Barbados 2011

Open problems session

Saturday February 5th

1 $\sharp$ extreme points

proposed by Xavier

Let D be a domain in $\mathbb{R}^2$

Let $f(n)$ expected number of extreme points amongst n points evenly distributed in D

Conjecture : $f(n)$ is non decreasing / increasing.

Segment : $f(1) = 1, f(n) = 2$

Convex curve : $f(n) = n$.

Consider 4 points in a square, you can label regions for the 5th point where $f(n)$ is increasing by 1 / stable / decreasing by 1.

2 Hypergraph

proposed by Xavier

H hypergraph on n vertices V $h \in H$ is a subset of V.

$f(k)$ = maximum on all subset X of k vertices of the number of subset of X induces by H.

$f(k) = max_{X \subset V, |X|=k} \sharp\{h \cap X;\ h \in H\}$

$\forall H f(k) \leq 2^k$

If $f(4) = 8 \Rightarrow |H| = O(n\sqrt{n})$.

find other results of the type.

e.g. $f(5) \leq 11 \Rightarrow |H| is O(n^2)\Omega(n\sqrt{n})$

3 Snap rounding

proposed by Sylvain

Given a set of polygons in the plane. Coordinates such that you can compute any given digt iin constant time.

You want to approximate coordinates, preserving the topology, such that the first p digits are correct and the k next digits are free (to preserve the topology).

Decision problem: Does such rounding exists?

Question : is decision problem NP hard ?

Milenkovic have similar result, when the polygon is defined as a sequence of lines, given by their equations.

Raimund : it is solved.

4 Delaunay of a sample of a p-manifold

proposed by Nina

Preleminary remark from a Quadtree you can construct (in $O(n)$ time) the NN graph and then the Delaunay triangulation.

[Nina, Dominique, Olivier] if a dim p polyhedron in dim d is carrefully sampled, then the size of the Delaunay triangulation has complexity $n^{\frac{p-k+1}{d+1}}$ where $k = \lceil \frac{d+1}{k+1} \rceil$.

Question 1: Can we design an algorithm which actually compute the triangulation with that complexity. (using RIC, using octree ?)

Question 2: Relax sampling hyptheses.

Question 3: Construct just the Delaunay restricted to the manifold.

Discussion about the possibility of adding more points to simplify the stuff.

5 following

proposed by Marc

Same notations as previous problem.

If instead of polytopes but generic manifold do we have the same bounds?

If the surface is non generic, we know that the bound do not apply. We have at least $\Omega \frac{d-p}{2}$: take the product of the moment curve in dimension $d-p$ by a sphere in dimension p.

6 Homological simplification

proposed by Dominique

$L \subset K$ simplicial complexes

X is the homological simplification of (K, L) if

i) $L \subset X \subset K$

ii) $H(L) \to H(X) \to H(K)$ where the first is surjective and the second is injective (H homology).

[Dominique,André]: this problem is NP-complete (by reduction to 3SAT)

from a 3SAT problem, we construct a (K, L) that admits an homological simplification iff the 3SAT is simplifiable.

Problem: the pair (K, L) can embedded in dim 4 but not a dimension 3.

New question: Is the existence of an homological simplification on $\mathbb{R}^3$ still NP-complete ?

In 2D it is easy to find an algorithm. (decide for each hole if it has to be filled).

7 Minimal triangulation

proposed by Raimund

Pb1: $S \subset \mathbb{R}^2$ set of n points. Find a triangulation T of S such that $\sum_{u,v \in S} d_T(u,v)$ where $d_T(u,v)$ = euclidean distance of the shortest path along the edges of T.

Pb2: P simple polygon, T triangulation of the inside of P. Same question as above.

8 Minimum matching

proposed by David

Two sets of points on a line.

Complete bipartite graph. weight of edge is the squared distance.

Find a subset of subgraph such that there is no isolated vertex that minimize total weight.

Now translate one of the set such that the total weight is minimized.

9 Average stretch factor

proposed by Christian

Given a polygon P in the plane.

The stretch between two vertices is the ratio distance along boundary over euclidean distance.

$s(u,v) = \frac{d_P(u,v)}{|u-v|}$

maximum stretch factor can be computed $O(n \log^5 n)$.

What about the average ?

Thus the question is: compute $\frac{1}{\binom{n}{2}} \sum_{u,v \in P} s(u,v)$ in subquadratic time.

Rk: $\frac{1}{\binom{n}{2}} \sum_{u,v \in P} s(u,v)^2$ can be computed in $O(n^{\frac{8}{5}}$.

10 Simultaneaous drawings without mapping

proposed by Beppe

Given two planar graphs such that $|V_1| = |V_2| = n$

Pb: find a set S of n points in $\mathbb{R}^2$ such that G_i can be drawn with linear edges using the points in S. (mapping for V_1 and V_2 can be different).

It is always possible if one graph is outerplanar.

Example: series parallel graphs of max degree 3. Is it possible for a pair of two such graphs ?

11 Hausdorff matching

proposed by Stefan

Transform one polytope so that it is close to another.

Given two convex compact polytopes $K, L \subset \mathbb{R}^d$ Find the transformation t such that $d_{\text{Hausdorff}}(K, L)$ is minimized.

If polytopes are given by their vertices, problem is polynomial.

If one is given as intersection of half spaces, then it is NP-hard.

Given two polytopes P, Q in vertex presentation, can we find an efficient algorithm to solve the problem for the class of homothetic transformations?

This means solving the minimzation problem $\min\{d_{\text{Hausdorff}}(\alpha P + c, Q) : \alpha \geq 0, c \in R^d\}$

12 ϵ goodness

proposed by Vida

P is ϵ-boundary good P is ϵ-area good

Take a point, its visibility polygon in P must it must see an ϵ portion of boundary/area

Question: if an object is ϵ-boundary (resp. area) good, is the perimeter bounded by the diameter times a constant (depending on ϵ).