How does ProVerif work?

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How to decide security for unlimited sessions ?

→ In general, it is undecidable !

(i.e. there exists no algorithm for checking e.g. secrecy)

How to prove undecidability ?

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How to prove undecidability ?

Post correspondence problem (PCP)

input $\{(u_i, v_i)\}_{1 \leq i \leq n}$, $u_i, v_i \in \Sigma^*$

output $\exists n, i_1, \ldots, i_n \quad u_{i_1} \cdots u_{i_n} = v_{i_1} \cdots v_{i_n}$

Example : $\{(bab, b), (ab, aba), (a, baba)\}$

Solution ?

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Solution ? $\rightarrow$ Yes, 1,2,3,1.

$bababab$

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How to encode PCP in protocols ?

Given $\{(u_i, v_i)\}_{1 \leq i \leq n}$, we construct the following protocol P :

$$\begin{aligned} A &\rightarrow B: \quad \{\langle \overline{u_1}, \overline{v_1} \rangle\}_{K_{ab}}, \ldots, \{\langle \overline{u_k}, \overline{v_k} \rangle\}_{K_{ab}} \\ B: \{\langle x, y \rangle\}_{K_{ab}} &\rightarrow A: \quad \{\langle \overline{x, u_1}, \overline{y, v_1} \rangle\}_{K_{ab}}, \{s\}_{\{\langle \overline{x,u_1}, \overline{x,u_1} \rangle\}_{K_{ab}}}, \\ &\qquad \ldots, \{\langle \overline{x, u_k}, \overline{y, v_k} \rangle\}_{K_{ab}}, \{s\}_{\{\langle \langle \overline{x,u_k}, \overline{x,u_k} \rangle\}_{K_{ab}}} \end{aligned}$$

where $\overline{a_1 \cdot a_2 \cdots a_n}$ denotes the term $\langle \cdots \langle \langle a_1, a_2 \rangle, a_3, \rangle \ldots a_n \rangle$.

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where $\overline{a_1 \cdot a_2 \cdots a_n}$ denotes the term $\langle \cdots \langle \langle a_1, a_2 \rangle, a_3, \rangle \ldots a_n \rangle$.

Then there is an attack on P iff there is a solution to the Post Correspondence Problem with entry $\{(u_i, v_i)\}_{1 \leq i \leq n}$.

How to circumvent undecidability ?

- Find decidable subclasses of protocols.
- Design semi-decision procedure, that works in practice
- ...

How to model an unbounded number of sessions ?

"For any x, if the agent A receives $\mathrm{enc}(x, k_a)$ *then A responds with x."*

$\longrightarrow$ Use of first-order logic.

Some vocabulary

First order logic

Atoms $P(t_1, \ldots, t_n)$ where t_i are terms, P is a predicate

Literals $P(t_1, \ldots, t_n)$ or $\neg P(t_1, \ldots, t_n)$

closed under $\vee, \wedge, \neg, \exists, \forall$

Clauses : Only universal quantifiers

Horn Clauses : at most one positive literal

$$A_1, \ldots, A_n \Rightarrow B$$

where A_i, B are atoms.

Intruder

Horn clauses perfectly reflects the attacker symbolic manipulations on terms.

$I(x), I(y)$	$\Rightarrow$	$I(< x, y >)$	pairing
$I(x), I(y)$	$\Rightarrow$	$I(\{x\}_y)$	encryption
$I(\{x\}_y), I(y)$	$\Rightarrow$	$I(x)$	decryption
$I(< x, y >)$	$\Rightarrow$	$I(x)$	projection
$I(< x, y >)$	$\Rightarrow$	$I(y)$	projection

Protocol

Protocol :

$$A \rightarrow B \quad : \quad \{\text{pin}\}_{k_a}$$
$$B \rightarrow A \quad : \quad \{\{\text{pin}\}_{k_a}\}_{k_b}$$
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Horn clauses :

$$\Rightarrow \quad I(\{\text{pin}\}_{k_a})$$
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Secrecy property is a reachability (accessibility) property

$$\neg I(\text{pin})$$

Then there exists an attack iff the set of formula corresponding to
Intruder manipulations + protocol + property
is NOT satisfiable.

How to decide satisfiability ?

$\longrightarrow$ Resolution techniques

Binary resolution

A, B are atoms and C, D are clauses.

An intuitive rule

$$\frac{A \Rightarrow C \quad A}{C}$$

In other words

$$\frac{\neg A \vee C \quad A}{C}$$

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Generalizing a bit more

$$\frac{\neg A \vee C \quad B \vee D}{C\theta \vee D\theta} \; \theta = mgu(A, B) \quad \text{Binary resolution}$$

Binary resolution and Factorization

$$\frac{\neg A \vee C \quad B \vee D}{C\theta \vee D\theta}\ \theta = \mathrm{mgu}(A, B) \qquad \text{Binary resolution}$$

$$\frac{A \vee B \vee C}{A\theta \vee C\theta}\ \theta = \mathrm{mgu}(A, B) \qquad \text{Factorisation}$$

Theorem (Soundness and Completeness)

Binary resolution and factorisation are sound and refutationally complete,
i.e. a set of clauses C is not satisfiable if and only if $\perp$ (the empty clause) can be obtained from C by binary resolution and factorisation.

Exercise : Why do we need the factorisation rule ?

Example

$$\mathcal{C} = \{\neg I(s), \quad I(k_1), \quad I(\{s\}_{\langle k_1,k_1\rangle}), \\ I(\{x\}_y), I(y) \Rightarrow I(x), \qquad I(x), I(y) \Rightarrow I(\langle x, y\rangle)\}$$

$$\dfrac{\neg I(s) \qquad \dfrac{\dfrac{I(\{s\}_{\langle k_1,k_1\rangle}) \quad I(\{x\}_y), I(y) \Rightarrow I(x)}{I(\langle k_1, k_1\rangle) \Rightarrow s} \qquad \dfrac{I(k_1) \qquad \dfrac{I(k_1) \quad I(x), I(y) \Rightarrow I(\langle x, y\rangle)}{I(y) \Rightarrow I(\langle k_1, y\rangle)}}{I(\langle k_1, k_1\rangle)}}{I(s)}}{\bot}$$

But it is not terminating !

$$\dfrac{\dfrac{I(y) \Rightarrow I(\langle s, y\rangle) \qquad \dfrac{I(y) \Rightarrow I(\langle s, y\rangle) \qquad \dfrac{I(s) \qquad \dfrac{I(s) \quad I(x), I(y) \Rightarrow I(\langle x, y\rangle)}{I(y) \Rightarrow I(\langle s, y\rangle)}}{I(\langle s, s\rangle)}}{I(\langle s, \langle s, s\rangle\rangle)}}{I(\langle s, \langle s, \langle s, s\rangle\rangle\rangle)}$$

$\dots$

$\rightarrow$ This does not yield any decidability result.

Ordered Binary resolution and Factorization

Let $<$ be any order on clauses.

$$\frac{\neg A \vee C \quad B \vee D}{C\theta \vee D\theta} \quad \begin{array}{l} \theta = \mathrm{mgu}(A, B) \\ A\theta \not< C\theta \vee D\theta \end{array}$$

Ordered binary resolution

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Ordered factorisation

Theorem (Soundness and Completeness)

Ordered binary resolution and factorisation are sound and refutationally complete provided that $<$ is liftable

$$\forall A, B, \theta \qquad A < B \Rightarrow A\theta < B\theta$$

Examples of liftable orders

$$\forall A, B, \theta \qquad A < B \Rightarrow A\theta < B\theta$$

First example : subterm order

$P(t_1, \ldots, t_n) < Q(u_1, \ldots, u_k)$ iff any t_i is a subterm of $u_1, \ldots, u_k$

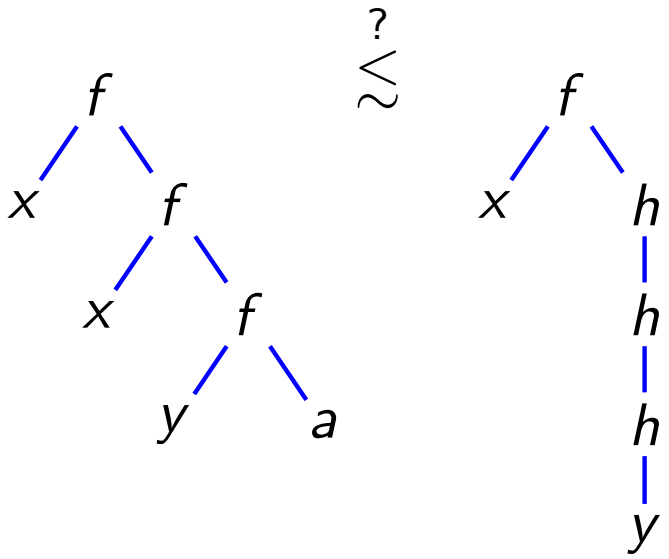
$\rightarrow$ extended to clauses as follows : $C_1 < C_2$ iff any literal of C_1 is smaller than some literal of C_2.

Exercise : Show that $\mathcal{C}$ is not satisfiable by ordered resolution (and factorisation).

Examples of liftable orders - continued

Second example : $P(t_1, \ldots, t_n) \lesssim Q(u_1, \ldots, u_k)$ iff

1. $\text{depth}(P(t_1, \ldots, t_n)) \leq \text{depth}(Q(u_1, \ldots, u_k))$
2. For any variable x,
   $\text{depth}_x(P(t_1, \ldots, t_n)) \leq \text{depth}_x(Q(u_1, \ldots, u_k))$



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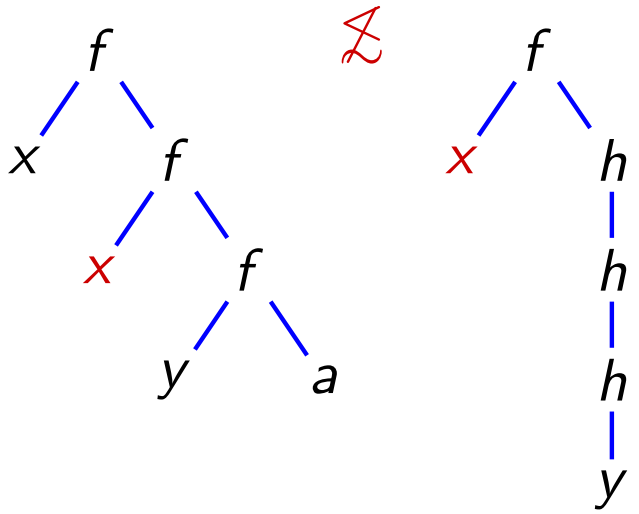

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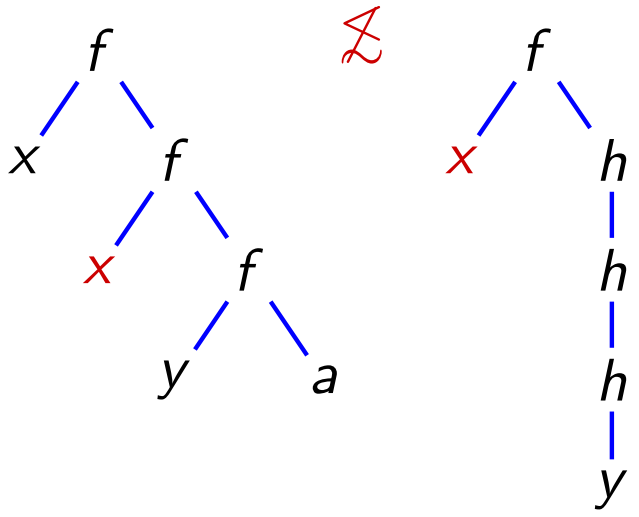

Exercise : Show that $\forall A, B, \theta \qquad A \lesssim B \Rightarrow A\theta \lesssim B\theta$

Back to protocols

Intruder clauses are of the form

$$\pm I(f(x_1, \ldots, x_n)),\ \pm I(x_i),\ \pm I(x_j)$$

Protocol clauses

$$\begin{aligned} & \Rightarrow && I(\{\mathsf{pin}\}_{k_a}) \\ I(x) & \Rightarrow && I(\{x\}_{k_b}) \\ I(\{x\}_{k_a}) & \Rightarrow && I(x) \end{aligned}$$

At most one variable per clause !

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Theorem

Given a set $\mathcal{C}$ of clauses such that each clause of $\mathcal{C}$

- *either contains at most one variable*
- *or is of the form $\pm I(f(x_1, \ldots, x_n)),\ \pm I(x_i),\ \pm I(x_j)$*

Then ordered ($\lesssim$) binary resolution and factorisation is terminating.

Decidability for an unbounded number of sessions

Corollary

For any protocol that can be encoded with clauses of the previous form, then checking secrecy is decidable.

But how to deal with protocols that need more than one variable per clause ?

ProVerif

Developed by Bruno Blanchet, Paris, France.

- No restriction on the clauses
- Implements a sound semi-decision procedure (that may not terminate).
- Based on a resolution strategy well adapted to protocols.
- performs very well in practice !
  - Works on most of existing protocols in the literature
  - Is also used on industrial protocols (e.g. certified email protocol, JFK, Plutus filesystem)

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  $\longrightarrow$ several tools for detecting attacks (Casper, Avispa, Scyther, ...)
- For an unbounded number of sessions
  - for one-copy protocols, secrecy is DEXPTIME-complete [CortierComon RTA03] [SeildVerma LPAR04]
  - for message-length bounded protocols, secrecy is DEXPTIME-complete [Durgin et al FMSP99] [Chevalier et al CSL03]

  $\longrightarrow$ some tools for proving security (ProVerif, Scyther)