

Homological simplification is NP complete, even in 3D

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Abstract

Keywords: homology

1 Introduction

Reduction to 3SAT.

2 The construction

We construct two complexes $L \subset K$.

2.1 The clause gadget

The gadget is the following, in K put one baloon for each clause. On these baloon we draw three edges with 2 vertices and thus creates three faces on the baloon, we add three hooks that connect the faces pairwise; in L we put only the 2 vertices, the hooks and six fragments of edges incident to the vertices (Figure 1-left). Then, the baloon is deformed to have three tentacles getting out from the baloon, and we pinch these tentacles in their extremities (Figure 1-middle). Each of the three edge is deformed to go through a tentacle, the part of edge not in L correspond exactly to the pinched extremity of the tentacle (Figure 1-right).

2.2 The variable gadget

The gadget is again a baloon in K , on this baloon we draw a cycle that is in L that separates it in two parts called the positive part and the negative part (Figure 2-left). Then in the positive hemisphere we draw a^- edges sharing a vertex, where a^- is the number of times that the variable appear negatively in a clause, symmetrically we draw on the negative hemisphere a^+ edges. Edges extremities are in L (Figure 2-right).

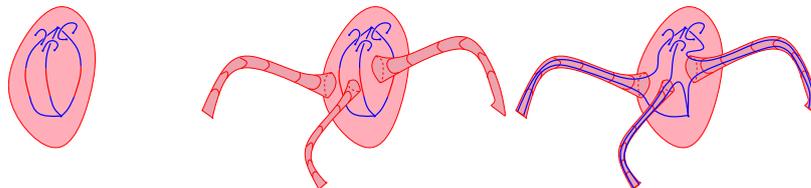


Figure 1: The clause gadget.

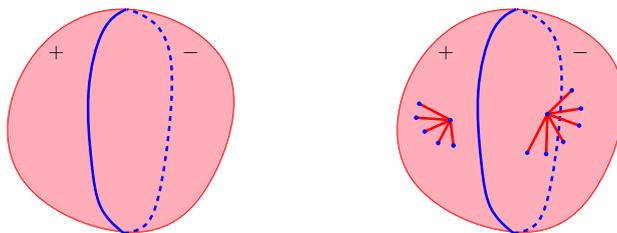


Figure 2: The variable gadget.

2.3 Connecting clauses and variables

Now for a clause $e_a a \vee e_b b \vee e_c c$, we connect the three tentacles to the variables a, b, c , more precisely, the pinched edge of the tentacle corresponding to $e_x x$ is identified with a small edge on the variable x gadget on the side $-e_x$. The edge of L that comes from the above part of the clause gadget is attached to the vertex of degree x^{e_x} .

3 From assignment to the homological simplification

Then we can construct X from an assignment.

If variable a is true, the positive hemisphere of the a gadget goes in X .

Then for each clause $e_a a \vee e_b b \vee e_c c$, if only one literal $e_x x$ is satisfied, then in X we just link the hooks to L . If two literals $e_x x$ and $e_y y$ are satisfied then we put in X the face of the clause gadget that share a part on the variables gadgets of x and y and we link the hooks between the two other faces. If all three literal are satisfied, then we use any two faces of the clause gadget.

4 From an homological simplification to an assignment

For each variable x , if the positive hemisphere of the gadget is entirely present assign *false* to x , otherwise assign *true*.

Then in the clause