic trajectory.

Give a non-optimal algorithm to do so, together with hints at how to improve it. More on that in a few minutes.

Early results presented at DIMACS Workshop, 2002.
3D Visibility complex = 4D set encoding visibility relationships between objects in space. Current algorithms for constructing it seem difficult [Durand] or are not (yet) implementable [Goaoec 04].

1. We prove a topological property of the 3D visibility complex.

2. And apply this property to a simple algorithm to construct the visibility complex. More on that in a few minutes.
Graphics Contributions

Computer graphics applications, related to visibility and motion:

- real-time rendering of large indoor scenes
- real-time rendering of shadows
Contrib #5: Automatic Cells-and-Portals Decomposition

Given an input polygonal scene, we give an algorithm that builds a cells-and-portals graph *suitable* for portal rendering (well sized cells).

INRIA Research Report 4898 (2003), S. Lefebvre and S. Hornus.
Contrib #5: Automatic Cells-and-Portals Decomposition

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Invisible polygons (portals) separate cells (“rooms”)

INRIA Research Report 4898 (2003), S. Lefebvre and S. Hornus.
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Contrib #5: Automatic Cells-and-Portals Decomposition

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“Portal rendering” is a graph traversal

INRIA Research Report 4898 (2003), S. Lefebvre and S. Hornus.
Contrib #7: ZP+, Correct Z-pass Stencil Shadows

Corrects a flaw in well-known algorithm. Generally faster than previous work. [Laine 05] combines best of ZP+ and previous work.

Doom 3

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3D visibility complex:
- *All* visibility relationships.
- Structures the 4D set of “light rays” between objects.

My contributions:
1. **Theorem:** boundaries of its 4-dimensional “cells” are path-connected.
2. **Applied to a simple algorithm to construct the visibility complex.**
Consider a set $\mathcal{O}$ of pairwise disjoint convex sets in space in 3D (figures are 2D).
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The free space $\mathcal{F}$ is outside the objects.
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Free segments (in green).
Consider a set $\mathcal{O}$ of pairwise disjoint convex sets in space in 3D (figures are 2D).

- The free space $\mathcal{F}$ is outside the objects.
- Free segments (in green).
- Maximal free segments (in blue).
Consider a set $\mathcal{O}$ of pairwise disjoint convex sets in space in 3D (figures are 2D).

- The free space $\mathcal{F}$ is outside the objects.
- Free segments (in green).
- Maximal free segments (in blue).
- Let $S$ be the set of maximal free segments. Each maximal free segment has 2 blockers in $(\mathcal{O} \cup \{\infty\})^2$
The visibility complex partitions $\mathcal{S}$ in maximal sets of segments having the same set of blockers.
The visibility complex partitions $S$ in maximal sets of segments having the same set of blockers.

And each set is separated in connected components.
In 2D, the visibility complex $\mathcal{VC}$ is a 2-dimensional cellular complex over $S$. Each $k$-cell, of dimension $k \leq 2$, is homeomorphic to a $k$-disc. [Pocchiola and Vegter 96].

- two 0-cells
- a part of a 1-cell
- one 2-cell, cell or face
In 3D, the visibility complex $\mathcal{VC}$ is **not** a cellular complex over $S$. Intuitively, tiny objects ‘create’ tunnels through 4-cells [Durand et al. 02].

The **green** loop of segments
is not contractible
in the 4-cell $AB$
Let $C$ be a 4-cell of the visibility complex. Let $\partial C$ be its boundary. Then, $\partial C$ is path-connected.

Remarks:

- The visibility complex is not a cell-complex, but $S$ is (e.g., arangement in Plücker space [Goa04]).
- $S$ is a Haussdorf space: in which tools from algebraic topology work well.
- The proof of the theorem uses 3 sub-lemmas...
In order to prove the theorem, we manipulate homology groups.

Let $X$ be a topological space.

$H_0(X)$ is the zeroth homology group. $H_0(X) = \mathbb{Z}^k$; $k$ is the number of connected components.

$H_1(X)$ is the first homology group. We have $H_1(X) = 0$ if $X$ is 1-connected.

Each lemma translates in an homological identity:

1. Segment space $S$ is path-connected $\Rightarrow H_0(S) = \mathbb{Z}$
2. Segment space $S$ is one-connected $\Rightarrow H_1(S) = 0$
3. The complement $C^C$ of 4-cell $C$ is path-connected $\Rightarrow H_0(C^C) = \mathbb{Z}$

We have $H_0(\partial C) = \mathbb{Z}^k$. We want to prove that $k = 1$. 
Proof of theorem

- We enlarge \( \partial C \) a little to obtain an open neighborhood \( B \) of \( \partial C \) with
  \[
  H_0(\partial C) = H_0(B) = \mathbb{Z}^k, \quad k \geq 1
  \]
- Define \( U = B \cup C \), \( V = B \cup C^C \) (\( C \), \( U \) and \( V \) are connected).

![Diagram showing \( C \), \( B \), \( U \), and \( V \)]
Proof of theorem

- We enlarge $\partial C$ a little to obtain an open neighborhood $B$ of $\partial C$ with
  \[ H_0(\partial C) = H_0(B) = \mathbb{Z}^k, \ k \geq 1 \]
- Define $U = B \cup C$, $V = B \cup C^C$ ($C$, $U$ and $V$ are connected).

Using Mayer-Vietoris sequence on $U$ and $V$, we obtain the following short exact sequence of morphisms of groups:

\[ 0 \xrightarrow{\phi_3} \mathbb{Z}^k \xrightarrow{\phi_2} \mathbb{Z}^2 \xrightarrow{\phi_1} \mathbb{Z} \xrightarrow{\phi_0} 0 \]

Generally, such a sequence $0 \to A \to B \to C \to 0$ is said to split, which means $B \approx A \oplus C$.
- In our case: $\mathbb{Z}^2 \approx \mathbb{Z}^k \oplus \mathbb{Z} \approx \mathbb{Z}^{k+1}$, therefore $k = 1$, that is, $\partial C$ is path-connected.
Application to the construction of the visibility complex

Assume we have constructed the 3-skeleton $\mathcal{VC}^{(3)}$ of the visibility complex $\mathcal{VC}$ (its cells of dimension 3 and lower, see manuscript).
We see $\mathcal{VC}^{(3)}$ as a graph whose edges are the 3-cells, and nodes are cells of dimension $< 3$. 
Application to the construction of the visibility complex

We label each 3-cell with its three adjacent 4-cells.
Let $C$ be a 4-cell. Its boundary is connected. So we can retrieve it as a connected component in the graph $\mathcal{VC}^{(3)}$. 

![Diagram of a graph with labeled 3-cells and <3-cells]
Application to the construction of the visibility complex

3-cell <3-cells

3-cell

<3-cells
Application to the construction of the visibility complex

3-cell <3-cells

3-cell

<3-cells
Application to the construction of the visibility complex

3-cell

<3-cells
Outline

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Problem statement

- $k$ disjoint convex polytopes in space.
- Viewpoint $V$. 
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- $k$ disjoint convex polytopes in space.
- Viewpoint $V$.
- Viewmap: partition of the sphere of directions around $V$. 
Problem statement

- \( k \) disjoint convex polytopes in space.
- Viewpoint \( V \).
- Viewmap: partition of the sphere of directions around \( V \).
- Goal: maintain the viewmap as \( V \) moves continuously.

The motion of \( V \) is given on-line as pseudo-algebraic trajectories. Polytopes can move too.
The viewmap, alone, does not contain enough information for its maintenance. We need additional information.
Visibility Polyhedra

- The viewmap, alone, does not contain enough information for its maintenance. We need additional information.
- The viewmap is the same as the visibility polyhedron: the set of all visible points

We extend the visibility polyhedron into a radial decomposition $R_V$ of the freespace, centered on $V$. Let us first describe the radial decomposition in 2D...
(a) Draw tangents at each silhouette points.
(b) Tangents (or walls) partition freespace in faces. Blue faces form the visibility polygon.
(c) Each cell can be seen as a one-dimensional set of segments. The 2D radial decomposition can be maintained without further data structure.
Radial Decomposition in 3D

We add walls in free space, supported by silhouette edges.

These walls partition the free space into 3D faces...
Radial walls and 3D Faces of the Radial Decomposition
Faces of $R_V$

A face $f$ of $R_V$ is

- a 3D set of points (blue)
Faces of $\mathcal{R}_V$

A face $f$ of $\mathcal{R}_V$ is
- a 3D set of points (blue)
- a 2D set of segments

Each face has a front blocker ($A$), and a back blocker ($B$):
- The front blocker is a polytope or the viewpoint $V$
- The back blocker is a polytope or the sky, $\infty$
Together, the faces of $R_V$ are self-maintenable.
Therefore $R_V$ is maintenable.
The visibility polyhedron (or viewmap) of $V$ is a subset of $R_V$.
Therefore the viewmap can be maintained by maintaining $R_V$. 
Maintaining a Face

Each face of $\mathcal{R}_V$ is also a 2D set of segments, each with a unique direction: A face of $\mathcal{R}_V$ can be described as a spherical polygon on the sphere of directions.

Perspective view from $V$  
The face $AB$
Maintaining a Face

Each face of $\mathcal{R}_V$ is also a 2D set of segments, each with a unique direction: A face of $\mathcal{R}_V$ can be described as a spherical polygon on the sphere of directions.

Perspective view from $V$  The face $AB$
Faces of $R_V$ as Spherical Polygons

$V$ is in the middle of the lot. Sphere of directions around $V$. 
EEE events (3 edges visually meet at a same point) are easy to detect.

VE events (1 vertex crosses an edge) are difficult to detect and correspond to topological change in a face.

In order to detect VE events, we triangulate each face:

VE event $\Leftrightarrow$ collapse of a triangle.
High triangle count in the triangulation of $\mathcal{R}_V$.
Yields a large event queue: approximately $O(s(\text{silhouette edges}) + m(\text{t-vertices}))$.
We present first steps to reduce the number of events.
Towards A Scene Sensitive Pseudo-Triangulation

Number of pseudo triangles: $O(m + k) + \text{separation sensitive term}$.
Conclusions

- **Exact visibility maintenance:**
  1. 2D with points: optimal algorithm.
  2. 2D with convex objects: new simpler proof.
  3. 3D with convex objects: arbitrary motion.

- **3D visibility complex:**
  1. New connexity result.
  2. Applied to visibility complex construction.

- **First (to my knowledge / together with [Haumont 03]) automatic decomposition of indoor scene suitable for real-time rendering.**

- **Stencil shadows: new technique (ZP+), “symmetrical” to previous work (Z-fail) — take advantage of triangle-strips for large meshes — generally faster — instrumental to new techniques [Laine 05].**
Future Work

- **3D visibility maintenance**: More work to do on maintaining pseudo-triangulation (e.g., canonical pseudo-triangulation).

- **3D visibility complex**: More to do on the topology of the 3D visibility complex? Maybe helpful for optimal visibility maintenance in 3D.
Future Work

- **3D visibility maintenance**: More work to do on maintaining pseudo-triangulation (e.g., canonical pseudo-triangulation)

- **3D visibility complex**: More to do on the topology of the 3D visibility complex? maybe helpful for *optimal* visibility maintenance in 3D.

The end
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Samuel Hornus, Jared Hoberock, Sylvain Lefebvre, and John C. Hart.
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Special issue devoted to the proceedings of the 9th Annual ACM Symposium on Computational Geometry (SoCG’93).