Abstract

We study the problem of optimizing the conflict-free coverage of a set of transmitters for which we are allowed to vary the power (radius of transmission). We give the first algorithmic results on the problem, proving that it is NP-hard to solve exactly and giving approximation results, including simple constant-factor approximation algorithms and a PTAS.

1 The Problem

Let $S = \{p_1, \ldots, p_n\}$ be a set of n points in the plane.

Let $B(p, r)$ denote the ball of radius r centered on p. Given a collection of sets $\mathcal{A} = \{A_1, \ldots, A_k\}$, we define the *1-coverage* of $\mathcal{A}$ to be the set $X_1(\mathcal{A}) = \{p \in \Re^2 : p$ lies in exactly one of the sets of $\mathcal{A}\}$.

We define the Max-1-Coverage problem as follows. The input is the set $S = \{p_1, \ldots, p_n\}$. We are to determine radii $r_i \leq 1$ for each point p_i in order to maximize the area of the set $X_1(\{B(p_1, r_1), \ldots, B(p_n, r_n)\})$.

In the Max-Packing problem, we require that the nontrivial (positive radius) balls $B(p_i, r_i)$ be pairwise disjoint, i.e., that if r_i and r_j $r_i + r_j \leq d(p_i, p_j)$, where $d(p, q)$ denotes the distance from p to q.

Summary of Results So Far

- We study the 1D problem, where we show that the Max-Packing problem and the Max-1-Coverage problem are solvable (exactly) in polynomial time. In the Max-Packing version, one can always achieve coverage of at least $\frac{1}{2}$ of the measure of the union of unit balls (intervals); further, this bound is tight. In the Max-1-Coverage version, one can always achieve coverage of at least $\frac{2}{3}$ of the measure of the union, and this fraction is also tight.

- We study the 2D versions of the problem. Both Max-Packing and Max-1-Coverage constant-factor approximation algorithms; the trick is to get the best possible constants (and these keep changing). For the packing version, we have examined both the general case (allowing "shrinking", i.e., allowing radii less than 1) and the case in which no shrinking is permitted (i.e., finding a maximum independent set in a unit disk graph).

  We have an efficient PTAS too: There is an $O(n)$ algorithm to compute a $(1 - \epsilon)$-approximate solution to Max-1-Coverage for equal radii balls, for any fixed $\epsilon > 0$. This result extends readily to the case in which we are to select radii r_i that obey $r_{min} \leq r_i \leq 1$, for some fixed $r_{min} < 1$. It also extends to higher dimensions, though the constant factors grow exponentially in dimension.