Barbados 2007

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Problems and progress.

1. [Hazel Everett] *Layered manufacturing.* Which polyhedra can be manufactured by a laser that cannot be turned off?

   Person in charge: Sylvain Lazard. Participants : Marc, Csaba, Hazel, John, Kim, Jeff

   Progress: If the laser can be turned on and off, the best approximation of the object that can be carved without cutting through the original polyedron is the visual hull viewed either from infinity or from outside the convex hull of the polyedron (see eg S. Petitjean 98). It can be computed in $O(n^9 \log n)$.

   Here the laser cannot be turned off. The best approximation in the above sense of the input polyedron is then the complement of the connected component of the set of free lines (ie lines that do not intersect the interior of the polyhedron) that contains the lines at infinity. We call this connected component the outer cell of free lines.

   Prop 1. The complexity of outer cell of free lines in line space is $O(n^{3+\epsilon})$ and $\Omega(n^3)$.

   Proof: Lower bound: consider the usual $\Omega(n^4)$ lower bound example for the visibility complex that consist of two pairs of two very close combs, and remove one of the comb. It can be easily observed that all the lines transversal to three edges can be moved to infinity.

   Upper bound. The outer cell of free line is one connnected component of an arrangement of patches of hypersurfaces in four-dimesional line space; each patch corresponds to the set of lines that intersect an edge of the polyhedron. A paper by Basu (The comb. and topol. complexity of a single cell) gives directly that one connected component of this arrangement has complexity $O(n^{d-1+\epsilon}) = O(n^{3+\epsilon})$

   Def: The carved approximation of a polyhedron is the connected component of the complement of the union in 3D of all the lines in the outer cell of free lines.

   Lemma. The boundary of the carved approximation consists of pathes of ruled surfaces that are the set of transversals to three edges of the polyhedron (possibly one vertex and one edge).
   Prop 2. The carved approximation has worst-case complexity $O(n^{6+\epsilon})$ and $\Omega(n^6)$.

   Proof. The above lemma and the above paper by Basu gives that the carved approximation is one cell of an arrangement of N surface pathes in 3D which has complexity $O(N^{2+\epsilon})$. Here N is the nummber of surface patches which is $O(n^{3+\epsilon})$ by the previous proposition.

   The lower bound is obtained by gluing 3 shoebox together that have O(n) slit on their sides (se Figure). Three sets of slits create $\Omega(n^3)$ pieces of quadrics. Same for the other three sets of slits. These patches intersect pairwise in $\Omega(n^6)$ arcs. No piece gets disconnected as they remain "attached" at the bottom of the shoebox.

Prop 3. The complement of the union in 3D of the set of lines in the outer cell of free lines has complexity $\Theta(n^9)$ in the worse case.

Proof. Upper bound. There are $O(n^3)$ surface patches generated by three edges, hence the complexity of the arrangement is $O(n^9)$.

Lower bound. A set of three comb generates $\Omega(n^3)$ surface patches. Create three set of three combs that creates three sets of $\Omega(n^3)$ surface patches that pairwise intersect (inducing an arrangement of size $\Omega(n^9)$. Now, put everything in a box so that the region of complexity $\Omega(n^9)$ is not carved out by free lines with other directions. Finally add some thin slits on the box so that all the lines tritangent to a triple of comb can be moved freely to infinity.

Prop 4. The 1-skeleton of the outer cell may consist of $\Omega(n^2)$ components.

Proof. Example of Prop 1

2. [Suresh Venkatasubramanian] *Drawing a tree in a strip.* Given a tree, draw it (on a regular grid) such that vertices are drawn as rectangles and edges are encoded as non-trivial boundary contacts. What is the complexity of deciding if a tree can be drawn in a strip of height h? This is not known to be NP-hard (it is NP-hard for planar graphs); $h = O(\log n)$ is always known to be sufficient.

   Person in charge : John Iacono

3. [Lata Narayanan] *Routing in 3D unit sphere graph using face-routing.* Let P denote a set of points in $\mathbb{R}^3$ such that P is contained within some slab of thickness λ. Let $USG(P)$ denote the unit sphere graph of P. That is, points p_1 and p_2 in P are adjacent in $USG(P)$ iff $||p_1 - p_2|| \leq 1$. For a given source-target pair (s, t) in P, we describe a distributed and memoryless algorithm that guarantees delivery of a message from s to t. The algorithm is based on the two-dimensional face routing algorithm of Bose, Morin, Stojmenovic, and Urrutia (Wireless Networks, 2001) in unit disc graphs.

   Person in charge : Stephane Durocher

   Observations and progress. Without loss of generality, assume the slab is parallel with the xy-plane such that the z-coordinate of each point in P is in the range $[0, \lambda]$. Let $e_1 = (u_1, v_1)$ and $e_2 = (u_2, v_2)$ be two edges in $USG(P)$ such that e_1 and e_2 cross in the projection of $USG(P)$ onto the xy-plane. The minimum distance between endpoints of e_1 and e_2 is maximized when both edges have unit length, the relative position of the edges is perpendicular in the xy-plane, each endpoint is a distance $1/2$ from the edge crossing in the projection, and the edges are parallel to opposite planes on the boundary of the slab. Without loss of generality, assume $u_1 = (0, 0, 0)$, $v_1 = (1, 0, 0)$, $u_2 = (1/2, 1/2, \lambda)$, and $v_2 = (1/2, -1/2, \lambda)$. The minimum distance between endpoints of e_1 and e_2 is $\sqrt{\lambda^2 + 1/2}$. Therefore, for any e_1 and e_2, some endpoint of e_1 must be adjacent to some endpoint of e_2 in $USG(P)$ if $\lambda \leq 1/\sqrt{2}$. If $\lambda > 1/\sqrt{2}$, then it is possible for e_1 and e_2 to cross in the projection of $USG(P)$ even if no edge exists between any endpoints of e_1 and any endpoints of e_2 in $USG(P)$. Therefore, assume $\lambda \leq 1/\sqrt{2}$. This ensures that edge crossings in the projection of $USG(P)$ can be detected locally. In particular, given an edge e_1 we can enumerate all edges that cross e_1 in the projection by considering the set of edges adjacent to neighbours of either endpoint of e_1.

We now describe the routing algorithm. Consider the projection of $USG(P)$ onto the xy-plane. The initial direction in which to send the message is given by the face routing algorithm. Each step in the route is determined as follows. Say the message is to be passed from node u_1 to node v_1. Let u' denote an intermediate point along the line segment u_1v_1 that was determined in the last step of computation (u' may be equal to u_1). Assume the face routing algorithm is presently following a "left hand" rule (the face is being traversed in a clockwise diretion). By examining the set of edges adjacent to neighbours of u_1 and v_1, determine the edge that crosses line segment $u'v_1$ for which the edge crossing is nearest to u'. Denote this edge by (u_2, v_2) such that v_2 lies inside the current face. The message is passed to v_1 and then to u_2. The algorithm repeats to pass the message from node u_2 to v_2 with the new value of u' set to the intersection of (u_1, v_1) and (u_2, v_2). If no edge crosses line segment $u'v_1$, then the first anticlockwise edge from v_1 is followed and the algorithm repeats with $u' = v_1$. When considering the segment $u'v_1$, the algorithm observes the same rules as the 2D face routing algorithm regarding crossing the line segment st. Unlike the 2D algorithm which operates on the subset of edges given by the Gabriel graph of P (which can be calculated locally), our algorithm considers all edges of $USG(P)$ and resolves edge crossings by creating a virtual vertex (u') at every edge crossing. In 3D, the Gabriel graph is not sufficient to eliminate all edge crossings in the projection.

Open questions. Can we bound the stretch factor? That is what is the worst-case ratio between the length of route taken by the message between s and t as compared to the length of the shortest path between s and t, where both path lengths are measured by total number of edges.

Can any memoryless distributed routing algorithm guarantee delivery when $\lambda > 1/\sqrt{2}$?

4. [Panos Giannopoulos] *Lawn-mowing.* Given a graph on the grid, can you find a spanning path with k turns? This problem became, more generally, representing geometry in logic;

   Person in charge : Nathan Yu and Christophe Paul

5. [Mike Fellows] *Parameterized complexity of guarding a postmodern art gallery.* Given a simple polygon P, find k points inside the polygon that guard P. Is that FPT? Is it even NP-hard (Csaba Toth)?

   Given a simple polygon P, find k points that cannot see each other. Is that FPT?

   Person in charge : Sue Whitesides and Nathan Yu

6. [Mike Fellows] *The Geometry toolkit : variations on set cover*

   k-IndependetSet and k-DominatingSet for certain geometric graphs (like t-interval graphs). Problems are $W[1]/W[2]$-hard for general graphs.

   Person in charge : Christian Knauer and Mike Fellows

7. [Helmut Alt] *Approximation of polylines by curves of bounded curvature*

   Person in charge : Helmut Alt Participants : David, Rolf, Marc

8. [David Kirkpatrick] *Finding a quick exit from a building*

   Person in charge : David Kirkpatrick and Rolf Klein

9. [Stephane Durocher] *Facility location in a unit disc graph.*

   Given a set of points P in the plane, choose one additional point f such that $\max_{p \in P} d(p, f)$ is minimized, where $d(u, v)$ denotes the unweighted graph distance between vertices u and v in the unit disc graph induced by $P \cup \{f\}$.

   Person in charge : Lata Narayanan

   Observations and Progress. If no new points can be added, f corresponds to a vertex 1-centre of the unit disc graph of P. The addition of a single point f can improve the radius of the unit disc graph of P by a factor of at most five. This bound is tight.

   The set of possible choices for f can be reduced to a discrete set by considering the partition of the plane induced by distinct regions of intersections of discs in the unit disc graph of P. There are $\Theta(n^2)$ such regions in the worst case. However, only regions of maximum local depth need to be considered. A region of maximum local depth is a region that is contained within k unit discs such that every adjacent region is contained in at most $k-1$ discs. A region has maximum local depth iff it is convex. Unfortunately, there are $\Theta(n^2)$ regions of maximum local depth in the worst case. For example, n points can be arranged in a cross to produce a grid of $n/2 \times n/2$, i.e., $\Omega(n^2)$, convex regions.

   Although the algorithmic details have not been worked out, it should not be difficult to construct the set of regions. An optimal position for f can be found in polynomial time by considering each of the $O(n^2)$ regions. For each region r, we calculate the graph distance from f to the furthest vertex of P in the unit disc graph that results from adding f to r. For every $p \in P$, $d(f, p) = 1 + \min_{q \in N_r} d_*(q, p)$, where N_r is the set of points of P at the centres of unit discs that contain r, d_* denotes the graph distance function in the unit disc graph incuded by P, and d denotes the graph distance function in the unit disc graph induced by $P \cup \{f\}$. This algorithm runs in time $O(\delta n)$ per region, resulting in an overall time of $O(\delta n^3)$, where δ is the maximum depth of any region.

10. [Jie Gao] *Minimum length separating cycle of matched pairs*

    Given a matching of n pairs of points in the plane, find a minimum length separating cycle that separate each pair of points in the matching.

    Person in charge: Jie Gao

    (a) The problem is NP-hard by reduction from TSP. Take each pair of points and place them arbitrarily close to a city, then the minimum length separating cycle is exactly the minimum length traveling salesman tour that visits each city.
    (b) Conjecture: $O(1)$-approximation in small polynomial time should be do-able.

    A similar problem (but simpler) is the red-blue separation problem. Given k red points and n blue points in the plane, find a minimum length cycle that includes all red points and excludes all blue points.

    (a) The problem is NP-hard, again, by reduction from TSP.
    (b) It omits a PTAS by Sanjeev Arora and Kevin L. Chang, published as *Approximation Schemes for Degree-restricted MST and Red-Blue Separation Problem, Algorithmica,*

40(3):189-210, 2004. A preliminary version appeared in ICALP 2003. (thanks to Csaba Toth and Joe Mitchell).

11. [Rolf Klein] *Race track problem*

    Person in charge : Rolf Klein

Other problems that were mentionned by that we didn't report progress on.

1. [Jochim Giesen] *Mesh smoothing.* Given a triangulated surface embedded in R^3, move the vertices to minimize the absolute Gaussian curvature. Such a surface is *tight*; whenever you slice it with a hyperplane, it breaks into at most 2 pieces. $\kappa_V = 2\pi - \sum \alpha_i$ $\kappa_V^+ = 0$ if V is in the CH of its neighbourhood and $2\pi - \sum \beta_i$ (β_i are the angles of the conver hull of the neighbourhood) otherwise. $\kappa_V^- = -\kappa_V + \kappa_V^+$. $|\kappa_V| = \kappa_V^+ + \kappa_V^-$.

2. [Marc Glisse] *Free lines tangent to polytopes.* Given k polytopes in $\mathbb{R}^3$ of total complexity n. What is the maximum number of free lines tangent to 4 polytopes? For maximal free line segments the number is $\Theta(n^2k^2)$. The upper bound also holds for lines; there the lower bound is $\Omega(n^2 + nk^3)$.

3. [David Kirkpatrick] *Path planning on a mountain road.* Bounded curvature, bounded vsibility motion planning. Given a curve (marked middle of a street), a street width (street boundary occludes view), decide if it is possible to drive from s to t on a path with bounded curvature. Variations: bound the view.

4. [Ethan Kim] *External limited visibility watchman tour.* Given a simple polygon P and a point robot R with visibility 1. Plan a closed route of R that avoids P and sees each point on the boudary of P. There is a PTAS of J. Mitchell. For convex polygons with unlimited visibility it can be done in polynomial time for $k = 1, 2, 3$ polygons. The case $k > 3$ is open.

5. [Lata Narayanan] *Minimum connected dominating set in a unit disc graph.* Without the connectedness it is $W[2]$-hard by a result of D. Marx.

6. [Christophe Paul] *Navigable graphs.*

7. [Christophe Paul] *Dynamic operations on graph representations.* Given a graph G and a geometric representation R, compute the representation R' of the graph G' you get from G via some modification (add/remove an edge/vertex).

8. [Jie Gao] *Geographic routing in sensor networks.* Greedy forwarding: Route to a neighbor that gets you closer to the target.

   (a) Given G, find an embedding that supports greedy forwarding. Not every graph has such an embedding (e.g., a 7-star). Papadimitriou conjectures that every 3-connected graph has.

   (b) Given an abstract unit disc graph, find an embedding, such that edges have length ≤ 1 and non-edges have length > 0.1 (or any other constant; this is known to be hard for large enough constants, maybe $1/\sqrt{2}$).