Theoretical threshold for quantum expander codes

Antoine Grospellier* Anthony Leverrier* Omar Fawzi†

Quantum error correction is a technique used in order to prevent the decoherence of quantum states. A quantum error correcting code encodes some logical qubits into a larger number of physical qubits. They allow us to recover an initial state when some small errors have been applied on it. We study quantum expander codes ([3]) and in particular we proved that there exists a threshold for them. These codes have several nice properties: they are LDPC, they have a strictly positive rate, their minimal distance is good and there is an efficient decoding algorithm to decode them. We proved that these codes can almost surely correct random errors (in the so called locally stochastic noise error model) assuming that the probability of error is below some strictly positive constant called "threshold". We use the same kind of tools than [1] and [2] and in particular we use percolation theory. A standard result of percolation theory states that if we pick randomly and independently vertices in a degree bounded graph then the induced sub-graph has small connected components. We proved a generalized version of this theorem and applied it to prove that there is a threshold for quantum expander codes (picked vertices represent errors on qubits). We have been able to prove the existence of a threshold when there is no error in the syndrome measurement and we are now investigating the case where the syndrome is noisy.
An application of our work is efficient fault-tolerant quantum computation with constant overhead, using a previous work of Gottesman ([1]). Fault-tolerant quantum computation is an essential tool in the designing of a quantum computer since it allows us to fight against the noise in quantum circuits.

References

[1] Daniel Gottesman. Fault-tolerant quantum computation with constant overhead. *arXiv preprint arXiv:1310.2984*, 2013.

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*Inria, SECRET Project, 2 Rue Simone Iff, 75012 Paris Cedex, France. antoine.grospellier@inria.fr, anthony.leverrier@inria.fr

†ENS de Lyon, UMR 5668 LIP - CNRS - UCBL - INRIA - Université de Lyon, 69364 Lyon, France. omar.fawzi@ens-lyon.fr