Cast as Intended in voting protocols

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Joint work with Alexandre Debant, Pierrick Gaudry, Stéphane Glondu, Anselme Goetschmann, Sophie Lemonnier

EVoteID, October 2023





What is a good voting system?

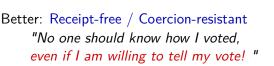
Confidentiality of the votes

Vote privacy "No one should know how I voted"



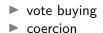
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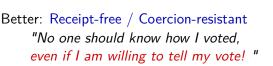






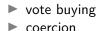
Confidentiality of the votes

Vote privacy "No one should know how I voted"











Everlasting privacy: no one should know my vote, even when the cryptographic keys will be eventually broken.

Verifiability

Individual Verifiability: a voter can check that

- cast as intended: their ballot contains their intended vote
- recorded as cast: their ballot is in the ballot box.

Universal Verifiability: everyone can check that

- ▶ tallied as recorded: the result corresponds to the ballot box.
- eligibility: ballots have been casted by legitimate voters.



You should verify the election, not the system.

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Even better: accountability

- the system tells whom to blame
- eases dispute resolution

And many more properties

- Availability: servers available at any time
- Accessibility: easy to use, adapted to people with various issues
 ...

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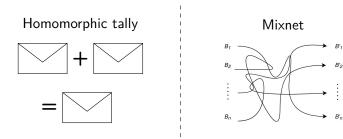
In this talk, focus on verifiability.

- cast ast intended
- recorded as cast
- tallied as recorded
- eligibility verifiability

Tallied as recorded

The result corresponds to the ballot box.

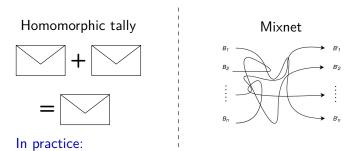
✓ Well studied academically, with two main techniques:



Tallied as recorded

The result corresponds to the ballot box.

✓ Well studied academically, with two main techniques:



- Many deployed solutions use such techniques: Estonia, France, Switzerland, ...
 - Many national evoting companies are still behind

Recorded as cast

 \checkmark easy in theory: the voter simply checks that their ballot appear on the bulletin board

Not so easy in practice

- require a public bulletin board
- voters do not check

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Alternative approaches

- delegation: voters send their ballot to a third party (eg French Legislative system, Polyas, ...)
- Swiss Post: the verification is embedded in the cast-as-intended mechanism, requires distributed servers
- Estonia: the ballot is sent (by the server) to another system component

Eligibility verifiability

✓academically: just sign but...

► require a PKI

public voter list? everlasting privacy?

Eligibility verifiability

✓academically: just sign but...

require a PKI

public voter list? everlasting privacy?

In practice

- Estonia: voters sign with their id cards
 - ✓ strong and verifiable eligibility
 - no public board
- $\pmb{\mathsf{X}}$ login/password sent by mail, SMS \rightarrow no eligibility verifiability
- distributed trust between authorities, eg Belenios (OTP + asymmetric key credential)

Cast as intended (Cal)

Few academic protocols

??? but yet a lot of systems in practice!

Cal in Australia

iVote system in the 2015 state election in New South Wales



What is my vote?

v (in clear!)



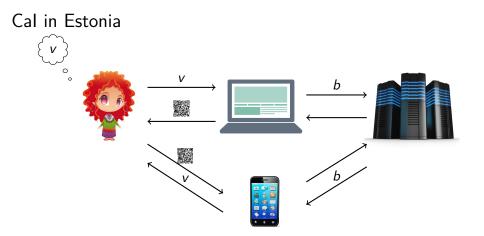
Cal in Australia

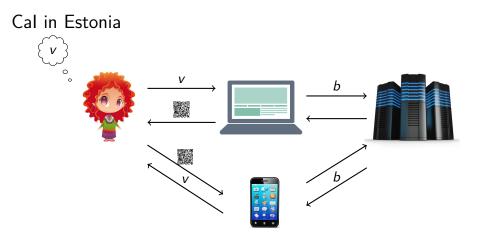
iVote system in the 2015 state election in New South Wales





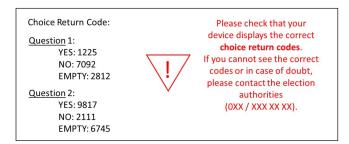
- ✓ simple
- ✓ cast-as-intended
- 🗴 no vote privacy 🔼
- X no cast-as-recorded





- ✓ cast-as-intended
 - some vote-buying threats (mitigated)
 - proxy cast-as-recorded
 - heavy infrastructure (two independent servers)

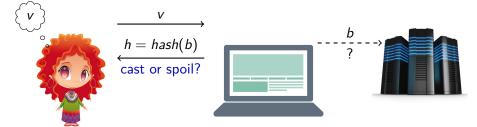
Cal in Switzerland



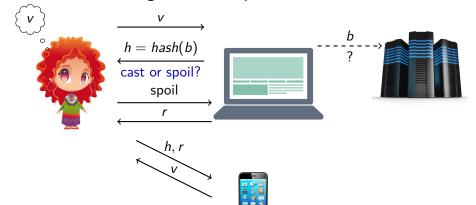
cast-as-intended

- proxy cast-as-recorded
- heavy infrastructure (four independent servers)

Benaloh's challenge: cast or spoil

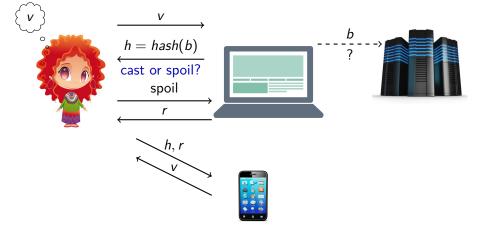


Benaloh's challenge: cast or spoil



13/54

Benaloh's challenge: cast or spoil



✓ simple principle

 \checkmark can be adapted to many systems

requires a second device

Helios Voting Booth

Choice of the EVoteID nicest location

exit

To vote, follow these steps:

- 1. Select your preferred options.
- 2. Review your choices, which are then encrypted.
- 3. Submit your encrypted ballot and authenticate to verify your eligibility.

Start

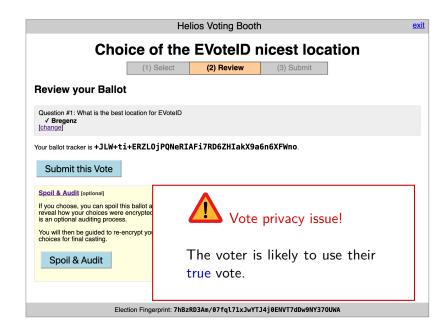
You can email for help.

Election Fingerprint: 7hBzRD3Am/07fql71xJwYTJ4j0ENVT7dDw9NY370UWA

| | Heli | os Voting Booth | ı | exit | |
|---|------------------|-----------------|------------|---------|--|
| Choice of the EVoteID nicest location | | | | | |
| | (1) Select | (2) Review | (3) Submit | | |
| What is the best loca #1 of 1 - vote for 1 | tion for EVoteID | | | | |
| Bregenz | | | | | |
| Luxembourg | | | | | |
| | | | | | |
| | | | | Proceed | |
| Election Fingerprint: 7hBzRD3Am/07fql71xJwYTJ4j0ENVT7dDw9NY370UWA | | | | | |

| | Helios Voting Boo | th | exit | |
|---|----------------------|------------|------|--|
| Choice of the EVoteID nicest location | | | | |
| (1 | I) Select (2) Review | (3) Submit | | |
| Review your Ballot | | | | |
| Question #1: What is the best location | for EVoteID | | | |
| Your ballot tracker is +JLW+ti+ERZL0jPQNeRIAFi7RD6ZHIakX9a6n6XFWno. | | | | |
| Submit this Vote | | | | |
| Spoil & Audit [optional] | | | | |
| | | | | |
| Election Fingerprint: 7hBzRD3Am/07fql7lxJwYTJ4j0ENVT7dDw9NY370UWA | | | | |

| Helios Voting Booth exit | | | | | |
|--|------------|------------|---|--|--|
| Choice of the EVoteID nicest location | | | | | |
| (1) Select | (2) Review | (3) Submit |] | | |
| Review your Ballot | | | | | |
| Question #1: What is the best location for EVoteID ✓ Bregenz [change] | | | | | |
| Your ballot tracker is +JLW+ti+ERZL0jPQNeRIAFi7RD6ZHIakX9a6n6XFWno. | | | | | |
| Submit this Vote | | | | | |
| Spoil & Audit [optional] | | | | | |
| If you choose, you can spoil this ballot and reveal how your choices were encrypted. This is an optional auditing process. | | | | | |
| You will then be guided to re-encrypt your choices for final casting. | | | | | |
| Spoil & Audit | | | | | |
| | | | | | |
| Election Fingerprint: 7hBzRD3Am/07fql71xJwYTJ4j0ENVT7dDw9NY370UWA | | | | | |



Benaloh: voter strategy

A voter should

- 1. decide at random if they will truly vote or audit
- $2. \quad \rightarrow \text{ if vote, then vote} \\$
 - $\rightarrow\,$ if audit, decide at random then audit and go to step 1
- 🗡 usability
- which probabilities to use?
- ▶ is it truly cast-as-intended? (see e.g. Jamroga's talk)

Other Cal solutions

Select, Selene, Hyperion: votes appear in clear on the ballot box

- ✓ simple for the voters
- specific systems
 - adversary caught to late
 - \rightarrow strong accountability needed

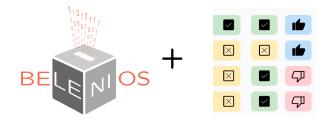
Other Cal solutions

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Two device solutions: Du-Vote, CAISED (this Friday!)

Our proposal: BeleniosCal



- based on Belenios
- could be adapted to other protocols
- ▶ no second device (except to read BB), no paper material
- on the fly detection
- one server

Voting protocol Belenios



- variant of Helios, designed by Ben Adida
- developed at Loria, teams Pesto and Caramba (P. Gaudry)
 Developer: Stéphane Glondu
- used in 2000+ elections, with a total of 100 000+ voters

http://www.belenios.org/

- confidentiality of the votes
- verifiability of the voting process
 - \rightarrow The ballot box is public at any time.
 - \rightarrow All the operations (tally, ...) can be checked by anyone.

How Belenios works (simplified)

Phase 1: vote



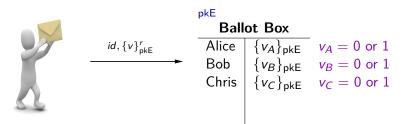
pkE

Ballot Box

| Alice | $\{v_A\}_{pkE}$ | $v_A = 0 \text{ or } 1$ |
|-------|-----------------|-------------------------|
| Bob | $\{v_B\}_{pkE}$ | $v_B = 0$ or 1 |
| Chris | $\{v_C\}_{pkE}$ | $v_C = 0 \text{ or } 1$ |
| | | |

pkE: public key, the private keys are shared among the authorities. $^{23/54}$

How Belenios works (simplified) Phase 1: vote



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pkE

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| Chris | $\{v_C\}_{pkE}$ | $v_{C} = 0 \text{ or } 1$ |
| David | $\{v_D\}_{pkE}$ | $v_D = 0$ or 1 |
| | - | |

pkE: public key, the private keys are shared among the authorities. 23/54

How Belenios works (simplified)

Phase 1: vote



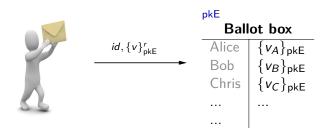
| pkE | | |
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| | | |

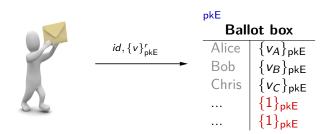
Phase 2: Tally - homomorphic encryption (El Gamal)

 $\{v_1\}_{\mathsf{pkE}} \times \cdots \times \{v_n\}_{\mathsf{pkE}} = \{v_1 + \cdots + v_n\}_{\mathsf{pkE}} \text{ since } g^a \times g^b = g^{a+b}$

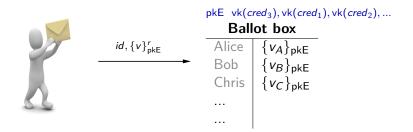
 \rightarrow Only the final result needs to be decrypted! And proved.

pkE: public key, the private keys are shared among the authorities. 23/54



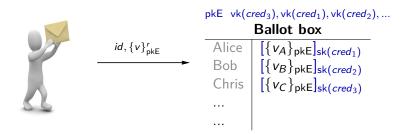


The ballot box could add ballots!



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1. During the setup phase, a Registrar generates private signing keys, one for each voter



The ballot box could add ballots!

- 1. During the setup phase, a Registrar generates private signing keys, one for each voter
- 2. The voters sign their ballot with a "credential" they have received (a credential = a right to vote)

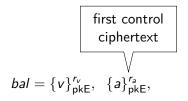
$$Alice' vote$$

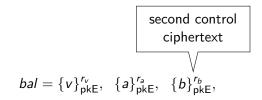
$$vote$$

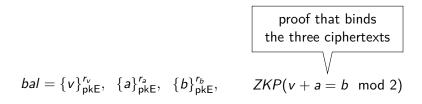
$$vote$$

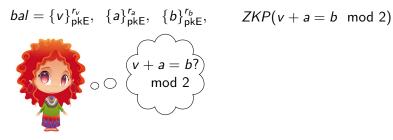
$$vote$$

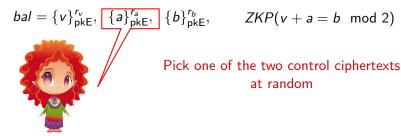
$$vote$$

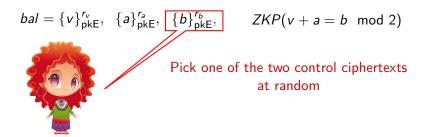


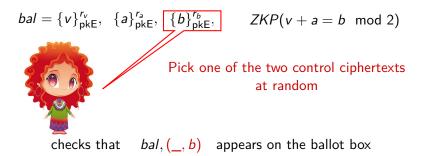


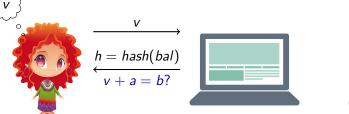




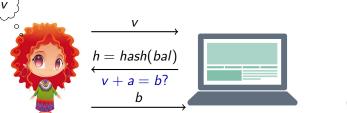




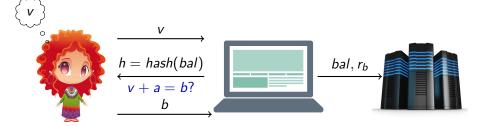


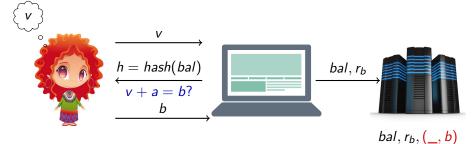






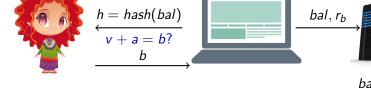






BeleniosCal's principle - continued v h = hash(bal)bal, r_b v + a = b? b $bal, r_b, (_, b)$ check check





check



check



- 🗸 no paper material
- ✓ the audited ballot is cast
- Alice needs to check the bulletin board
- X Alice needs to compute modulo 2!

Can voters compute modulo 2 ?!?



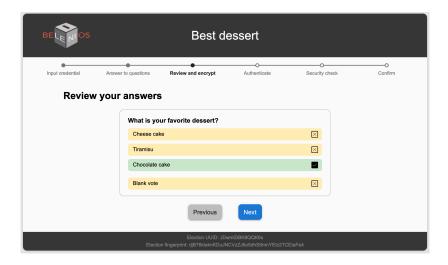
Can voters compute modulo 2 ?!?

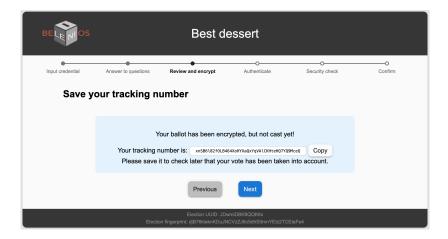


Let see how we propose to implement it.

| BELENIOS | | Best de | essert | | |
|------------------|----------|---|---------------|--------|-----------|
| Input credential | -O | Review and encrypt | OAuthenticate | -O | O Confirm |
| | | Please enter you | | | |
| | Election | Election UUID: JDw n fingerprint: djB76kleknKDuJN0 | | ElaFe4 | |

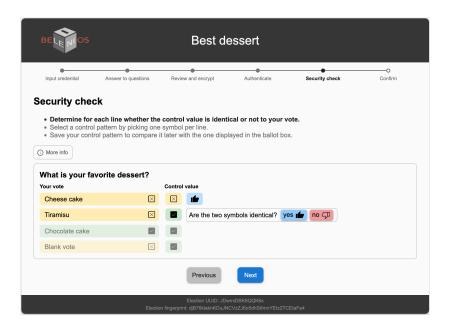
| BELENIOS | | Best de | essert | | |
|------------------|----------------------|--|--------|------|--------------|
| Input credential | Answer to questions | O Review and encrypt | O | O | O Confirm |
| | our favorite dessert | | | , | |
| | Cheese cal | ke | | | |
| | Tiramisu | | | | |
| | Chocolate | cake | | | |
| | Blank vote | | | | |
| | Question 1 of 1 | | | Next | |
| | | Election UUID: JDv n fingerprint: djB76kleknKDuJN | | | |







| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | Confirm |
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| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | Confirm |
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| | | it later with the one disp | played in the ballot boy | с. | |
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| More info That is your fav ur vote <mark>Cheese cake</mark> | vorite dessert? ⊠ | Control value | | | |
| That is your fav ur vote Cheese cake | | | | | |
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| That is your fav ur vote Cheese cake Tiramisu | \boxtimes | | wmbols identical? ye | s 👉 по 🖓 | |
| /hat is your fav ^{ur vote} | | | rymbols identical? ye | s 💼 no 🖓 | |

| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | Confirm |
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| Determine for o | each line whether th | e control value is iden | tical or not to your v | ote. | |
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| BELENIOS | | Best d | essert | | |
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| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | O Confirm |
| Select a control | CK reach line whether the ol pattern by picking one trol pattern to compare i | symbol per line. | - | | |
| What is your fa | avorite dessert? | Control value | | | |
| Cheese cake | \boxtimes | | | | |
| Tiramisu | \boxtimes | | | | |
| Chocolate cake | | | | | |
| Blank vote | \boxtimes | | | | |
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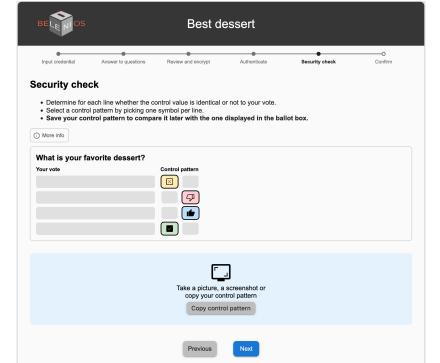
| BELENIOS | | Best de | essert | | |
|-----------------------------------|--|--|--------------|----------------------|-----------|
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| Select a cont | eck each line whether the co rol pattern by picking o trrol pattern to compare it | ne symbol per line. | | ς. | |
| What is your f Your vote | avorite dessert? | Control pattern | Pick on | e of the two symbols | |
| | | | | | |
| | | Previous | Next | | |
| | Election | Election UUID: JDv fingerprint: djB76kleknKDuJN | | ElaFe4 | |

| | Answer to questions | Review and encrypt | Authenticate | Security check | Confirm |
|------------------------------|---|---|------------------------|----------------------|---------|
| Input credential | Answer to questions | Review and encrypt | Aumeniicate | Security check | Comirm |
| ecurity che | ck | | | | |
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| | each line whether the c rol pattern by picking | control value is identical of one symbol per line. | or not to your vote. | | |
| | | it later with the one disp | layed in the ballot bo | κ. | |
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|) More info | | | | | |
| | | | | | |
| | | | | | |
| Vhat is your fa | avorite dessert? | | | | |
| - | avorite dessert? | Control pattern | | | |
| - | avorite dessert? | Control pattern | | | |
| What is your fa ′our vote | avorite dessert? | | Pick of | e of the two symbols | |
| - | avorite dessert? | | Pick or | e of the two symbols | |
| - | avorite dessert? | | Pick or | e of the two symbols | |
| - | avorite dessert? | | Pick or | e of the two symbols | |

| BELENIOS | | Best d | essert | | |
|-----------------------------|---|----------------------------|------------------------|-----------------------|--------------|
| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | O Confirm |
| ecurity che | ck | | | | |
| Save your con More info | rol pattern by picking trol pattern to compare avorite dessert? | it later with the one disp | layed in the ballot bo | х. | |
| Your vote | | Control pattern | | | |
| | | | Pick or | ne of the two symbols | |
| | | Previous | Next | | |
| | Electio | Election UUID: JD | | :ElaFe4 | |

| BELENIOS | | Best d | essert | | |
|-----------------------------------|--|---|--------------|-----------------------|--------------|
| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | O Confirm |
| Select a cont | eck each line whether the c rol pattern by picking ntrol pattern to compare | one symbol per line. | | х. | |
| What is your f Your vote | avorite dessert? | Control pattern | | | |
| | | | Pick or | ne of the two symbols | |
| | | Previous | Next | | |
| | Electi | Election UUID: JDv on fingerprint: djB76kleknKDuJN | | ElaFe4 | |

| | | Best de | essert | | |
|------------------|--|--|-------------------------|----------------|--------------|
| Input credential | Answer to questions | Review and encrypt | Authenticate | Security check | O Confirm |
| ecurity che | ck | | | | |
| | rol pattern by picking out of the second sec | one symbol per line. it later with the one disp | layed in the ballot box | κ. | |
| | avorite dessert? | | | | |
| Vhat is your fa | avorite dessert? | Control pattern | | | |
| What is your fa | avorite dessert? | | | | |
| What is your fa | avorite dessert? | | | | |
| | avorite dessert? | | | | |
| Vhat is your fa | avorite dessert? | | | | |



| BEL _{E pros} Best dessert | | | | | |
|---|---|---|--------------|----------------|---------|
| Input creden | tial Answer to questions | Review and encrypt | Authenticate | Security check | Confirm |
| | | Thank you | for voting! | | |
| | Next steps • Follow the link in your confirmation email • Verify your control pattern • If your ballot is missing or the control pattern does not match, contact the administrator: cortier | | | | |
| About your ballot Voter Tracking number Status Revote Email sent | | veronique.cortier@loria.fr HoVF28p19LEUtK0EkoNaitT7RzhM6AM1BJQLnClic8g accepted yes yes | | | |
| | | Go back to | election | | |

| BELENIOS | Best dessert - Accepted ballots | |
|---|--|----------|
| | Search tracking number | |
| Showing 1 out of 1 ballot. | | |
| Tracking number HoVF3 Hid Control pattern | | Raw data |
| | Back to election | |
| | Election UUID: JDwmiDBK8QQK6x Election fingerprint: djB76kleknKDuJNCVzZJ6o5dhStImnYEIz2TCEIaFe4 | |

How to analyse ${\sf BeleniosCal}$?

Formal analysis of e-voting systems

Why a formal analysis of an e-voting system?

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 \longrightarrow Because formal methods can find attacks before implementations

 \rightarrow Now a current practice for many protocols (TLS, 5G, ...)

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 \rightarrow Legal requirements in Switzerland to provide symbolic and cryptographic proofs of e-voting protocols.

- 2.14 Proofs of compliance with the cryptographic protocol requirements
- 2.14.1 A symbolic and a cryptographic proof of compliance must demonstrate that the cryptographic protocol meets the requirements in Numbers 2.1–2.12.
- 2.14.2 The proofs of compliance must directly refer to the protocol description that forms the basis for system development.
- 2.14.3 The proofs of compliance relating to basic cryptographic components may be provided according to generally accepted security assumptions and constructions (e.g. «random oracle model», «decisional Diffie-Hellman assumption», «Fiat-Shamir heuristic»).

Two main models for security

| | Formal approach | Computational approach |
|------------|---|---|
| Messages | $\begin{cases} \\ \langle , \rangle \\ k \\ A \\ N_A \end{cases} k$ | 0101000101110101 1101010110101010 001110101110110 |
| Encryption | terms | algorithm |
| Adversary | idealized | any polynomial algorithm |
| Guarantees | some attacks missed | stronger |
| Proof | often automatic | mostly by hand difficult for complex protocols |

Good tools in practice for formal / symbolic models

$\begin{array}{c} \text{ProVerif} \\ \hline \\ \text{Process} \longrightarrow \text{Translation into Horn clauses} \longrightarrow \text{Saturation of Horn clauses} \\ & \downarrow \\ & \text{Verification of the query} \end{array}$

- fully automatic
- axioms, lemmas, and restrictions [S&P'22]
- framework for verifiability [CSF'23]
- many voting protocols
 Swiss Chancellery requirements: Swiss Post. CHVote
 - Helios, Belenios, ...

semi automatic

Tamarin

- exclusive or
- voting protocols (Belenios, Selene, ...)

Two major issues for analyzing BeleniosCal

1. Addition modulo 2

$$egin{array}{rcl} 0+0&=&0\\ 0+1&=&1\\ 1+0&=&1\\ 1+1&=&0 \end{array}$$

 \rightarrow state explosion

 \rightarrow non termination

Two major issues for analyzing BeleniosCal

1. Addition modulo 2

 \rightarrow state explosion

- \rightarrow non termination
- 2. Probabilistic model
 - ► Alice checks either *a* or *b* at random
 - ► Intuition: An attacker may modify k votes without been detected with proba (¹/₂)^k.

Model for addition modulo - trace properties

Follows the approach introduced in CCS'22

Trace properties (verifiability)

Introduction of two predicates isSum(x, a, b) and isNotSum(x, a, b)

$$\begin{split} \texttt{isSum}(x, a, b), \quad \texttt{isSum}(x, a, b') &\Rightarrow b = b' \\ \texttt{isSum}(x, a, b), \quad \texttt{isNotSum}(x, a, b') &\Rightarrow b \neq b' \end{split}$$

- sound over-approximation
- another arithmetic operator could be used

Model for addition modulo - equivalence properties Vote secrecy

 $Alice(0) \mid Bob(1) \approx Alice(1) \mid Bob(0)$

over-approximation would be unsound

 \rightarrow Exactly the same tuples (x, a, b) are created on the left and on the right.

(Lemma) $isSum(x, a, b) \in fst(tr_b) \Leftrightarrow isSum(x, a, b) \in snd(tr_b)$

 \rightarrow allow to conclude by hand

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privacy relies on the following property: for all x₁, x₂, a₁, a₂, there exist b₁, b₂, b₃, b₄ such that

$$x_1 = a_1 + b_1 = a_2 + b_2$$

 $x_2 = a_2 + b_3 = a_1 + b_4$

 \rightarrow Encoded in the privacy ProVerif query

Model for probabilities

- to be done! for a real model with probabilities
- for the moment, for verifiability, the model assumes that Alice asks for opening both ciphertexts.

To conclude

Many challenges remain! (which is fun 🐸)

Strong demand for Cast as Intended

- many systems are currently proposed
- usability (two devices? computation in the head?)
- trust assumptions?
- vote secrecy

Better formal verification

- decision procedures for larger equational theory classes
- further improve tools
- account for probabilities