Golden codes, regular quantum codes built from regular tessellations of hyperbolic 4-manifolds

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Quantum information is fragile. Quantum error correcting codes offer a promising solution to protecting quantum information. LDPC (low density parity check) stabilizer codes are of particular interest because they lead to efficient decoding algorithms. Three parameters summarise the properties of such a code. The number of physical qubits of the code is n. The number of logical qubits of the code is k: it gives the quantity of quantum information that the code can carry. The minimal distance of the code is d. It is proportional to the number of physical qubits that can be corrupted without loss of information.
Geometry offers an efficient way to construct LDPC stabilizer codes. From a tessellation and a dimension i, a code is constructed by identifying qubits with i-faces, and stabilizers with $(i-1)$-faces and $(i+1)$-faces of the tessellation. It is then possible to understand n, k and d geometrically. The number of physical qubits n is proportional to the volume of the tessellated manifold. The number of logical qubits k equals the i^{th} Betti number of the manifold. The minimal distance d is proportional to the minimum of the i^{th}-systole and the $(m-i)^{th}$-systole of the manifold, m being the dimension of the manifold. Such codes coming from geometry are called homological codes.
We present an improvement over a construction of Guth and Lubotzky [1] of a family of homological codes whose parameters satisfy the following asymptotic relations: k is linear in n and d grows at least like $n^{0.2}$. Such parameters are beyond the reach of tessellations of two-dimensional manifolds such as the toric code and other surface codes. Our construction exhibits the same excellent asymptotic relations as [1] and benefits from a regular local structure. More precisely a building block of our construction is the regular tessellation of hyperbolic 4-space by 4-cubes, i.e. the tessellation with Schläfli symbols {4,3,3,5}.
The regularity of the local structure is exploited to design explicit and efficient decoding algorithms.

References

[1] Larry Guth and Alexander Lubotzky. Quantum error correcting codes and 4-dimensional arithmetic hyperbolic manifolds. *Journal of Mathematical Physics*, 55(8):082202, 2014.

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