A Stochastic Hybrid Model for a Soccer Player

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Abstract

In this paper, the authors propose an stochastic model for a soccer player and his quest in scoring a goal. This model is based on *Stochastic Hybrid System* theory. A stability analysis is given along with a model generated by an identification process in hopes of comparing the two models. The sampled data for identification purposes come from a video game

Keywords: *Stochastic Hybrid Model, RoboCup, Robotic Soccer.*

1 Introduction

The field of robotic has penetrated in several real-life applications as it evolves throughout time. A famous application is playing a soccer match, this is mainly because a soccer match integrates various aspects and challenges from control theory, agent systems [1] and robotic.

This application has spawned several international competitions in hopes of advancing robotic towards a more mature and human-behavior condition. *RoboCup* is one of these competitions where robot teams designed by groups from all the world compete each year [2]. As this relatively new robotic-soccer increases in popularity, many applications are devised to create a control-scheme sufficient enough to model all significant aspects of reality as well as give insight or tactics into a winning-strategy.

Here, the authors detail a stochastic hybrid model which represents a soccer player trying to score a goal. A probabilistic approach is taken in order to model the pseudo-randomness of the effectiveness of the opponent team into the action of stealing the ball from the player.

Moreover, a player on the soccer field can have various roles depending on the state of the game or the state of the ball, stances such as *defense* or *attack* are based on the current possession of the ball and they exhibit completely different behaviors and objectives. In order to model this kind of discrete activation of roles, hybrid control theory offers mathematically founded structures combining the continuous behavior of the player with the role switching condition. Other aspects of the player can also be controlled by hybrid systems like in [3], and insights of the player's tactics as well as strategies and/or physical conditions of the player(such as mean velocity) can be show their efficacy through simulation of the model.

This work is organized as follows, in this section we give an introduction as well as some background theory on the concepts used throughout the paper, section 2 explains in detail the stochastic hybrid model along with some remarks on what was assumed and simplified. Section 2.1 does a stability analysis on several parts of the model and reflects on these results, section 3 generates a deterministic linear model by a technique of identification in order to compare it with the proposed model and point out the differences in behavior through simulation. Finally, section 4 gives some concluding remarks on the matter and some possible directions of work.

1.1 SHS background

Stochastic Hybrid Systems(SHS for short) is a relatively new probabilistic approach to Hybrid systems [4] that has been applied successfully to air traffic control problems [5] and even has analysis tools and solvers [6]. Intuitively, an SHS is a deterministic hybrid system with the stochastic component added into: (a) the dynamics of each state, through a Brownian motion; (b) the reset of the state in a discrete step through a probabilistic mass function, and; (c) a random time to *trigger* a discrete step into some known state .

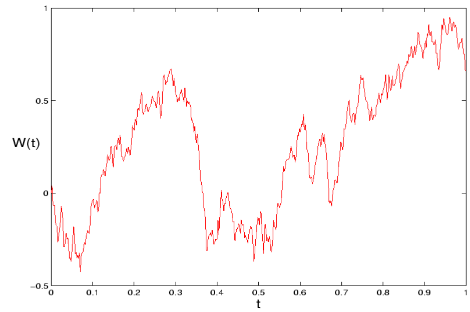


Figure 1: A one-dimensional wiener process

We begin by giving the main definitions to construct the SHS background:

Definition 1. ***(Wiener Process).***

A wiener process W *is a function characterized by three properties:*

(i) $\mathrm{W}_0 = 0$.

(ii) W *is continuous everywhere.*

(iii) if $0 \leq t_1 < t_2$ *then* $\mathrm{W}_{t_2} - \mathrm{W}_{t_1} \approx N(0, t_2 - t_1)$ *where* $N(\mu, \sigma^2)$ *is a normal distribution with mean and variance* μ *and* σ^2 *respectively.*

A wiener process can be thought of as random walk in its scaling limit, figure 1 shows an example of a 1D wiener process.

Definition 2. ***(Stochastic Hybrid System).*** *[7, 8]*

A stochastic hybrid systems is a collection $\mathcal{H} = \langle Q, X, Dom, f, g, Init, G, R\rangle$ *where:*

- Q *is set of discrete states.*
- X *is the state space, generally* $\mathbb{R}^n$.
- $Dom : Q \rightarrow 2^X$ *assigns to each state* Q_i *an open subset of* X *(a domain).*
- $f, g : Q \times X \rightarrow \mathbb{R}^n$ *are the vector fields.*
- $Init : \mathcal{B}(Q \times X) \rightarrow [0, 1]$ *is an initial probability measure concentrated on* $\bigcup_{i \in Q} i \times Dom(i)$.
- $G : Q \times Q \rightarrow 2^X$ *assigns a guard* $G(i, j) \subset X$ *to each transition* $(i, j) \in Q \times Q$ *such that:*
  - *For each transition* $(i, j) \in Q \times Q$, $G(i, j) \subset \partial Dom(i)$, *the guard* G *may be empty (meaning there does not exist a transition from state* i *to state* j*).* ∂ *is the* boundary *of the topological set.*
  - *For each* $i \in Q$, $\{G(i, j) | j \in Q\}$ *is a disjoint partition of* $\partial Dom(j)$.
- $R : Q \times Q \times X \rightarrow \mathcal{P}(X)$ *assigns to each transition* $(i, j) \in Q \times Q$ *and each state* $x \in G(i, j)$ *a reset probability kernel* $\mathcal{P}$ *concentrated on* $Dom(j)$.

Intuitively, an SHS has a set of discrete states each with a continuous evolution on $\mathbb{R}^n$, transitions between these states may occur provided that the current continuous state is in the guard between the states of the transition. When a discrete transition occurs, the continuous state is reset according to some probability function.

Definition 3. ***(SHS Execution).*** *[7]*

A stochastic process $\alpha_t = (q(t), x(t))$ *is called an SHS execution if there exists a sequence of stopping times* $\tau_0 = 0 \leq \tau_1 \leq \tau_2 \leq \ldots$ *such that for each* $i \in \mathbb{N}$*:*

- $\alpha_0 = (q(0), x(0))$ *is a* $Q \times X$ *value randomly extracted according to the probability measure in* $Init$.
- *for* $t \in [\tau_i, \tau_{i+1})$, $q(t) = q(\tau_i)$ *is constant and* $x(t)$ *is a continuous solution for the Stochastic Differential Equation(SDE):*

$$dx(t) = f\left(q(\tau_i), x(t)\right) dt + g\left(q(\tau_i), x(t)\right) d\mathrm{W}_t$$

  where W_t *is a* $1D$ *wiener process.*
- $\tau_{i+1} = inf\{t \geq \tau_i : x(t) \notin Dom(q(\tau_i))\}$.
- $x(\tau_{i+1}) \in G\left(q(\tau_i), q(\tau_{i+1})\right)$.
- *The probability of* $x(\tau_{i+1})$ *is governed by the law* $R\left(q(\tau_i), q(\tau_{i+1}), \lim_{t \rightarrow \tau_{i+1}} x(t)\right)$.

A valid run in a SHS has an initial random value according to $Init$, continuously evolves according to the state SDE, note that the SDE is a common ODE but with an stochastic part g added. Whenever the run hits a guard the discrete step happens and the continuous state changes according to the probabilistic rules in R.

2 The Stochastic Model

In this section we explain in detail the stochastic model used to represent our soccer player in the field. We first constraint the soccer game with a set of assumptions for simplicity reasons: (a) the player has no capacity of passing the ball to another peer, (b) the player has constant linear and angular velocity and

(c) the player is represented by a dot in the field, meaning he occupies no area.

Taking the idea of player representation as a *posture* from [9, 10], we can model the player as an augmented posture that keeps track of the time the player has had the ball. The game state can be represented as:

$$s^T = \begin{bmatrix} x_p & y_p & \theta_p & t_p & x_b & y_b & \theta_b & t_b \end{bmatrix}$$

Where (x_p, y_p) and (x_b, y_b) are the player and ball positions respectively and θ_p and θ_b are their orientation in counterclockwise starting from the x axis and, as said before, t_p is the time of average ball possession before *stealing* and t_b is the time of non-possession. Taking figure 2 as a reference, the model state space accounts for the position of the *ball* and the *player*. The model is structured as follows:

Definition 4. ***(SHS Soccer Model).***

Let $\mathcal{H}_{soccer} = \langle Q, X, Dom, f, g, Init, G, R\rangle$ *where:*

- $Q = \{\boldsymbol{attack}, \boldsymbol{defense}\}$
- $X = \mathbb{R}^8$
- $Dom(\boldsymbol{attack}) = Dom(\boldsymbol{defense}) = \left\{ s \in \mathbb{R}^8 | \begin{bmatrix} 0 \le s_1 \le width \\ 0 \le s_2 \le height \\ 0^\circ \le s_3 \le 360^\circ \\ 0 \le s_4 < \infty \\ 0 \le s_5 \le width \\ 0 \le s_6 \le height \\ 0^\circ \le s_7 \le 360^\circ \\ 0 \le s_8 < \infty \end{bmatrix} \right\}$
- $- \; f(\boldsymbol{attack}) = \begin{bmatrix} v_p \cdot \cos(\theta_p) \\ v_p \cdot \sin(\theta_p) \\ \omega_p \cdot dir\,(\theta_p, x_p, y_p, goal_x, goal_y) \\ -1 \\ v_p \cdot \cos(\theta_p) \\ v_p \cdot \sin(\theta_p) \\ \omega_p \cdot dir\,(\theta_p, x_p, y_p, goal_x, goal_y) \\ 0 \end{bmatrix}$

  $- \; f(\boldsymbol{defense}) = \begin{bmatrix} v_p \cdot \cos(\theta_p) \\ v_p \cdot \sin(\theta_p) \\ \omega_p \cdot dir\,(\theta_p, x_p, y_p, x_b, y_b) \\ 0 \\ v_b \cdot \cos(\theta_b) \\ v_b \cdot \sin(\theta_b) \\ \omega_b \cdot dir\,(\theta_b, x_b, y_b, post_x, post_y) \\ 1 \end{bmatrix}$

  $- \; g(\boldsymbol{attack}) = g(\boldsymbol{defense}) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

  where $v_p, \omega_p, v_b, \omega_b$ *are the linear and angular velocity of the player and the ball respectively and:*

  $dir\,(\theta, x_1, y_1, x_2, y_2) = 1 - (\cos(\theta), \sin(\theta)) \cdot (x_2 - \widehat{x_1, y_2} - y_1)$

- $Init = \left(\boldsymbol{attack}, [\frac{width}{2} \; \frac{height}{2} \; 0 \; mean_t \frac{width}{2} \; \frac{height}{2} \; 0 \; 0]\right) \to 1$
- $- \; G(\boldsymbol{attack}, \boldsymbol{defense}) =$
  $(0 \ge x_b \vee x_b \ge width) \cup$
  $(0 \ge y_b \vee y_b \ge height) \cup$
  $t_p \le 0$

  $- \; G(\boldsymbol{defense}, \boldsymbol{attack}) =$
  $(0 \ge x_b \vee x_b \ge width) \cup$
  $(0 \ge y_b \vee y_b \ge height) \cup$
  $||\,(x_b - x_p, y_b - y_p)\,|| \le \varepsilon$

  where ε *is the radius of vicinity where the player can* steal *the ball from the opponent*
- $- \; R(\boldsymbol{attack}, \boldsymbol{defense}, s) =$
  $s_{\left(x_p \leftarrow foul, y_p \leftarrow \frac{height}{2}\right)}$

  $- \; R(\boldsymbol{defense}, \boldsymbol{attack}, s) = s_{(t_p \leftarrow N(mean_t, 1))}$

  where $mean_t$ *means the average ball possession time of the player and* $V_{(x \leftarrow x')}$ *means the vector* V *with the* x *coordinated replace by* x'. $foul$ *is the distance the player must take from his own post.*

Some remarks are in order:

Our player has only **attack** and **defense** poses where just one is active at any instant, the state space keeps track of both: the *player* and the *ball*. Also, the domain of these two objects are restricted to the field area and positive times.

A significant aspect is the dynamics of each state, in **attack** we have the player heading for the opponent post and the ball goes *along* with him, that is why their dynamics are almost the same. The time t_p has a special meaning, it decreases linearly reflecting his chances of *losing* the ball as time passes due to tiredness or his opponents catching up to him.

If his possession time t_p hits zero or he leaves the field, the model switches to state *defense* where the opposite team tries to score a goal in his post, the

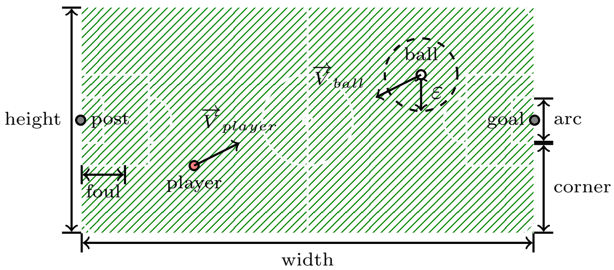


Figure 2: The soccer field and its components

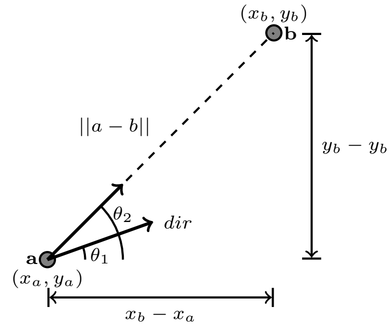


Figure 3: General Orientation Scheme

dynamics of the ball is then headed to the player post. The player is trying to *recover* the ball and is directed to its position, the function *dir* models this dynamic of changing the orientation in pursuit of another position. The fact that $g = 1$ lets the player have a random walk in his displacement because it is never perfect.

When passing to the **defense** state, we have the player taking a defense position that is directly ahead of his post at a distance $foul$. On the contrary, when passing to state **attack**, either by catching up to the opponent under a distance ε thus *recovering* the ball or when the ball leaves the field, the player keeps the current state, but his possession time is reset according to a normal random value centered in $mean_t$. The initial state is in the center of the field in **attack** mode.

2.1 Stability Analysis

Stability analysis of mathematical control models as the one presented here is frequently common in order to study key aspects of the application. The model presented here is tailored specifically to a sport, and as such, stability analysis in its movement dynamics is irrelevant to its description.

In spite of this argument, some elements of the model may have equilibrium points where some properties are seen. In this case, our system has an orientation dynamic applied to the player in order to align its displacement direction headed to the a particular objective. This objective may be the ball, the player post, or the opponent post. It is express as:

$$\omega\left(1 - (\cos\theta_1, \sin\theta_1)\cdot(x_b - \widehat{x_a, y_b} - y_a)\right)$$

This orientation dynamic reaches equilibrium when the player angle is aligned to the objective angle formed with the horizontal axis. Figure 3 shows this scheme in general terms.

Lemma 1. *$\theta_1 = \theta_2$ is an equilibrium point of the orientation scheme*

Proof.

$$0 = \omega\left(1 - (\cos\theta_1, \sin\theta_1)\cdot(x_b - \widehat{x_a, y_b} - y_a)\right) \tag{1}$$

$$1 = (\cos\theta_1, \sin\theta_1)\cdot\left(\frac{x_b - x_a}{||a-b||}, \frac{y_b - y_a}{||a-b||}\right) \tag{2}$$

$$||a-b|| = (x_b - x_a)\cos\theta_1 + (y_b - y_a)\sin\theta_1 \tag{3}$$

$$\frac{||a-b||}{x_b - x_a} = \cos\theta_1 + \frac{y_b - y_a}{x_b - x_a}\sin\theta_1 \tag{4}$$

$$\frac{1}{\cos\theta_2} = \cos\theta_1 + \tan\theta_2\sin\theta_1 \tag{5}$$

$$1 = \cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1 \tag{6}$$

$$1 = \cos(\theta_2 - \theta_1) \tag{7}$$

$$0 = \theta_2 - \theta_1 \tag{8}$$

$$\theta_1 = \theta_2 \tag{9}$$

□

This orientation equilibrium is confined to each discrete state of the model. When switching occurs the equilibrium is disrupted because the player position and, possibly, the objective position suddenly change.

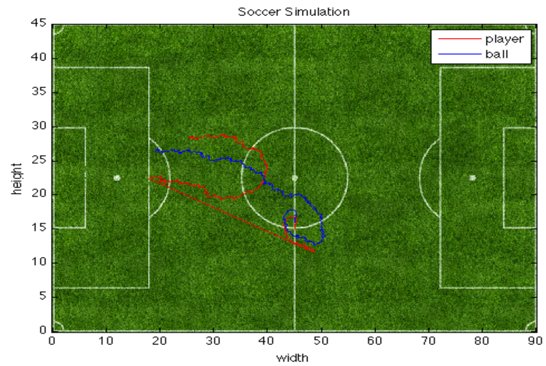
3 Identification

For purposes of having a comparison ground, we produced a linear hybrid model by means of identification. The key idea behind the id process is exposed in [11] and we took data from the software *Football Manager 2011* [12], a video game published by the company `SEGA`.

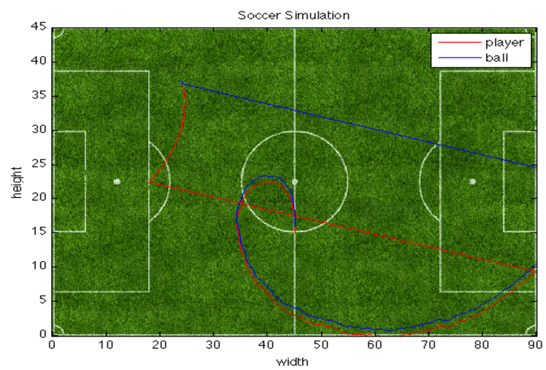
The data was taken from two matches previously recorded in video, the x, y position of a selected player as well as the ball were sampled from these sources. The position tracking was done by hand and the positions were taken relatively to the lower-left corner of the field and normalized to the range $0 - 100$ in order to abstract the measurements of the field as they can vary according to the FIFA regulations.

The identification process involved a hybrid component that, in our case, was easy to solve because of the nature of the problem. The *attack* state was assumed when the player team was performing an offensive set of movements and the *defense* state when the opposite team was doing so.

Each continuous state was estimated using `MATLAB`'s `n4sid` identification algorithm and were of the form:



(a) Simulation 1



(b) Simulation 2

Figure 4: Model Simulation

$$\frac{dx}{dt} = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

Where x is the state-space of the system, y(the output) is in $\mathbb{R}^4$ containing the position of the player and the ball. The input is the function u and e is noise in the input as well as from the measurements of the output.

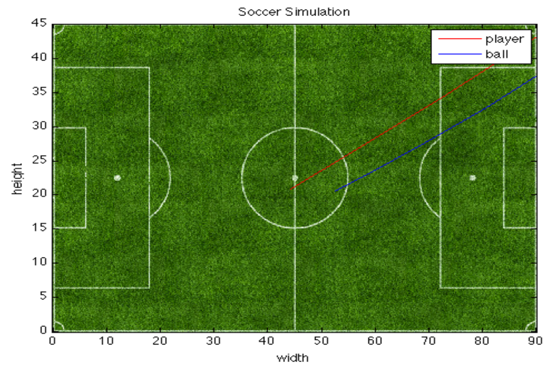
3.1 Simulation, Analysis and Comparison

In here we plot some simulated scenarios for both models, figures 4 and 5 show the simulations for our model and the identified model respectively.

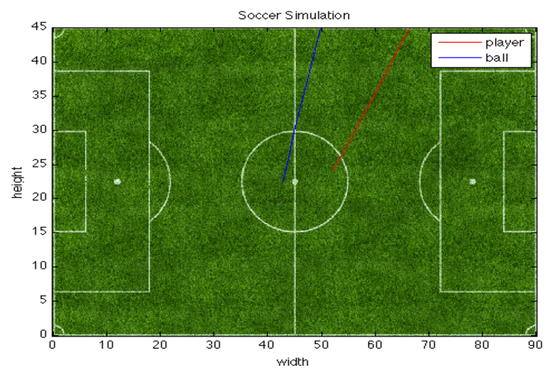
In figure 4a the initial state is **attack** and the player and ball position are near the center. We can see the player player controlling the ball until he loses it and the opponent tries to score a goal by driving the ball near the player post as a defensive player aims to intercept the ball in order to steal it.

Figure 4b, again with the player and ball initially near the center and in state *defense* we can observe an uninterrupted movement of the player towards the opponent post before losing the offensive stance after reaching the off-court line. The opponent team then tries to reach the player post and the defensive player successfully intercepts him near the defensive area. The reader may see that in both simulations, the stochastic factor of the player movement is present in the plotted displacements.

Figures 5a and 5b plot the identified model in **attack** and **defense** state respectively. While in state **attack** the player is clearly trying to reach the opponent post along with the ball, in **defense** state there is no clear intention of the player to intercept and try to steal the ball from the opponent and even their trajectories have no interception point. Both simulations show the linear nature of the identified model



(a) Attack State



(b) Defense State

Figure 5: Identified Model Simulation

in the player and ball trajectories.

4 Concluding Remarks

The use of hybrid systems theory to model problems constrained by physical rules offer a consistent and extensible framework for designing, modeling and analyzing real life application. In this article we have devised a hybrid model of a soccer match where two states (**attack** and **defense**) dictate the continuous evolution of the player as well as the ball.

An stochastic hybrid model was used for modelling the slight randomness in the player and opponent displacement, specifically, a wiener process was used. Because the nature of sports is dynamically active in movement, an equilibrium study is more apt to aspects like player orientation.

As the simulations presented here show, our model is accurate enough to predict common soccer scenarios as ball interception, goal scoring, off-court trespassing and opponent chase by the player. The hybrid approach used in the model fits in convenient way the role-switching nature of a soccer team and enables us to study their behaviour in the same model and the same simulation.

As a future extension to this work, we propose a bigger state-space where continuous evolution allows to keep track of more than one player and opponent, and also define different continuous evolution for each player in order to model defensive, midfielder and striker roles. Also, the discrete states can be expanded to more attack and defense states where each one reflects a certain strategy according to its goal.

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