Human-in-the-loop stability analysis of haptic rendering of a virtual stiffness with delay – the effect of arm impedance.

Reut Nomberg, *Student Member IEEE*, and Ilana Nisky, *Senior Member, IEEE*

***Abstract*— The uncoupled stability of haptic rendering of virtual stiffness was analyzed extensively, predominantly using passivity considerations. Yet, the role of the operator in improving the stability of haptic systems received less attention. Here, towards human-in-the-loop analysis of stability in haptic rendering, we study the effect of the impedance of the human operator on the stability boundaries of haptic rendering of a virtual stiffness with time-delay. We employ a method that counts the $j\omega$ crossings of the roots of a characteristic second order equation of a coupled system that includes the impedance of the operator and the haptic device, the rendered virtual stiffness, and the time delay. We found that the added impedance of the operator brings to significantly less conservative stable time delay margins, which for certain regions in the parameters space of the coupled system may allow for: (1) Assure a delay-independent stability (2) Increasing the delay to induce stability.**

I. INTRODUCTION

Haptic interfaces enable a sense of touch to human operators during interactions with a virtual or remote environment, providing them with an interaction that is similar to natural. Guaranteeing stability in haptic systems with time delay is challenging. In the same time, it is important in many haptic applications, such as force reflecting teleoperation, where the communication delays are substantial and unavoidable. State of the art approaches focus on the stability of uncoupled haptic systems, without the human operator, and mostly employ passivity considerations. These often require an increase of the damping of the uncoupled system to assure stability and result in a distortion in the haptic feedback. We propose an alternative approach to examining the stability boundaries of time-delayed haptic systems that focuses on the analysis of a coupled system – the haptic system and the human operator motor control.

II. STABILITY ANALYSIS

Fig. 1, shows the conventional schematic model, which describes the uncoupled system (Fig. 1a), and the alternative scheme, which we proposed for describing the coupled system. The coupled system included the dynamics of the operator's arm impedance, the haptic device and the virtual environment properties (Fig. 1b) is described by the characteristic equation:

$$(m_a+m_d)s^2+(b_d+b_a)s+k_a+(b_{VE}s+k_{VE})e^{-T_ds}=0 \quad (1)$$

*This research was supported in part by the Helmsley Charitable Trust through the Agricultural, Biological and Cognitive Robotics Initiative, the Marcus Endowment Fund both at Ben-Gurion University of the Negev, the Israeli MOST within the Israel-Italy Virtual lab on artificial somatosensation, the ISF (grant number 823/15), and the Kreitman Fellowship (to RN).

R. Nomberg and I. Nisky are with the Biomedical Engineering Department and with the Zlotowski Center for Neuroscience, Ben-Gurion University of the Negev, Beer-Sheva, Israel nombergr@post.bgu.ac.il

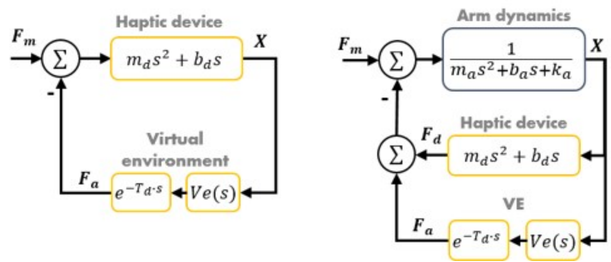

Fig. 1. ***a.*** A schematic model of the *uncoupled* system. The operator exerts a force $F_h(s)$. The haptic device is modeled as a mass (m_d) and damping (b_d), and the rendered virtual environment is a stiffness k_{VE} with a time delay T_d [1]. ***b.*** A schematic model of the *coupled* system. The muscles of the operator apply forces $F_m(s)$ on the arm, modeled by its inertia m_a, and impedance (k_a and b_a). The haptic device applies forces on the arm resulting from the dynamics of the device $F_d(s)$ and the virtual environment $F_a(s)$.

which also can be denoted as

$$m_is^2+b_is+k_i+(b_{VE}s+k_{VE})e^{-T_ds}=0 \quad (2)$$

We limit the analysis to cases where k_{VE} and b_{VE} are positive.

To determine the stability boundaries of (2), we followed the method in [2]. This method counts the number of the right half plane roots of a characteristic equation of increasing values of a time delay T_d. All the inertia, damping, and stiffness coefficients are positive, and hence all the roots of the delay-free version of (2) are within the left half plane and the system is stable. As T_d increases, the analysis can be divided into three regions, according to the number of the crossover frequencies (C_i, $i\in\{0,1,2\}$) - the positive real solutions of (2), as follows:

C_0*: Delay independent stable* (DIS). None of the roots of (2) crosses the $j\omega$-axis from the left to the right as T_d increases, if the following two conditions :

$$\begin{cases} k_i>k_{VE} \\ \dfrac{b_i^2-b_{VE}^2}{2m_i}>k_i-\sqrt{k_i^2-k_{VE}^2} \end{cases} \quad (3)$$

C_1*: Single critical delay*. The system is stable until some $T_d^*>0$, for which the roots of (2) cross the $j\omega$-axis, leading to instability, if:

$$k_i<k_{VE} \quad (4)$$

C_2*: Stability / Instability intervals*. Pairs of roots migrate between LHP and RHP and back several times, if:

$$\begin{cases} k_i>k_{VE} \\ \dfrac{b_i^2-b_{VE}^2}{2m_i}\le k_i-\sqrt{k_i^2-k_{VE}^2} \end{cases} \quad (5)$$

.

In contrast to the coupled system, the uncoupled system is characterized only by the C_1 region.

III. STABILITY BOUNDARIES

A. Effect of the virtual parameters (K_{VE}, B_{VE})

Fig. 2 demonstrates the delay boundaries of the uncoupled and the coupled systems, for different virtual environments' parameters. Since the impedance values of the arm are significantly larger than the impedance values in the haptic device, the boundaries in the coupled system (Fig. 2b) are much wider than the boundaries in the coupled system (Fig. 2a). Furthermore, as the virtual environment (k_{VE}) increases the coupled system transitions between the three different stability regions. For the low values of virtual stiffness $k_{VE} < 237[N/m]$, the system is DIS. For $237 < k_{VE} < 336[N/m]$ the system remains stable until some delay value. However, beyond that minimal value, increasing the delay can restore the stability. For example, for $k_{VE} = 280[N/m]$ the system is stable when $T_d \in [0, 0.1) \bigcup (0.4, 0.5)$. Increasing the virtual damping b_{VE} reduces the range of the DIS and the stability / Instability intervals regions (Fig. 2c).

B. Effect of the arm impedance components (k_i, b_i, m_i)

The effect of increasing the arm's impedance components on the delay boundaries depends on the virtual environment properties. These results are summarized in (Fig. 3b).

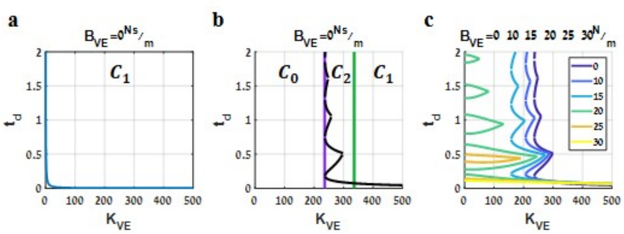

Fig. 2. ***a*** The maximal stable (MS) delay as a function of the virtual stiffness, with a damping-free virtual environment, for the *uncoupled system*. ***b*** The MS delay as a function of the virtual stiffness, with a damping-free virtual environment, for the c*oupled system*. The vertical lines divide the parameters space (from left to right) into Delay-independent stable, Stability/instability intervals, and Single critical delay regions. ***c*** The MS delay as a function of the virtual stiffness, with different damping virtual environment values, for the *coupled system*. The parameters for the uncoupled model correspond to the PHANTOM Desktop haptic device, with $b_d = 0.1[Ns/m]$, and $m_d = 0.045[kg]$, and the coupled model parameters correspond to the median values, evaluated in different experiments found in the literature $k_i = 336.5[Ns/m]$, $b_i = 19.5[Ns/m]$, and $m_i = 1.95[kg]$, see Fig. 3*a* .

IV. DISCUSSION

We present a new approach for the analysis of the stability of haptic systems. We used this approach to analyze the stability of a coupled haptic system, taking into account the impedance of the human operator, the passive dynamics of the haptic device, and a rendered virtual stiffness with delay.

The operator augments the system's damping and adds a stiffness component. Increasing the damping in the uncoupled system is limited since it reduces the haptic feedback efficiency, and hence the operator's damping is highly

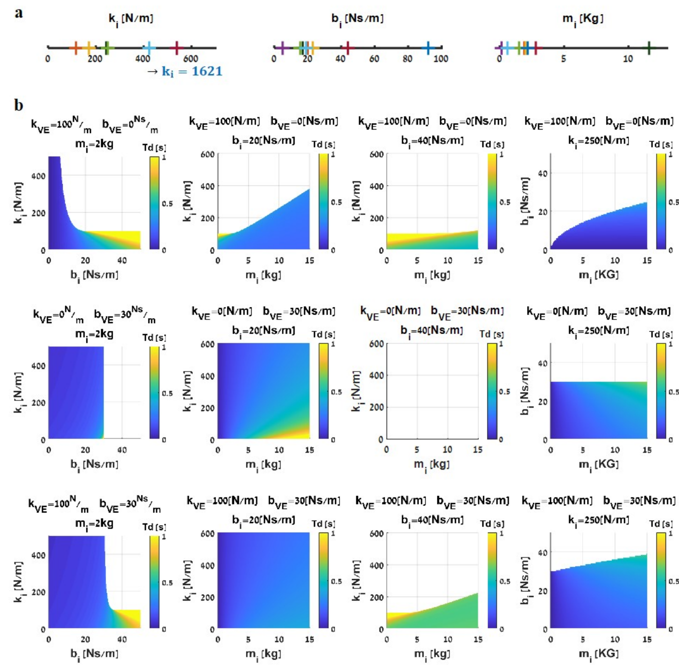

Fig. 3. ***a*** Literature values for the arm impedance components– stiffness k_i, damping (b_i) and mass (m_i). ***b***. The effect of the arm impedance components – The maximal stable time delay of the coupled system, depending on the – stiffness, damping and mass of the operator, for different virtual environment conditions: In a virtual damping free case (first row) to increase the range of stable delays, it is preferred $k_i \uparrow$, $b_i \uparrow$, $m_i \downarrow$. In a virtual stiffness free case (second row) to increase the range of stable delays, it is preferred $k_i \downarrow$, $b_i \uparrow$, $m_i \uparrow$, also when $(b_i > b_{VE})$ the system is DIS. In a virtual environment, composed of both stiffness and damping (third row) to increase the range of stable delays, it is preferred $b_i \uparrow$ and if $(b_i < b_{VE})$ $ki_i \downarrow$, $m_i \uparrow$, else $ki_i \uparrow$, $m_i \downarrow$. The white area represents an infinite allowed delay value (DIS region).

advantageous to the stability of the system. Importantly, the stiffness that is added by the operator introduces two unique stability ranges in the parameters space of the virtual environment and the operator's impedance. One is delay independent stability, in which the system remains stable regardless of the inherent delay. In the other, certain values of delay may render the system unstable, but increasing the delay can restore stability. This adds a practical tool for dealing with instability when delay cannot be mitigated, and without compromising the control gains.

This work is our first step towards a human-centered stability analysis for time-delayed haptic systems. Next, we will analyze the contribution of additional components that characterize human sensorimotor control, e.g. feedback and feed-forward control, to the stability of haptic interaction.

REFERENCES

[1] Minsky, M., Ming, O. Y., Steele, O., Brooks Jr, F. P., Behensky, M. (1990, February). Feeling and seeing: issues in force display. In Proceedings of the 1990 symposium on Interactive 3D graphics (pp. 235-241).

[2] Malakhovski, E., Mirkin, L. (2006). On stability of second-order quasi-polynomials with a single delay. Automatica, 42(6), 1041-1047.