Abstract Self Modifying Machines

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joint work with Jean-Yves Marion
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Plan

Plan

Introduction

Framework

Applications

Program? Data?

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- Example : Python

Low level case: nothing is forbidden!

- Programs and data are totally indistinguishable
- They belong to the same space (memory)

Example

```
 1  move 10 2
 2  jz $\langle 10 \rangle$ 6
 3  move$(10 - \langle 10 \rangle)$ $((\langle 10 - \langle 10 \rangle \rangle) - 42)$
 4  move 10 $(\langle 10 \rangle - 1)$
 5  jump 2
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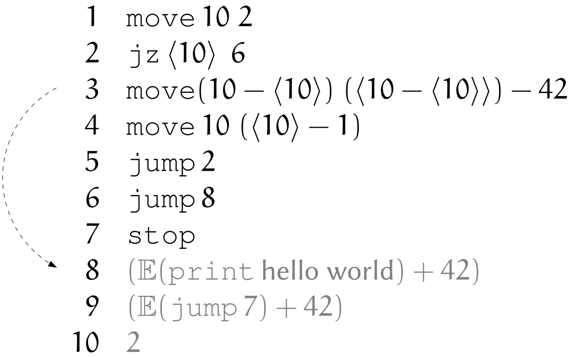

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7 $\texttt{stop}$
8 $\texttt{print}$ **hello world**
9 $(\mathbb{E}(\texttt{jump}\ 7)+42)$
10 2

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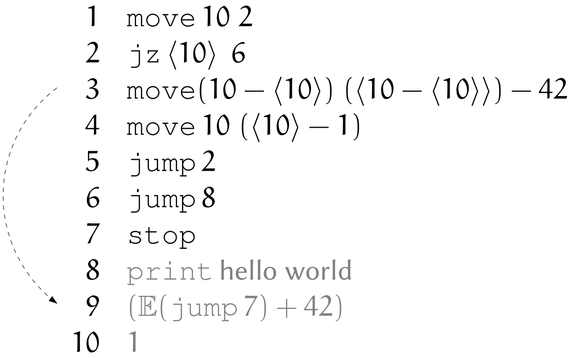

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General problematics

Compilation & certification

- From non SM programs to SM programs (obfuscation/optimisation)
- Certify compilation

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Recover high-level semantics from low-level SM semantics

- Recover non SM program from SM program…
- …wrt existing models of self-modification (wave semantics)

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Compilation & certification

- From non SM programs to SM programs (obfuscation/optimisation)
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Recover high-level semantics from low-level SM semantics

- Recover non SM program from SM program…
- …wrt existing models of self-modification (wave semantics)

Program abstraction

- Find abstract model specifically taking about self-modification.

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Current frameworks

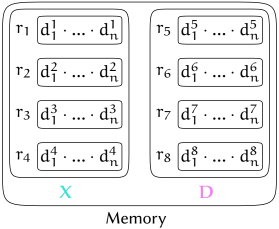
- Turing machine
- RAM (Cook & Reckhow, 1973)
- Cellular automaton (Neumann, 1966)
- Blob (Jones, 2010)
- RASP (Elgot & Robinson, 1964)
- SRM (Marion, 2012)

Language ASM$_2$

Language over data in $\mathcal{D}$, addresses in $\mathcal{A}$ and registers in $\mathcal{R}$:
$\forall r \in \mathcal{R}, \langle r \rangle : \mathcal{A} \to \mathcal{D}$

Abstract machine

- Register pointer: $\mathtt{RP} \in \mathcal{R}$
- Instruction pointer: $\mathtt{IP} \in \mathcal{A}$
- Executable zone: $\mathbf{X} \in \wp(\mathcal{R})$

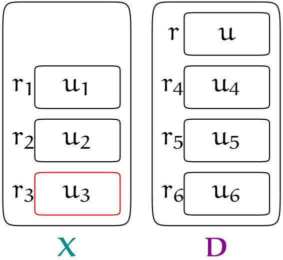


Instruction

The set of data $\mathcal{D}$ contains codes of the following instructions:

Instruction	Meaning
`move` r, d	Write the data d at the end of $\mathbf{D}[r]$
`input` r	Write the top of the input at the end of $\mathbf{D}[r]$
`pop` r	Pop the data on the top of $\mathbf{D}[r]$
`jump` a	Go to the instruction at address a
`case` r	Conditional jump depending on $\mathbf{D}[r]$
`exec` r	Control transfer to register $\mathtt{RP} = r$ and $\mathtt{IP} = 0$
`activate` r	Activate $\mathbf{D}[r]$
`inactivate` r	Inactivate $\mathbf{X}[r]$

Instruction `activate`



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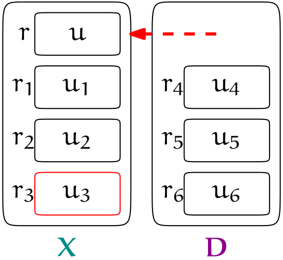

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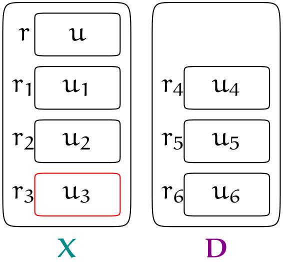

$$
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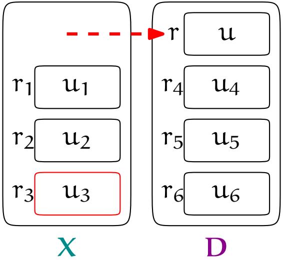

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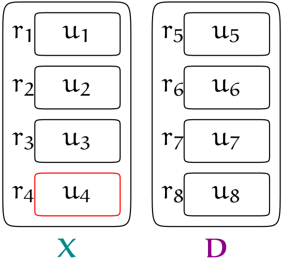
Instruction `exec`



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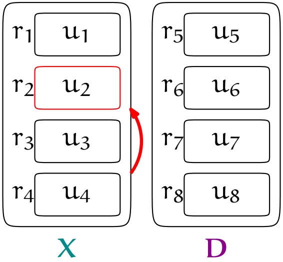
Instruction exec



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Cinematic

a_1 `inactivate` r_4
a_2 `move` r_4 2
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a_4 `case` r_4
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► r_1 a_7 `move` r_3 $\langle r_3 \mid 10 - \langle r_4 \mid 10 \rangle \rangle - 42$
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r_2 a_{14} `stop`

r_4 a_{17} `exec` r_1

X

r_3 a_{15} $(E($`print` hello world$) + 42)$
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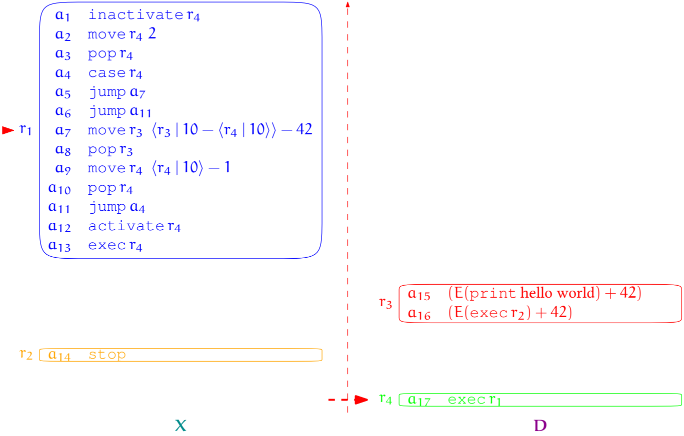

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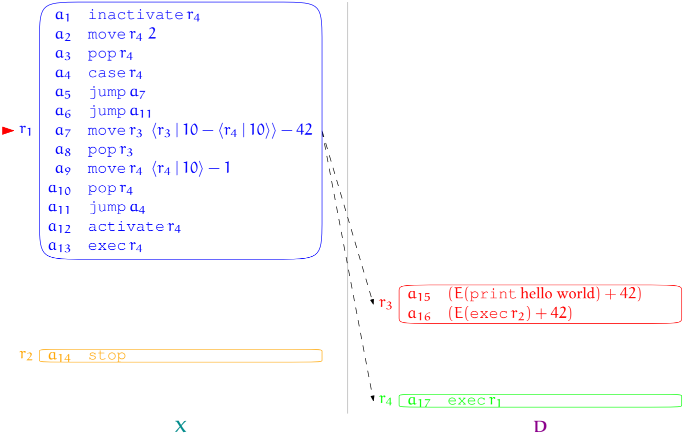

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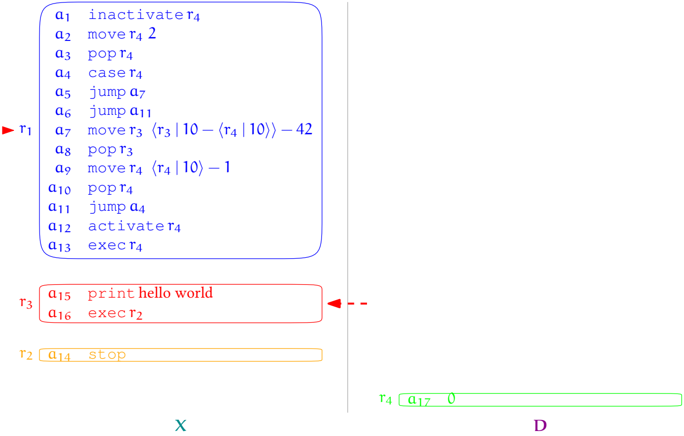

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X **D**

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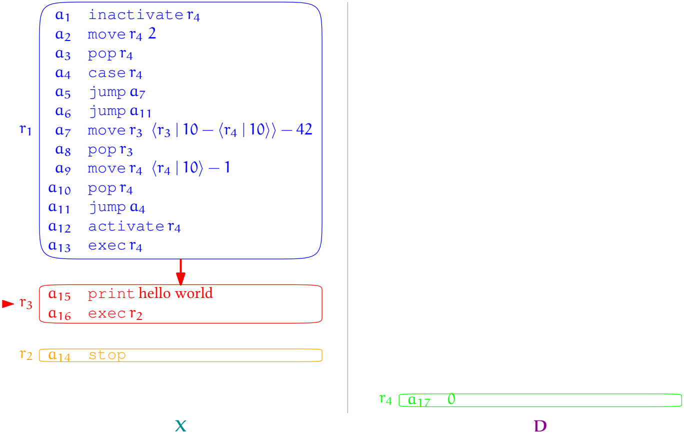

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r_2

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r_4

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X

D

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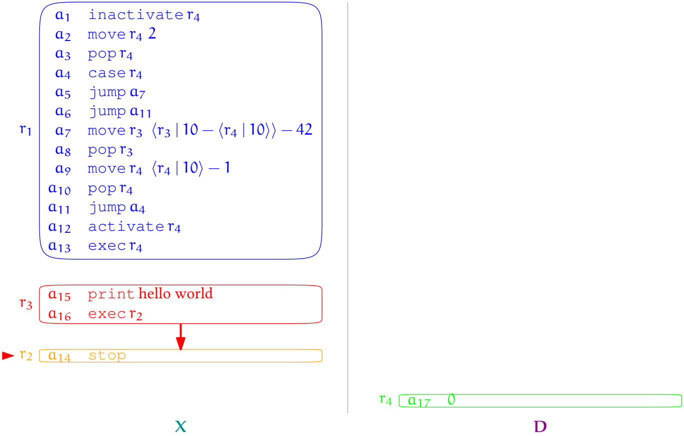

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a_{13} `exec` r_4

r_3

a_{15} `print` hello world

a_{16} `exec` r_2

► r_2

a_{14} `stop`

r_4

a_{17} 0

Computation

- Valuation $v_R \in \mathcal{V}$ with $R \subset \mathcal{R}$:

$$v_R = \{\langle r \rangle \mid r \in R\}$$

- State $s \in \mathcal{S}$:

$$(\underbrace{\mathtt{RP}, \mathtt{IP}, \mathbf{X}, v_{\mathbf{X}}}_{p \in \mathcal{P}}, v_{\mathbf{D}})$$

- Transition $\triangleright \in \wp(\mathcal{S}^2)$
- Interpretation of p (set of traces):

$$[\![p]\!] \overset{\text{def}}{=} \{s_1 \cdots s_n \in \mathcal{S}^* \mid s_1 = (p, v) \land \forall i \in [\![1, n-1]\!]\ s_i \triangleright s_{i+1}\}$$

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Writing relation

Given a trace $\tau \in [\![p]\!]$

$$\begin{array}{cccccccccccccccccccccccccccc} 1 & \rightarrow & 2 & \rightarrow & 3 & \rightarrow & 4 & \rightarrow & 5 & \rightarrow & 2 & \rightarrow & 3 & \rightarrow & 4 & \rightarrow & 5 & \rightarrow & 2 & \rightarrow & 6 & \rightarrow & 8 & \rightarrow & 9 & \rightarrow & 5 \\ \mathit{1} & & \mathit{2} & & \mathit{3} & & \mathit{4} & & \mathit{5} & & \mathit{6} & & \mathit{7} & & \mathit{8} & & \mathit{9} & & \mathit{10} & & \mathit{11} & & \mathit{12} & & \mathit{13} & & \mathit{14} \end{array}$$

Writing relation: $\dashrightarrow_\tau \in \wp(\mathbb{N}^2)$

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\mathit{1} & & \mathit{2} & & \mathit{3} & & \mathit{4} & & \mathit{5} & & \mathit{6} & & \mathit{7} & & \mathit{8} & & \mathit{9} & & \mathit{10} & & \mathit{11} & & \mathit{12} & & \mathit{13} & & \mathit{14}
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Example

Steps *2* and *11* write on address 5.

Writing relation

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$$\begin{tikzcd}[column sep=small]
1 \arrow[r] & 2 \arrow[r] \arrow[rrr, dashed, bend left] \arrow[rrrrrrr, dashed, bend left] & 3 \arrow[r] & 4 \arrow[r] & 5 \arrow[r] & 2 \arrow[r] & 3 \arrow[r] & 4 \arrow[r] & 5 \arrow[r] & 2 \arrow[r] & 6 \arrow[r] \arrow[rrr, dashed, bend left] & 8 \arrow[r] & 9 \arrow[r] & 5 \\
\mathit{1} & \mathit{2} & \mathit{3} & \mathit{4} & \mathit{5} & \mathit{6} & \mathit{7} & \mathit{8} & \mathit{9} & \mathit{10} & \mathit{11} & \mathit{12} & \mathit{13} & \mathit{14}
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Program:

r_1	a_1	$\texttt{inactivate}\ r_4$
	a_2	$\texttt{move}\ r_4\ \ 2$
	a_3	$\texttt{pop}\ r_4$
	a_4	$\texttt{case}\ r_4$
	a_5	$\texttt{jump}\ a_7$
	a_6	$\texttt{jump}\ a_{11}$
	a_7	$\texttt{move}\ r_3\ \ \langle r_3 \mid 10 - \langle r_4 \mid 10 \rangle\rangle - 42$
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r_2	a_{14}	$\texttt{stop}$
r_3	a_{15}	$\mathbb{D}(\mathbb{E}(\texttt{print}\ \text{hello world}) + 42)$
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Trace:

1 → 2 → 3 → 4 → 5 → 7 → 8 → 9 → 10 → 11 → 4 → 5 → 7 → 8 → 9 → 10 → 11 → 4 → 6 → 15 → 16 → 14

Example

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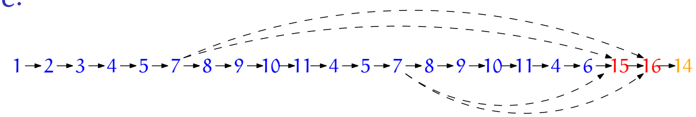
	a_7	$\texttt{move}\ r_3\ \ \langle r_3 \mid 10 - \langle r_4 \mid 10 \rangle\rangle - 42$
	a_8	$\texttt{pop}\ r_3$
	a_9	$\texttt{move}\ r_4\ \ \langle r_4 \mid 10 \rangle - 1$
	a_{10}	$\texttt{pop}\ r_4$
	a_{11}	$\texttt{jump}\ a_4$
	a_{12}	$\texttt{activate}\ r_4$
	a_{13}	$\texttt{exec}\ r_4$
r_2	a_{14}	$\texttt{stop}$
r_3	a_{15}	$\mathbb{D}(\mathbb{E}(\texttt{print}\ \text{hello world}) + 42)$
	a_{16}	$\mathbb{D}(\mathbb{E}(\texttt{exec}\ r_2) + 42)$
r_4	a_{17}	$\texttt{exec}\ r_1$

Trace:



1 → 2 → 3 → 4 → 5 → 7 → 8 → 9 → 10 → 11 → 4 → 5 → 7 → 8 → 9 → 10 → 11 → 4 → 6 → 15 → 16 → 14

Trace interpretation

The monotone level $\eta_\tau(i)$:

> *“ The number of necessary self-modifications before executing step i”*

$$\begin{aligned}
\eta_\tau(1) &\overset{\text{def}}{=} 1 \\
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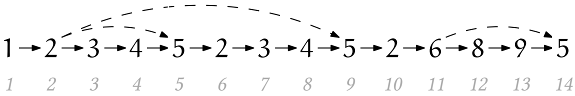

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$\eta_\tau = 1$

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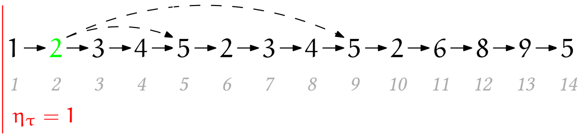

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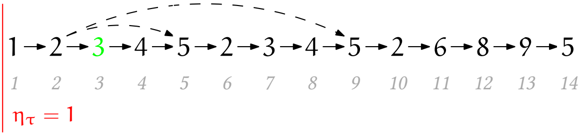

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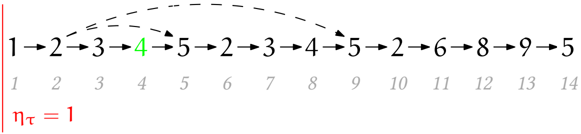

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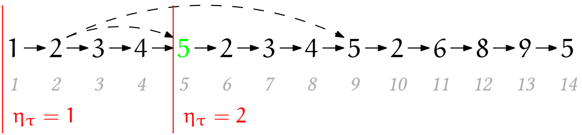

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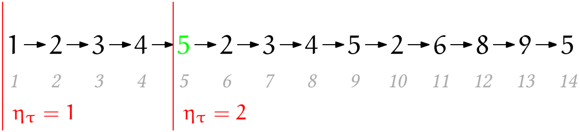

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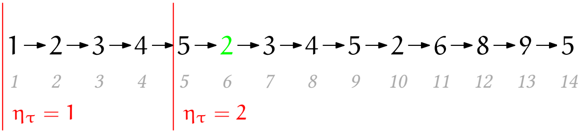

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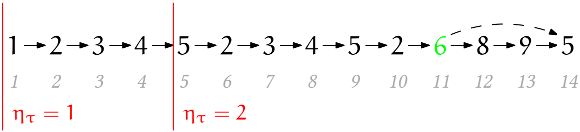

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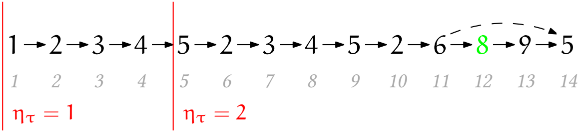

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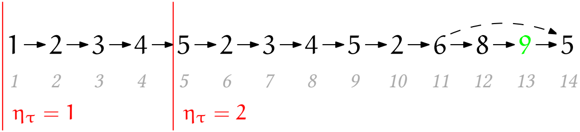

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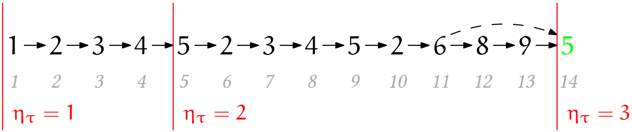

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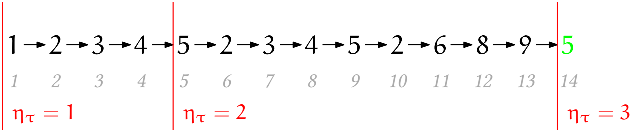

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A wave of level n:

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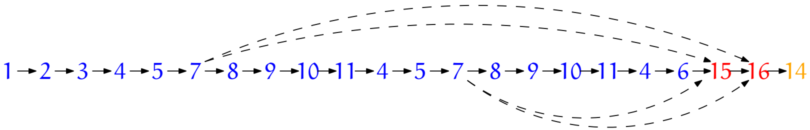

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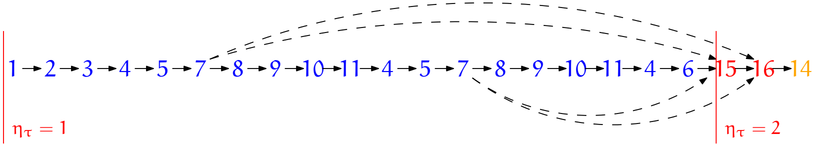

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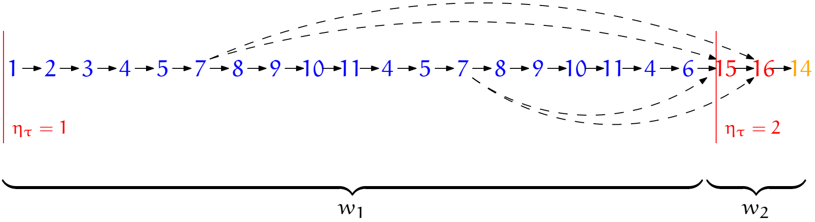

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w_1

w_2

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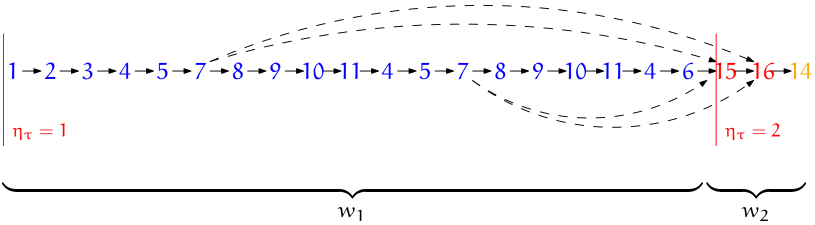

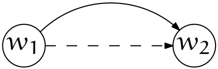

Sum up

- Measure self-modification.
  - → the writing relation.
- Interpretation wrt self-modification
  - → the trace-oriented notion of wave.

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Question

Reconstruct non self-modifying program?

Plan

Introduction

Framework

Applications

- Measure
- Program extraction
- Abstraction

Intuition

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How do I switch from a wave to another?

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How do I switch from a wave to another?

Answer

When I execute something I wrote.

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Answer

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ASM_2 answer

When I execute a register which was not activated when I began.

→ A good witness of a wave is thus **X** when the wave begins.

Witness

Given a trace $\tau = s_1 \cdots s_n$, a wave $w = [\![i, j]\!]$ of τ, the witness of w is

$$\operatorname{prog}_\tau w \stackrel{\text{def}}{=} p_{\min w} \text{ where } \forall i, s_i = (p_i, v_{\mathbf{D}_i})$$

$\operatorname{prog}_\tau w$ is a snapshot of the executable memory at the beginning of w.

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Soundness

$$s_i \cdots s_j \in [\![\mathrm{prog}_\tau(w)]\!] \,.$$

Idea

All registers executed in w are already present in $\mathrm{prog}_\tau(w)$: otherwise a register have to be activated (so written) and we change of wave when it is executed.

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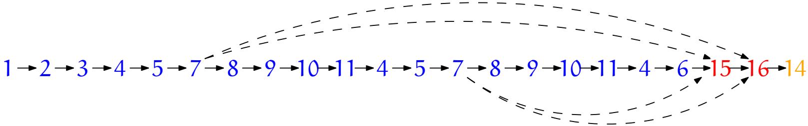
Abstract execution

Given a trace $\tau \in [\![p]\!]$



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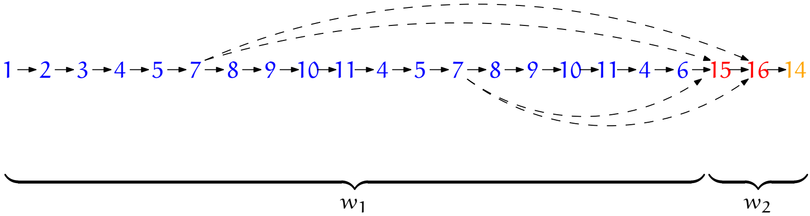

$$\underbrace{1 \to 2 \to 3 \to 4 \to 5 \to 7 \to 8 \to 9 \to 10 \to 11 \to 4 \to 5 \to 7 \to 8 \to 9 \to 10 \to 11 \to 4 \to 6}_{w_1} \to \underbrace{15 \to 16 \to 14}_{w_2}$$

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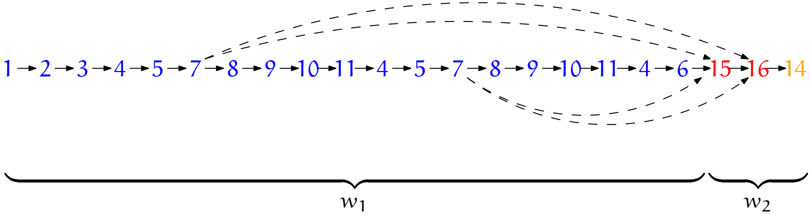

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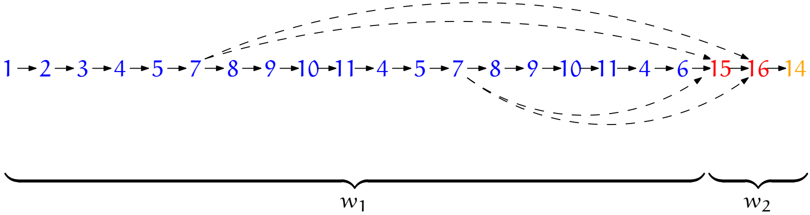

we group steps into waves:

$$w_1 \cdot w_2$$

and recover program for each wave:

$$p_1 \cdot p_2.$$

This is the abstract execution $\alpha(\tau)$.

Abstract semantics

Construct a correct abstract semantics wrt α:

Abstract semantics

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Abstract semantics of p

$$[\![p]\!]^{\#} \quad : \quad \text{set of sequences of programs}$$

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such that

$$\alpha([\![p]\!]) \subseteq [\![p]\!]^\#$$

Abstract semantics

Abstract transition $\triangleright^{\#} \in \wp(\mathcal{P}^{\mathcal{P}})$:

$p \triangleright^{\#} p' \iff p'$ is the witness of the 2^{nd} wave of a $\tau \in [\![p]\!]$

Abstract interpretation $[\![p]\!]^{\#}$:

$$[\![p]\!]^{\#} \overset{\text{def}}{=} \{p_1 \cdots p_n \in \mathcal{P}^* \mid p_1 = p \land \forall i \in [\![1, n-1]\!], p_i \triangleright^{\#} p_{i+1}\}$$

Example: $[\![p]\!]^\#$

Valuations $v_1, v_2, v_3, v_4 \in \mathcal{V}$

p

Example: $[\![\mathrm{p}]\!]^{\#}$

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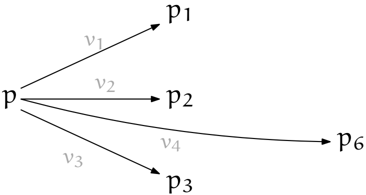

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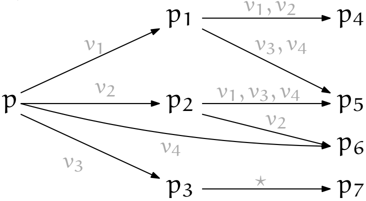

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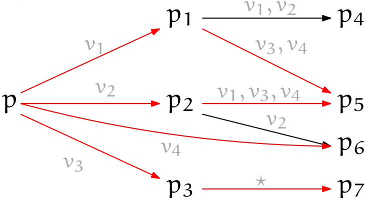

$\alpha([\![\mathrm{p}]\!]) \subseteq [\![\mathrm{p}]\!]^{\#}$

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We have…

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We will…

- define non monotone waves,
- improve symmetry of the definition (read/write),
- take advantage of intermediate granularity.

Bibliography

M. D. Preda, R. Giacobazzi, and S. Debray.
Modeling metamorphism by abstract interpretation.
Theoretical Computer Science, 2012.

D. Reynaud.
Analyse de codes auto-modifiants pour la sécurité informatique.
PhD thesis, INPL, 2010.

C. C. Elgot and A. Robinson.
Random-access stored-program machines, an approach to programming languages.
Journal of the Association for Computing Machinery, 11(4):365–399, October 1964.

C.-K. Hur and D. Dreyer.
A kripke logical relation between ml and assembly.
Principles of programming languages, 2011.

Annexe: non monotone waves

The set of waves:

$$\mathcal{W}_\tau \overset{\text{def}}{=} \mathbb{N}/_{\sim_\tau}$$

The wave relation $\sim_\tau$:

- $\star \dashrightarrow_\tau i$ if i is written by nobody
- $\star \sim_\tau \star$
- $i \sim_\tau j \iff \exists i' \sim_\tau j', i' \dashrightarrow_\tau i \wedge j' \dashrightarrow_\tau j$

Properties

$\sim_\tau$ is an equivalence relation.

Annexe: abstract interpretation

$$\triangleright^{\#}\, p \overset{\text{def}}{=} \{p' | \exists\, \tau = s_1 \cdots s_{n+1} \in [\![p]\!]\,, p' = \mathrm{prog}(s_{n+1}) \wedge \forall i, j \in [\![1, n]\!]\,, i \sim_{\tau} j\}$$

$$\mathsf{T}^{\#}\, \mathsf{X} \overset{\text{def}}{=} \mathsf{X} \cup \{p_1 \cdots p_n p_{n+1} \mid p_1 \cdots p_n \in \mathsf{X} \wedge \triangleright^{\#}\, p_n = p_{n+1}\}$$

$$[\![p]\!]^{\#} \overset{\text{def}}{=} \mathsf{Fix}_{\{p\}}\, \mathsf{T}^{\#}$$