Two sentences are called contradictory in a given theory if whenever one of them denotes 1 in the theory, the other denotes 0 (e.g. Mary is not tall and Mary is tall). We may also talk about two contradictory readings/structures of sentences, which is especially useful when sentences are structurally ambiguous.

Consider the following abstract syntax together with its associated Montague-like semantics:

\[
\begin{align*}
\text{ALICE} & : \text{np} & [\text{ALICE}] &= \lambda P. P \text{ alice} \\
\text{EVERYONE} & : \text{np} & [\text{EVERYONE}] &= \lambda P. \forall x. P x \\
\text{RUN}, \text{RAN} & : \text{np} \rightarrow \text{s} & [\text{RUN}] &= [\text{RAN}] = \lambda S. S (\lambda x. \text{run} x) \\
\text{WAS STANDING} & : \text{np} \rightarrow \text{s} & [\text{WAS STANDING}] &= \lambda S. S (\lambda x. \text{stand} x) \\
\text{WAS LYING} & : \text{np} \rightarrow \text{s} & [\text{WAS LYING}] &= \lambda S. S (\lambda x. \text{lie} x) \\
\text{DIDN’T} & : (\text{np} \rightarrow \text{s}) \rightarrow \text{np} \rightarrow \text{s} & [\text{DIDN’T}] &= \lambda r. \lambda x. \neg (r x)
\end{align*}
\]

where:

\[
\begin{align*}
\text{alice} & : \text{e} & \forall : (\text{e} \rightarrow \text{t}) \rightarrow \text{t} \\
\text{stand}, \text{lie}, \text{run} & : \text{e} \rightarrow \text{t} & \neg : \text{t} \rightarrow \text{t}
\end{align*}
\]

1. Consider the sentences Alice was standing (Alice était debout) and Alice was lying (Alice était couchée).
   
   (a) Can you think of a theoretical assumption that would make these sentences contradictory?
   
   (b) How do you then express their denotations?
   
   (c) Are these denotations arbitrary? Why?

2. What is the semantic interpretation of the syntactic type \( \text{np} \)? Of the syntactic type \( \text{s} \)?

3. We now consider the sentence everyone didn’t run whose abstract syntax is given by \( t \) such that:

\[
t = ((\text{DIDN’T}) \text{ RUN}) \text{ EVERYONE}
\]

(a) Is \( [\text{DIDN’T}] \) well-typed?

(b) What is the interpretation \( u \) of \( t \)?

(c) What can you say about the respective scopes of the universal quantifier and of the negation in \( u \)?

(d) What would be a reading inverting these scopes? Which one is the strongest reading (that implies the other one)? With its weakest reading, are everyone didn’t run and everyone ran still contradictory?

(e) Propose another interpretation for \( \text{DIDN’T} \) that inverts the scope. Check that it is well-typed and compatible with the given type definitions and that the result is as you expect.