A Complete Axiomatisation of the ZX-Calculus for Clifford+T Quantum Mechanics

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When it comes to reasoning on quantum evolutions, it is convenient to turn to quantum circuits, for they provide a graphical and compact visualisation of said evolutions. However two different circuits might represent the same evolution – or matrix: for instance two consecutive Hadamard gates or CNOT gates result in the identity, respectively on one and two qubits. One might then want to equip circuits with a set of transformation rules, that change the circuit, but not the corresponding matrix.

ZX-Calculus is another diagrammatic language well-fitted for quantum computation. Even though it is closely related to circuits, it is laxer, which simplifies the research for transformation rules. Indeed, here, some non-unitary operations are allowed, and the wires can be bent at will, which greatly changes our conception of the mathematical objects we handle, and that we call here diagrams.

ZX-Calculus can be used for reasoning on quantum circuits: these can easily be transformed into ZX-diagrams, and there exists a method that characterises "circuit-like" diagrams. ZX-Calculus can hence be used to simplify a circuit, or check if two circuits are equivalent. The applications of the language are not limited to circuitry. For instance the generators of the ZX-Calculus match exactly the operations of lattice surgery, a promising model designed for error correction.

If the language were *complete*, then we would not need matrices at all to prove that two diagrams are equivalent. The language itself would be enough. Some restrictions of the language have been proven to be complete, however, up to our contribution, none of them were approximately universal. Hence we propose the first axiomatisation which is provably complete for an approximately universal restriction of the ZX-Caclulus, namely Clifford+T.

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