Barbados 2012

Open problems session

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The team

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1 Lombardi drawings of planar graph

proposed by Stephen

Lombardi drawing = Every edge is circular arc, tangent to edges around each vertex are equally spaced (in angle).

There exists planar graph with no Lombardi drawing (three nested triangles).

Question: Do outer-planar graph have plane Lombardi drawing ?

(outer planar= dual is a tree)

2 Two graphs planar drawing

proposed by Will

Given two planar graphs G_1 and G_2.

$|G_1| = n$, $|G_2| = k$, $k \leq n$.

For which k (depending on n) is it possible to find a point set of size n in the plane to embed both graphs (straight line drawing).

Known : $k = 4$ works.

3 Linear time RIC ?

proposed by Olivier

Randomized incremental constructions can be managed either with the conflict graph [Clarkson-Shor] or the history graph[BDSTY]. Mixing these two graphs and using some extra information one can be able to get $O(n \log^\star n)$ algorithms, e.g. for triangulating a simple polygon [Raimund] or the Delaunay triangulation of points in the plane knowing a bounded degree spanning subgraph [Olivier].

BRIO (Biased random insertion ordering) [Nina] allow to use a non completely random order and still get a randomized good complexity.

Question: If we apply BRIO to an ordering given by a a simple polygon or a spanning subgraph of Delaunay, can we kind of mix everything and get linear time algorithms ?

Motivation: may be to build an algorithm for deleting a point in 3d Delaunay.

Raimund: this is $\Omega(n \log^\star n)$ for trapezoidal map of a simple polygon.

4 Finding convex configuration

proposed by Christian

Erdös Szekeres Theorem:*If you have more than $n(k)$ points in the plane, there always exists a convex k-gon (not necessarily empty).*

Given n points $P \subset \mathbb{R}^2$ and $k \leq n$ decide if P contains an (empty) convex k-gon?

In 3D the problem is NP-hard. Approximation algorithm to find the maximal k ?

5

proposed by Beppe

Family of binary trees with n_r red vertices and n_b blue vertices.

Is there a bicolored universal point set for straight line drawing of such binary trees?

Known if $n_r = 1$.

Known if the binary trees contains only degree 1 or 3 nodes.

6 Shape modelization

proposed by Brian

Field fonction (potential). Add or subtract such functions (CSG). Look at iso-surfaces.

Given some convex polygon, draw an aesthetic curve inside it with such tools?

7

proposed by Xavier

Let $[n] = \{1, \ldots, n\}$ and let $Y = \{y_1, \ldots, y_m\} \subseteq [n]$, with $y_1 < y_2 < \ldots < y_m$. The *restriction* of a permutation σ on $[n]$ to Y is the permutation $\sigma_{|Y}$ on $[m]$ such that $\sigma_{|Y}(i) < \sigma_{|Y}(j)$ if and only if $\sigma(y_i) < \sigma(y_j)$. If a permutation σ does not contain a permutation τ as a restriction then we say that σ *avoids* τ.

Marcus and Tardos proved that for any permutation τ on $[m]$ there is a constant c such that the number of permutations on $[n]$ that avoid τ is $O(c^n)$ (here m is fixed and $n \to \infty$). It was a well-known conjecture of Stanley and Wilf.

Let us fix a permutation τ on $[m]$. What is the complexity of sorting n numbers assuming that they are given in an initial order that avoids τ? The classical argument that gives a lower bound of $\Omega(n \log n)$ no longer holds.

Some very partial results are given in the paper "Fast Sorting and Pattern-Avoiding Permutations" by David Arthur (ANALCO 2007).

8 Path planning in disks

proposed by Hyo-Sil

Given an arrangement of disks in $\mathbb{R}^2$ and two points s and t.

Find a path from s to t than minimize the number of disks crossed ($\sharp$ disks you have to removed so that the path is free).

This problem is NP-hard for line segments [Helmut].

Open for disks and for unit disks.

9 Triangle touching graph

proposed by Stephen

Represent a graph:

A node is represented by a polygon An edge is represented by adjacency.

Restrict the kind of polygons used : rectangles, triangles.

You can represent with rectangles graph that does not have a filled triangle.

Unknown for triangles. (Complexity of decision algorithm, characterization of classes of graph).

10 Distance between piecewise linear functions

proposed by Guillaume

Given two triangulations of a convex polygon in the plane.

Let f be a piecewise linear function, linear on each blue triangle. and g linear on red triangles.

Compute the distance between f and g that is $\sqrt{\int (f-g)^2}$ can be solved in $O(n \log^4 n)$ time.

Question: do it for L_1 norm $\sqrt{\int |f-g|}$ in subquadratic time.

Simplified case: grid.

11 Separate points with k lines

proposed by Christian

Red and green point sets R and G.

You can decide if there is a line separating R and G in linear time.

If you cannot separate by a line, try to separate by more lines (faces of the arrangement are monochromatic).

Problem: Given G and R can you separate with k lines ?

Remarks:

Interesting lines are spanned by the points.

$k = 1$ linear programming.

$k = 2$ look at all pair of lines $O(n^4)$.

Is there an algorithm that runs in $O(f(k)polylog(n)$?

Considering all bicolored segments, you get an instance of

Transformed problem: Given m lines, is it possible to intersect all of them with k lines.

This transformed problem is known to be $W[1]$ hard.