A Certified Compiler for Verifiable Computing

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### Outsourcing computations to untrusted parties

<table>
<thead>
<tr>
<th><strong>Verifier</strong></th>
<th><strong>Worker</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>- Wants some program to be executed</td>
<td>- Has the required power and secrets</td>
</tr>
<tr>
<td>- Has limited resources</td>
<td>- Is neither reliable, nor trusted</td>
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</tbody>
</table>
Pinocchio: protocols for verifiable computation

- **Run**
  - program $f$
  - public input $x$
  - output $z$

- **Prove**
  - secret input $y$
  - witness $w$

- **Verify**
  - proof $\pi$
  - $\exists y, f(x, y) = z$
Pinocchio: practical verifiable computation

**Witness: all intermediate values**
- Too large
- Reveals all secrets

**Proof: nine points on elliptic curves**
- 288 octets (verifiable in \( \sim 10\text{ms} \))
- Look like random points

**Verify: divisibility check between polynomials**
- Program execution is encoded as polynomial of (very) high degree
- Proof validity expressed as a divisibility relation
- Test this relation at a single (random) point
Example program

```c
int main(void) {
    int N = public-input();
    int a = private-input();
    int b = private-input();
    int r = (a != 1) * (b != 1) * (N == a * b);
    output(r);
    return 0;
}
```

If the output is one, then $N$ is not prime and the worker *most probably* knows how to factor it.
Example program

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int main(void) {
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If the output is one, then N is not prime and the worker *most probably* knows how to factor it.

QAP: System of quadratic equations in a field.

\[
\begin{align*}
(1 - x_0) \times (a - 1) &= 0 \\
 x_1 \times (a - 1) &= x_0 \\
(1 - x_2) \times (b - 1) &= 0 \\
 x_3 \times (b - 1) &= x_2 \\
a \times b &= x_4 \\
x_5 \times (N - x_4) &= 0 \\
x_6 \times (N - x_4) &= 1 - x_5 \\
x_0 \times x_2 &= x_7 \\
x_7 \times x_5 &= r
\end{align*}
\]
QAPs are not for programming

Pinocchio theorem

If the proof verification succeeds, then the worker most probably knows a solution to the QAP.

- Can we use a general-purpose programming language (e.g., C) and compile it into a QAP?
- Can we get similarly strong guarantees on the source program?
A certified compiler for Pinocchio

- CompCert front-end from C to RTL
- A RTL *interpreter* to:
  - compile to a QAP;
  - run to produce outputs and witnesses.
- Geppetto engine to compute proofs
- A proof verifier specification

Theorem (formalized in Coq)

If the proof verification succeeds w.r.t. some public I/Os, then there exists an execution of the *source* program with said I/Os.
The CompCert C compiler

- Optimizing C compiler to x86, PowerPC and ARM
- Various intermediate representations
- Formal semantics of all the languages involved
- Machine-checked proof of a correctness theorem

Theorem

The compiler does not introduce new behaviours:

$$\forall p, p', \text{compile}(p) = \text{OK}(p') \implies \llbracket p \rrbracket \subseteq \text{notWrong} \implies \llbracket p' \rrbracket \subseteq \llbracket p \rrbracket.$$  

$$\llbracket p \rrbracket : \text{set of behaviours of program } p \text{ (not empty)}$$
Behaviours and traces

In CompCert

A behaviour is a trace (i.e., a possibly infinite list) of visible events:

- call to external function;
- volatile memory access.

Some behaviours are wrong:
reaching a (non-final) state from which the execution is stuck.

In PinocchioQ

Finite list of input or output events
(modeled through volatile memory accesses).
A modular interpreter

Three layers for independent proofs

1. **Values and arithmetic operations**
   Two implementations:
   - Symbolic values with quadratic equations (compilation)
   - Machine integers (evaluation)

2. **Memory loads/stores and pointer arithmetic**
   Resolves addresses at compile-time

3. **Control-flow and events**
   Builds an execution trace with symbolic values from the first layer.
   Resolves branches at compile-time: loops are completely unrolled
### From RTL to QAP

#### Symbolic values
Linear combinations of wires (QAP variables).
The interpreter is stateful: remember a set of quadratic equations.

#### Easy case
Linear operations (add, sub): direct symbolic computation.
Multiplications: add fresh wire for the result, add equation.

#### Beyond multiplication, e.g., \( r = a \neq 0 \)
Two fresh wires: \( q, r \); two equations:
\[
\begin{align*}
(1 - r) \times a &= 0 \\
q \times a &= r
\end{align*}
\]

#### Proof invariant
State only \textit{increases}: solutions to the QAP at the end of every step are solutions to the QAP at the beginning of the step.
Some encodings are only valid within some range

\[
\begin{aligned}
(1 - r) \times a &= 0 \\
q \times a &= r
\end{aligned}
\]

only works if \(a \in ] - 2^{32}; 2^{32} [\).

Indeed \(2^{32}\) is not null in the field, but null as 32-bit machine integer.

Public inputs are annotated with a range

For each value, we over-approximate it with an interval

Proof invariant

Assuming a solution complies with the intervals attached to public inputs, the computed ranges ensure that for every value there is exactly one integer in the range that represents this value.
Some operations have no simple quadratic encoding, e.g., right shift. We can perform a (costly) binary decomposition and cache the result.

Knowing: \( \text{range}(v) \subseteq [0; 2^{b+1} - 1] \)

\[
x_0 \times (1 - x_0) = 0 \\
\vdots \\
x_b \times (1 - x_b) = 0 \\
v = \sum_{i=0}^{b} 2^i x_i
\]

Proof invariant

Cached binary decompositions are consistent with the decomposed value.
Simulation proof

Theorem

If the interpretation of a program $P$ produces an symbolic trace $tr$ and QAP $q$, if $\rho$ is a solution to the QAP, then the concrete trace $\rho \cdot tr$ is a behaviour of $P$.

Simulation lemma

Conclusions in red.

$\sigma_0$, $\sigma_1$, interpreter states (QAP)
$\rho$ solution to $\sigma_1$
$s_0$, $s_1$, RTL states
$\sigma_0$ and $s_0$ are related (given $\rho$)

Then induction on the length of the interpretation.
Proof verification

- A divisibility check between polynomials
- New Coq library for Lagrange polynomials
- Formal link between the divisibility relation and QAP solutions

**Theorem**

If the divisibility relation between the polynomials constructed from the QAP and its putative solution holds, then it is an actual solution to the QAP.
Experimental evaluation

- Extract and run our compiler on various C programs
- Connect to the Geppetto engine for the cryptography part

<table>
<thead>
<tr>
<th>Case</th>
<th>RTL instructions</th>
<th>Size (#wire)</th>
<th>Degree</th>
<th>Compile (s)</th>
<th>Evaluate (s)</th>
<th>KeyGen</th>
<th>Prove</th>
<th>Verify</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>static</td>
<td>dynamic</td>
<td>I/O</td>
<td>private</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First</td>
<td>21</td>
<td>34</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
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<tr>
<td>Factorization</td>
<td>40</td>
<td>167</td>
<td>2</td>
<td>19</td>
<td>20</td>
<td>0.00</td>
<td>0.01</td>
<td>0.04</td>
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<tr>
<td>Bachet</td>
<td>63</td>
<td>527</td>
<td>2</td>
<td>64</td>
<td>65</td>
<td>0.00</td>
<td>0.02</td>
<td>0.08</td>
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<tr>
<td>Matrix (10)</td>
<td>97</td>
<td>37,892</td>
<td>201</td>
<td>1800</td>
<td>1,900</td>
<td>0.22</td>
<td>0.37</td>
<td>0.90</td>
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<tr>
<td>Matrix (100)</td>
<td>97</td>
<td>17,251,542</td>
<td>20,001</td>
<td>1,080,000</td>
<td>1,090,000</td>
<td>143.72</td>
<td>178.62</td>
<td>229.37</td>
</tr>
<tr>
<td>SHA1 (4)</td>
<td>180</td>
<td>29,313</td>
<td>7</td>
<td>36,138</td>
<td>37,082</td>
<td>4.59</td>
<td>5.65</td>
<td>25.82</td>
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<tr>
<td>SHA1 (96)</td>
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<td>58,831</td>
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<td>77,004</td>
<td>78,925</td>
<td>10.02</td>
<td>12.26</td>
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<tr>
<td>SHA1 (159)</td>
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<td>88,251</td>
<td>46</td>
<td>116,325</td>
<td>119,209</td>
<td>15.33</td>
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<tr>
<td>SAT (20)</td>
<td>39,462</td>
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<td>4220</td>
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<td>SAT (50)</td>
<td>583,902</td>
<td>588,902</td>
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<td>63,800</td>
<td>63,800</td>
<td>5.67</td>
<td>1843.5</td>
<td>14.07</td>
</tr>
</tbody>
</table>
Conclusion

Summary

- Coq implementation and proof of a compiler from C to QAP, based on CompCert
- Linked to the Geppetto engine, complete implementation of the Pinocchio protocol (soon available at https://vc.codeplex.com/)
- Formal link between the proof verification and the execution of the source program

Future work

- Verify the implementation of the proof verifier
- Bootstrapping