Art Gallery and Watchman Route problems with limited visibilty

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Problem definitions

- *Art Gallery* problem: to find min number of guards needed to station in a polygonal gallery so that each point in the gallery is visible to at least one guard.

- *Watchman Route* problem: to find the shortest route from a point s back to itself so that each point in a given space is visible from at least one point along the route.

Watchman Routes under limited visibility

- By S. Ntafos, published in 1992
- d-visibility
- d-watchman problem: watchman wants to see only boundary of P
  - Finding the shortest route that visits a set of circular sectors of radius d centered at the vertices of the polygon P.
- d-sweeper problem: watchman wants to see all of P
  - Same as sweeping a polygonal floor with a circular broom of radius d so that the total travel of the broom is minimized.

d-watchman problem

- Same as finding the shortest route that visits a set of circular sectors of radius d centered at the vertices of the polygon P.
- The paper uses the **safari route problem** and **zoo-keeper route problem** to get an approximate solution
- **Safari route problem:** find the shortest route that visits a set of m convex polygons that lie in the interior and are attached to the boundary of P.
- **Zoo-keeper route problem:** find the shortest route that visits but does not enter the interior of the set of convex polygons.

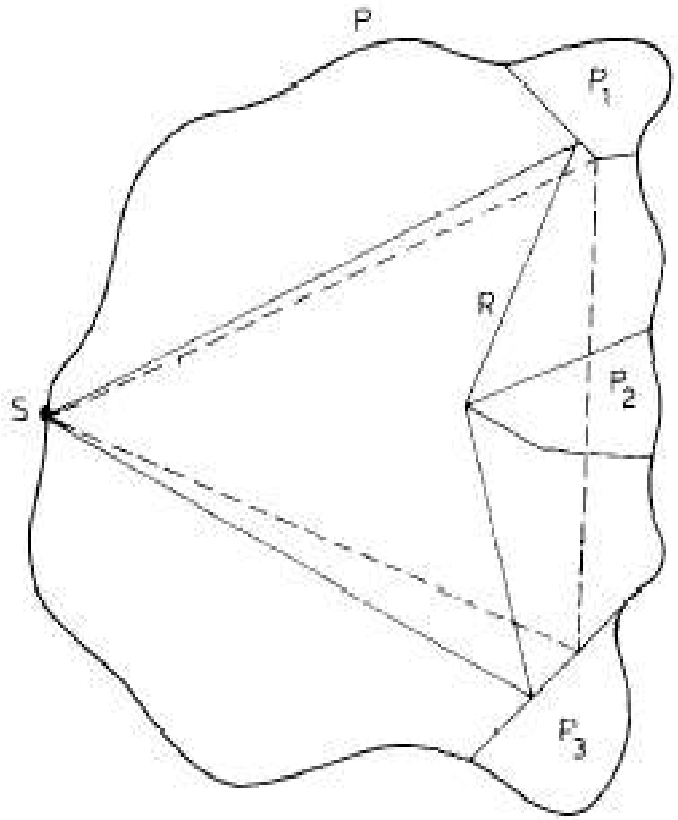


Fig. 1. Shortest zoo-keeper and safari routes.

- The zookeeper route *reflects* on some polygons and *wraps* around others, as the polygons cannot be entered.
- The safari route *reflects* on some polygons and *crosses* the others.

Safari Route algorithm

- P is a simple polygon, and P' is a set of convex polygons in P attached to the boundary.
- Lemma: Let R be a shortest zoo-keeper route for P, P'. If R reflects on all polygons in P', then R is the shortest safari route for P, P'
- Approach to solve the safari route problem:
  - Need to identify the subset of P' that the shortest safari route will reflect on.
  - For this, construct a shortest zookeeper route R.
  - If R does reflects on all polygs in P', done.
  - Else identify some polygon that R wraps around, so that the shortest s.r. will always cross it, and remove it from P', thus reducing the problem.

Approximating the solution to d-watchmen route problem

- Complexity:
  - Zoo-keeper route: $O(n^2)$
  - So, Safari route: $O(mn^2)$
  - where m = cardinality of P' and n = total no. of vertices in P, P'
- The shortest s.r. that visits the set of circular sectors of radius d centered at each vertex of P is the shortest d-watchmen route for P.
- Approximate solution found by modeling the circular sectors with inscribed regular k-gons and solving the resulting s.r. problem

The d-sweeper problem

- Requires viewing the whole interior of the polygon
- Related to TSP in grids
- Approximate solution: superimpose a grid of unit size 2d on the polygon, clip parts of grid on exterior and find a TSP route on the resulting simple grid.
- Claim approximate solution is within 33% of the optimum.

Short Inspection paths for polygons

- ICRA 2000
- Here, they restrict the watchman route problem to viewing only the boundary of the polygon.
- Addresses inspection problems with 2 visibility constraints to model real sensors
  - Limited visibility
  - Limited angle of incidence
- For a practical solution, their approach is to use a randomized algorithm.

2D inspection

- Internal inspection of polygon addressed.
- If external inspection required, enclosing rectangle used.
- Algorithm divides path computation into 2 parts:
  - Solve an art gallery problem to choose a set of sensing locations
  - Connect these to get a short path

Algorithm

- Selecting the guards
  - Algorithm proceeds as a loop
  - At each iteration, a point p on the boundary not yet guarded is selected at random.
  - A balanced tree keeps track of which sections of the border is already guarded
  - The region that can see p is constructed, clipped to the range constraint, then clipped to the incidence constraint.
  - k potential guards are chosen at random from this region, each evaluated as a new guard and the one guarding the most new length of border is chosen.
  - Border representation is updated to reflect the new guarded portion.
  - Loop is repeated till all of the border is guarded.

- Connecting the guards
  - once guards are selected, it is required to find an order to visit them.
  - Shortest path graph used to get the path

- Loop termination ensured, since the unguarded portion of the border decreases monotonically at each iteration.
- The algorithm performs poorly in polygons with sharp interior angles or narrow corners where guards are hard to place.

Fast placement of limited-visibility guards in 2d environments

- IEEE/RSJ 2002
- Deals with inspection too
- Only finds stationary guards using an approximate solution instead of finding a path
- Method works under the limited visibility constraint, but the field of view taken to be 360 degrees.

Method description

- Method used on 2D simple nonconvex polygons with holes.
- Solves under constrained visibility for the interior borders of the polygon
- Algorithm first decomposes the original non-convex polygon into convex polygons
- To cope with limited visibility, a successive division of every convex polygon into convex subpolygons is applied, until each can be locally inspected by 1 guard.
- Complex sweep algorithms are thus avoided.

- For decomposing the initial non-convex polygon with holes to a set of convex polygon, a fast randomized triangulation algorithm used.
- Then, to find the set of guards sufficient for the inspection of a convex polygon, given visibility range d, a potential guard or *observation point OP* inside the polygon is computed
  - optimum OP minimizes the max distance from the polygon's vertices
  - here, they use the point WS instead, where

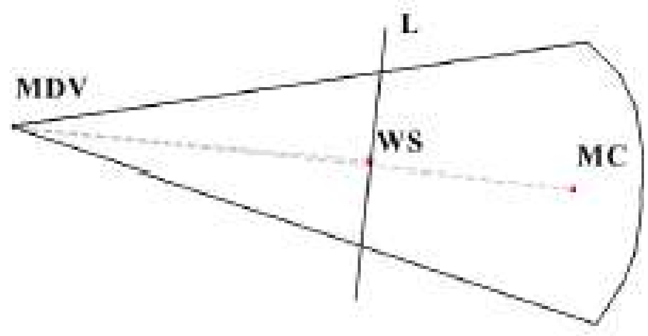
$$WS(x,y) = \sum_{i=1}^{A} \|E_i\| M_i(x,y) / \sum_{i=1}^{A} \|E_i\|$$

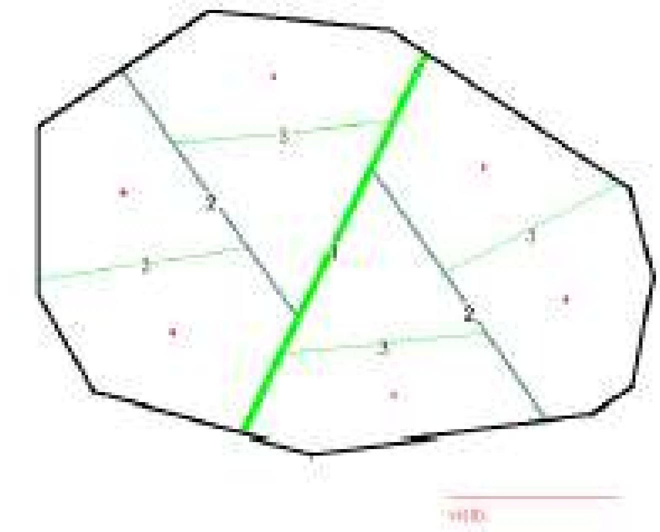
M_i = coordinates of midpoint of the i-th edge E_i of the polygon, $||E_i||$ = length of edge E_i.

  - This is to bias the OP towards the long polygon edges.

Polygon division

- Once a potential guard OP is computed, the polygon vertex with max distance r from the OP is determined (MDV).
- Iff r<d, OP is considered a guard capable of inspecting the polygon.
- Else a line L perpendicular to the line segment that connects OP and the MDV and passes through OP is computed.
- Line L divides this polygon into 2 convex subpolygons.
- Same procedure is recursively applied to each derived convex subpolygon.



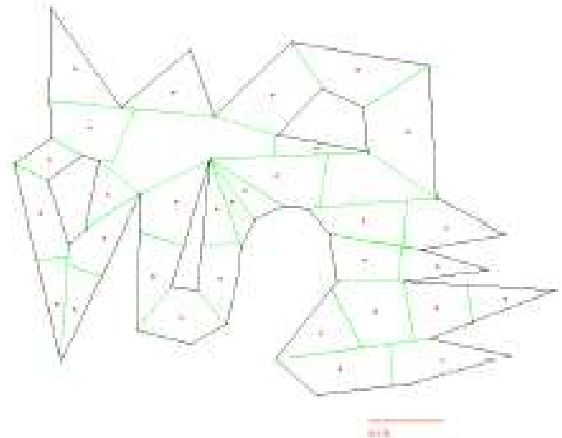


- It can be easily determined if subpolygons produced are attached to the border or not.
- At each level of recursion, only these exterior polygons are considered, speeding up computation substantially.
- For full area inspection, all polygons have to inspected.

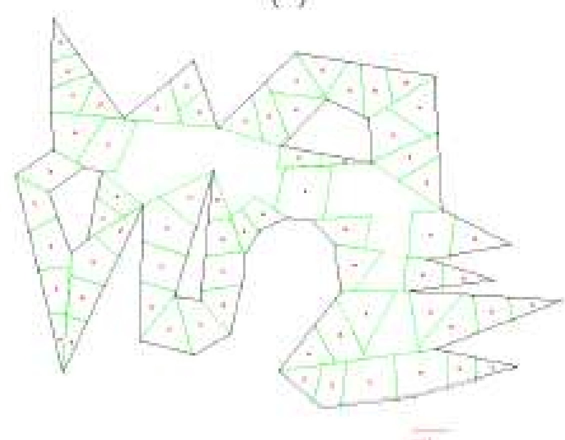
Quality of the solution

- The computational complexity is indicated by the total no. of vertices the algorithm needs to process until it converges to a final solution.
- So the larger the dimensions of the polygon w.r.t. the visibility range, the larger the no. of polygons and polygon vertices required to be processed.
- Advantage: ability to completely inspect the workspace, even narrow corridors, etc.
- Disadvantage: may assign unnecessary guards.
- So the solution is suboptimal but efficient for large workspaces.

Example of inspection of a givenworkspace with two different visibility ranges



(a)



(b)

References

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